Capital adjustment cost and bias in income based dynamic panel models with fixed effects

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Abstract

The fixed effects (FE) estimator of "conditional convergence" in income based dynamic panel models could be biased downward when capital adjustment cost is present. Such a capital adjustment cost means a rising marginal cost of investment which could slow down the convergence. The standard FE regression fails to take into account of this capital adjustment cost and thus it could overestimate the rate of convergence. Using a Ramsey model with long-run adjustment cost of capital, we characterize this bias that does not go away even for a longer time dimension. The size of the bias is greater in economies with a higher adjustment cost. The cross-country regression suggests that the size of this bias could be substantial.

Key words: Dynamic panel model, fixed effects, adjustment cost of capital, downward bias


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1. Introduction

In the empirical economic growth literature, a typical specification of an income based dynamic panel econometric model has the following form,

\[
\ln y_{it} = \gamma \ln y_{it-1} + x_{it}'\lambda + \eta_i + u_{it}
\]  

(1)

where \( y_{it} \) is the income of the \( i \)th cross section unit at date \( t \) (\( = 1, 2, 3, ..., T \)), \( x_{it} \) denotes a \( 1 \times J \) vector of control and interest variables; \( \lambda \) is a \( J \times 1 \) parameter vector, \( \eta_i \) is the fixed effects and \( u_{it} \) is the random disturbance term. The parameters \( \gamma \) and \( \lambda \) summarize the list of parameters to be estimated. In the convergence literature, \( \gamma \in (0, 1) \) is the crucial parameter of interest because \( \gamma - 1 \), (after subtracting \( \ln y_{it-1} \) from both of sides of (1)), measures the coefficient of conditional convergence known as \( \beta \)-convergence.\(^1\) Such a concept is used to understand the convergence of countries or regions to their own steady-state (see, for instance, Barro and Sala-i-Martin, 2004, Ch. 11).

The fixed effects (FE) estimator is among the main candidates for estimating eq. (1) when the time horizon \( T \) of the panel is sufficiently large. In particular, dynamic panel models with fixed effects have been widely employed in the literature for studying convergence among group of countries (e.g., Islam, 1995, Caselli et al., 1996 and Ho, 2006). Since the seminal work of Nickell (1981), it is well known that the fixed effects estimator of dynamic panel models is biased when \( T \) is short but

\(^1\) Note that (1) can be rewritten as: \( \triangle \ln y_{it} = -(1 - \gamma) \ln y_{it-1} + x_{it}'\lambda + \eta_i + u_{it} \)

\[ = -(1 - e^{-\beta}) \ln y_{it-1} + x_{it}'\lambda + \eta_i + u_{it}. \] This means that \( \beta = -\ln \gamma \simeq 1 - \gamma \) is a measure of \( \beta \)-convergence. See Barro and Sala-i-Martin (2004, p. 462).
unbiased and consistent when $T \to \infty$.

A sizable econometric literature investigates the nature of this FE bias and possible remedies in dynamic panel data models. For instance, following the work of Nickell (1981), Kiviet (1995), Judson and Owen (1999) and Hahn and Kuersteiner (2002) examine this bias in short and long dynamic panel FE models. Arellano and Bond (1991), Arellano and Bover (1995) and Blundell and Bond (1998) provide a handy way to correct the FE bias in short-time dynamic panel models applying internal instruments.\(^2\)

However, the literature has hardly directed any effort to understand the economic root of such a bias. In this paper, we examine the macroeconomic fundamental behind a FE bias of an income based dynamic panel model similar to (1). We identify a convex capital adjustment cost technology as one such fundamental which means a rising marginal cost of investment. The empirical evidence are abound that the adjustment cost of capital is present (Chirinko, 1993 and Hamermesh and Pfann, 1996). While diminishing returns to capital facilitate the process of convergence, a rising marginal cost of investment schedule slows it down. An FE regression fails to take into account of this capital adjustment cost and thus it overestimates the rate of convergence. We show this formally by establishing that the income based dynamic models has a negative first order moving average error when capital adjustment cost is present. A negative correlation between the lagged dependent variable $\ln y_{it-1}$ and the disturbance terms $u_{it}$ in (1) thus arises. An FE estimator does not take into account of this negative correlation and consequently becomes downward biased.

\(^2\)However, these are also criticized as sensitive to "instrumental proliferation" (Roodman, 2009).
To demonstrate this point, we derive a closed form expression for the FE bias employing a standard Ramsey growth model with a capital adjustment cost technology. A well known parametric form for the adjustment cost is borrowed from Lucas and Prescott (1971) that was subsequently used by Basu (1987), Hercowitz and Sampson (1991) and Basu et al. (2012). Such a capital adjustment cost differs from the investment adjustment cost (e.g. Christiano et al., 2005) in the sense that the adjustment cost persists even in the long-run. This explains why the bias stays even for infinite time horizon ($T \rightarrow \infty$). The size of this bias increases with the degree of the adjustment cost of capital.

Using the panel data for 18 OECD countries for the period 1970 to 2010 we compare the performances of the FE regression (1) with GMM estimation that corrects the bias. The size of the downward bias is estimated to range from 0.75 to 0.84. The implied Tobin’q (a measure of adjustment cost) ranges from 4.35 to 6.66 which is quite substantial.

The bias resulting from capital adjustment cost is not uncommon in the literature. Caballero (1994) establishes that the wealth elasticity estimate generally tends to be downward biased in a small sample when capital adjustment cost is present. However, there is hardly any attempt in the literature to explore the fact that country convergence could be seriously overestimated due to the presence of capital adjustment cost. To the best of our knowledge, our paper is the first attempt in the literature to understand the economic fundamentals behind the bias in the convergence regression in terms of a capital adjustment cost.

In the next section, we develop a Ramsey-type growth model with heterogeneous
countries in terms of initial wealth, tastes and productivity to characterize the adjustment cost bias. Section 3 reports cross country panel estimation. Section 4 concludes.

2. The model

2.1. Preference and technology

Consider a sequence of infinitely-lived heterogenous citizens, \( i = 1, 2, ..., N \) and \( t = 1, 2, ..., \infty \) where \( i \) stands for the country representative\(^3\) and \( t \) stands for time. Countries are heterogenous in terms of (i) initial capital stock \( (k_{i0}) \), (ii) preference (discount factor, \( \rho_i \)) and (iii) productivity shock \( (\xi_{it}) \).\(^4\) We let the preference parameter \( \rho_i \) vary across countries which could give rise to country specific fixed effects. Also, assume that cross country productivity shocks are i.i.d. Households are further assumed to be both consumers and entrepreneurs.\(^5\)

The production function facing the \( i \)th country resident is Cobb-Douglas with constant returns to scale as follows,

\[
q_{it} = \xi_{it} \left( \prod_{j=1}^{J} (g_{ijt})^{x_j} \right) (k_{it})^{\omega} (m_{it})^{\varphi} \tag{2}
\]

\[
\varphi + \omega + \sum_{j=1}^{k} x_j = 1 \tag{3}
\]

In (2), \( k_{it} \) is the country specific capital stock at period \( t \) and \( k_{i0} \) is given. \( g_{ijt} \)

\(^3\)Alternatively, \( i \) could represent a country in the world economy whereas each country is represented with a single representative consumer, as in Acemoglu and Ventura (2002).

\(^4\)Variables without subscript(s) \( t \) but (and) \( i \) represent individual (economy-wide) steady state values.

\(^5\)See Angeletos and Calvet (2006) for a similar type of entrepreneurship.
represents the $j$th input in the production function (e.g., infrastructure or a learning-by-doing knowledge spillover that is specific to the $i$th country condition, in the spirit of Arrow, 1962), which could potentially give rise to technological externality. In addition, $m_{it}$ is a flow imported intermediate input that the country finances by borrowing from the international credit market at a fixed interest rate, $r^*$. The $i$th country agent treats $g_{ijt}$ as given while choosing consumption and investment. The production technology thus exhibits private diminishing returns to reproducible input $k_{it}$, imported intermediate input $m_{it}$ and the exogenous inputs $g_{ijt}$ but aggregate constant returns to scale similar to Romer (1986) and Barro (1990).\footnote{Barro (1990) models the production function at the individual firm level as a function of private and public capital.}

The $i$th country borrows $m_{it}$ at the start of each period and fully pays off the loan with interest rate at the end of each period. The optimal purchase of intermediate input thus satisfies the condition:

$$\frac{\partial q_{it}}{\partial m_{it}} = 1 + r^* \quad (4)$$

This means that

$$m_{it} = \left[\frac{\varphi}{1 + r^*}\right]^{1/(1-\varphi)} \left(\prod_{j=1}^{J} \left(g_{ijt}\right)_{\chi_j/(1-\varphi)}\right) \left(k_{it}\right)_{\omega/(1-\varphi)} \left(\xi_{it}\right)^{1/(1-\varphi)} \quad (5)$$

which upon plugging into (2) and after netting out the loan retirement cost $(1+r^*)m_{it}$ gives the net value added ($y_{it}$),
\[ y_{it} = \epsilon_{it} \left( \prod_{j=1}^{J} (g_{ij})^{\lambda_j} \right) (k_{it})^\alpha \]  \hspace{1cm} (6)

where \( \epsilon_{it} \equiv (1 - \varphi) (\varphi / (1 + r^*))^{\varphi / (1 - \varphi)} (\xi_{it})^{1/(1 - \varphi)} \), \( \lambda_j \equiv \chi_j / (1 - \varphi) \) and \( \alpha \equiv \omega / (1 - \varphi) \).

The \( i \)th country agent maximizes her utility in accordance to the utility function,

\[ E_0 \left[ \sum_{t=0}^{\infty} (\rho_t)^t \ln c_{it} \right] \]  \hspace{1cm} (7)

subject to the budget constraint,

\[ c_{it} + s_{it} = y_{it} \]  \hspace{1cm} (8)

where \( c_{it} \) and \( s_{it} \) represent consumption and saving, respectively.

Following Lucas and Prescott (1971), Basu (1987) and Basu et al. (2012), the investment technology is given by the following specification:

\[ k_{it+1} = k_{it} (1 - \delta + s_{it} / k_{it})^\theta \]  \hspace{1cm} (9)

where \( \delta \in (0, 1) \) and \( \theta \in (0, 1) \) are rate of depreciation and degree of adjustment cost of capital \( (k_{it}) \), respectively. If \( \theta = 0 \), adjustment cost of capital is prohibitively high to change the capital stock. However, if \( \theta = 1 \), adjustment cost of capital is zero and we obtain a standard linear depreciation rule.

Supposing capital depreciates fully each period, we may then rewrite (9) as:

\[ k_{it+1} = k_{it} (s_{it} / k_{it})^\theta \]  \hspace{1cm} (10)
Applying standard methods of undetermined coefficient, the optimal policy functions for the $i$th agent are as follows, (see Appendix A for details in the derivation),\(^7\)

\[
c_{it} = (1 - \psi_i) y_{it} \tag{11a}
\]
\[
s_{it} = \psi_i y_{it} \tag{11b}
\]

where

\[
\psi_i \equiv \theta \alpha \rho_i / (1 - \rho_i (1 - \theta)) \tag{12}
\]

After substituting (6) and (11b) into (10), the optimal dynamic equation of capital stock of the $i$th country resident is given by,

\[
k_{it+1} = (\psi_i)^\theta \left( k_{it} \right)^\gamma \left( \prod_{j=1}^{J} (g_{ijt})^{\lambda_i} \right)^\theta (\epsilon_{it})^\theta \tag{13}
\]

where $\gamma \equiv 1 - (1 - \alpha) \theta$. Therefore, the optimal capital dynamics at the individual country level is a function of the country’s initial capital ($k_{it}$), the idiosyncratic shock ($\epsilon_{it}$), time-dependent country specific exogenous factors ($g_{ijt}$) and a time-independent country specific factor ($\psi_i$).

\(^7\)See also, Basu (1987) and Hercowitz and Sampson (1991) for a similar closed form solution.
2.2. Role of long-run adjustment cost in determining the bias

According to (13), the adjustment cost of capital \( \theta \neq 1 \) impacts not only the dynamics of capital at the individual country level but also the steady-state capital.\(^8\) To see this, set \( \lambda_j = 0 \) for all \( j \) (for simplicity) and verify that the long-run mean and variance of the log of capital stock with respect to the \( i \)th country are given by \( (\theta \ln \psi_i)/(1 - \gamma) \) and \( \nu^2 \theta^2 / (1 - \theta^2) \) respectively,\(^9\) which depend on the adjustment cost parameter \( \theta \). Thus the adjustment cost does not disappear in the long-run.\(^10\)

In the present context, this long-lasting nature of the adjustment cost is particularly reflected on its effect on the idiosyncratic shock. First, this idiosyncratic shock forms the disturbance term in an estimation equation (1). Second, the shock relates to country’s contemporaneous income (6), which appears as a lagged variable in dynamic panel regression models. Therefore, such effects of the idiosyncratic shock will manifest as a bias in the estimate of the lagged income in (1).

Based on the production function (6), the log of income of the \( i \)th country at date \( t \) is given by,

\[
\ln y_{it} = \alpha \ln k_{it} + \sum_{j=1}^{J} (\lambda_j \ln g_{ijt}) + \ln c_{it} \\
= \alpha \ln k_{it} + g_{it} \theta + \ln A_{it} \\
\tag{14}
\]

\(^8\)Note that in the present model, the country specific fixed effect arises solely due to differences in the taste parameter \( \rho_i \). A more general specification can allow for differences in technology which we do not pursue here.

\(^9\)Refer to footnote 10 below for details of the derivation of the variance.

\(^{10}\)Note that the adjustment cost functions of Christiano et al. (2005) do not have such long run effects. See Basu et al. (2012) and Groth and Khan (2010) for a discussion.
where \( \mathbf{g}_{it} \) is a \( 1 \times J \) vector of (exogenous) regressors, \( \mathbf{g}_{it} \equiv (\ln g_{it1}, \ln g_{it2}, \ldots, \ln g_{itJ}) \) and 
\( \lambda \equiv (\lambda_1, \lambda_2, \ldots, \lambda_J)' \) is a \( J \times 1 \).

Finally, applying (6) and (13) to (14), we obtain the following representation for
the dynamic panel model:

\[
\ln y_{it} = \gamma \ln y_{it-1} + \sum_{j=1}^{J} \lambda_j x_{ijt} + \eta_i + u_{it} \tag{15}
\]

where \( \eta_i \equiv \alpha \theta \ln \psi_i \) and,

\[
x_{ijt} \equiv (\theta - 1) \ln g_{ijt-1} + \ln g_{ijt} \tag{16a}
\]

\[
u_{it} = \ln \epsilon_{it} - (1 - \theta) \ln \epsilon_{it-1} \tag{16b}
\]

In vector form,

\[
\ln y_{it} = \gamma \ln y_{it-1} + \mathbf{x}_{it} \lambda + \eta_i + u_{it} \tag{17}
\]

where \( \mathbf{x}_{it} \equiv (x_{it1}, x_{it2}, \ldots, x_{itJ}) \) is a \( 1 \times J \), \( \lambda \equiv (\lambda_1, \lambda_2, \ldots, \lambda_J)' \) as \( x_{ijt} \) is defined in (16a) while \( u_{it} \) is given by (16b).

Eq. (17) is an ARMA (1,1) evolution of the income of the \( i \)th country which
looks observationally equivalent to (1). It represents the true specification for the
estimation model of conditional convergence based on the Ramsey growth model with
a non-zero long-run adjustment cost of capital. Such adjustment cost can be seen as
a permanent tax on capital imposed by mother nature. The long-run impact of such
idiosyncratic shock (13) is responsible for the negative moving average disturbance
term \( (u_{it}) \) in (17).

The income based dynamic panel regression model thus involves an error term which is negatively correlated with the lagged dependent variable \((\ln y_{it-1})\). Therefore, not only the OLS but also the FE estimators of \( \gamma \) are biased and inconsistent, even when \( T \to \infty \).

To see the magnitude of the bias involved in the lagged income term, set \( \lambda_j = 0 \) for all \( j \) to simplify exposition. Then, the following Proposition and Corollary can be stated for the univariate case:

**Proposition 1.** The bias from the FE estimator \( \hat{\gamma} \) of the coefficient \( \gamma \) with respect to (17), when \( \forall j \lambda_j = 0 \), is given by:

\[
\hat{\gamma} = \gamma + (\theta - 1) \frac{\sigma^2}{E_i \text{var}[\ln y_{it-1}]} \tag{18}
\]

where \( \sigma^2 = \text{var}(\epsilon_{it}) \).

**Proof.** See Appendix B. \( \blacksquare \)

**Corollary 1.** The size of the bias is given by

\[
\Phi = \frac{(1 - \gamma^2)(1 - \theta)}{\alpha^2 \theta^2 + (1 - \gamma^2)} \tag{19}
\]

**Proof.** Using (14), when \( \forall j \lambda_j = 0 \), we can rewrite the denominator in (18) as,

\[
E_i \text{var}[\ln y_{it-1}] = E_i \text{var}[\alpha \ln k_{it-1} + \ln \epsilon_{it-1}]
= \alpha^2 E_i \text{var}[\ln k_{it-1}] + \sigma^2 \tag{20}
\]
Then, from (13),

$$E_i \text{var} [\ln k_{it-1}] = E_i \text{var} [\ln \eta_i + \gamma \ln k_{it-2} + \theta \ln \epsilon_{it-2}]$$

$$= \gamma^2 E_i \text{var} [\ln k_{it-2}] + \theta^2 v^2$$

(21)

since $\eta_i$ is fixed over time and $\text{cov}(\ln k_{it}, \ln \epsilon_{it}) = 0$. Next note from (13) that for a generic $i$th country agent, $\lim_{t \to \infty} \text{var} [\ln k_{it-1}] = \text{var} [\ln k_{it-2}] = v^2 \theta^2 / (1 - \gamma^2)$ because $0 < \gamma < 1$. Since $\alpha, v^2$ and $\theta$ are the same for all $i$, all agents converge to the same variance of the capital stock which implies

$$E_i \text{var} [\ln k_{it-1}] = v^2 \theta^2 / (1 - \gamma^2)$$

(22)

Substitute (20) into (18) after substituting (22) into the former to derive the closed form solution for the bias.

Thus in the presence of capital adjustment cost in a growth model, the FE estimator of $\gamma$ in (1) is biased regardless of the time dimension of the panel. This overestimates the conditional convergence. It is straightforward to verify that this bias is greater in economies with a lower value of $\theta$ meaning a higher adjustment cost. The bias is absent if there is no capital adjustment cost ($\theta = 1$).

The downward bias in the FE estimator arises due to a negative contemporaneous

\[\text{cov}(\ln k_{it}, \ln \epsilon_{it})\]
correlation between $\ln y_{it-1}$ and $\ln \epsilon_{it-1}$. The size of this correlation is proportional to the degree of adjustment cost $(1 - \theta)$. To get the (economic) intuition further for such a negative correlation, let the $ith$ country experience a positive TFP shock ($\Delta \ln \epsilon_{it-1}$) at date $t - 1$. The optimal investment rule (11b) dictates that the $ith$ country resident’s contemporaneous investment rises by the same percent because the elasticity of $s_{it-1}$ with respect to $\epsilon_{it-1}$ is unity. Such a blip in investment ($\Delta \ln s_{it-1}$) increases the current capital stock ($\ln k_{it}$) by only $\theta\%$ (see (10)). The remaining $(1 - \theta)\%$ of the investment is lost due to the presence of the long-run capital adjustment cost. This loss enters the error term in (17) with a negative coefficient $-(1 - \theta)\ln \epsilon_{it-1}$). The standard regression equation (1) ignores this negative correlation between $\ln y_{it-1}$ and $\ln \epsilon_{it-1}$. As a result, the estimate of $\gamma$ will be downward biased and, hence, the "conditional convergence" will be upward biased.

In the multivariate case where $\exists j \lambda_j \neq 0$, the FE bias affect the estimators of all variables due to a correlation between lagged output and exogenous technological variables. Proposition 2 below demonstrates this.

**Proposition 2.** The bias from the FE estimators of $\hat{\gamma}$ and $\hat{\lambda}$ of the parameters $\gamma$ and $\lambda$ with respect to (17) are given by, when $\exists j \lambda_j \neq 0$:

$$\hat{\lambda} = \tilde{\lambda} + (E_i E [b_{it} b_{it}'])^{-1} p(\theta - 1) v^2$$

(23)

where $b_{it} \equiv \tilde{x}_{it} - \tilde{x}_i$, $\tilde{\lambda} \equiv (\gamma, \lambda)'$, $p \equiv (1, 0)'$ and $0$ is $1 \times J$ zero vector.

**Proof.** See Appendix C. ■

The last term in (23) is different from zero with a non-zero adjustment cost of capital ($\theta \neq 1$). Therefore, in the multivariate case all the coefficient estimates of the variables in (17) are affected from the bias when the adjustment cost of capital
is present.

3. Cross-country dynamic panel estimation

Our model predicts that a FE estimator overestimates the rate of convergence if in the true model the capital adjustment cost is present while the estimated model ignores it. Is this predication validated by the data? There are two ways to address this issue. First is a structural form approach by testing our parametric adjustment cost model directly against the data. Due to the highly stylized nature of the model and restrictive functional forms for preference and technology, this approach is unlikely to lead to any conclusive evidence.

The second approach that we actually follow here is a reduced form approach. We propose to run the FE and GMM regressions based on the specification (1) and check which estimation methodology yields realistic convergence coefficient. For a sufficiently large time horizon, the bias from FE dynamic panel data model is supposed to be small (Nickell, 1981) and hence the estimates from the two approaches should be close. Is it really the case with the data?

We address this question by using the cross country annual real GDP data for 18 OECD countries spanning a sample period of 1970-2010. The details of the data and list of countries are in the Appendix D. Since our model ARMA is based on a stationary specification of the real GDP, to make the data consistent with the model we filter the real GDP series to pick the business cycle component. To this end, we employ the Christiano-Fitzgerald (2003) filter (CF filtered hereafter) to the log real GDP of each country to identify the business cycle component (call it \( \ln \tilde{y}_{it} \) hereafter)
of the series. In the next step, we run a dynamic panel regression with an AR(1) specification for $\ln \tilde{y}_{it}$ which is consistent with our reduced form equation (17) of the model.

Table 1 reports the results of fixed effects regressions for alternative specifications. The first specification is a simple fixed effects AR(1) version of (1) without adding any control while the remaining three specifications add controls such as lagged CF filtered lagged log of population ($\ln \tilde{Pop}_{t-1}$), lagged CF filtered lagged log of imports ($\ln \tilde{Imp}_{t-1}$) which may have some bearing on the technology as per our production function (6).

What is noteworthy is that in all these three specifications, the lagged GDP is robustly 0.14 and statistically significant even at a 1% level. This suggests a rate of convergence of about 86% while Barro and Sala-i-Martin (2004) report that country $\beta$-convergence is of the order of 2% for annual real GDP. The FE estimation thus predicts an implausibly high rate of country convergence even though the time horizon is sufficiently large to offset any bias from the fixed effects.

The issue arises whether this allegedly fast rate of convergence in FE regression basically is an artifact of a classic omitted variable bias or it supports our hypothesis of an adjustment cost bias. Since the convergence coefficient is remarkably insensitive to the inclusion of additional explanatory variables in the regression, we do not ascribe this bias to the omitted variable specification.\footnote{We have also added controls such as investment-GDP ratio (not reported here for brevity) and the convergence coefficient does not change much. Moreover, omitted variables could give rise to either upward or downward bias while in our case the bias is robustly downward.}

Table 2 reports the estimates of convergence for GMM dynamic panel specifi-
Table 1: Dynamic FE Panel Regression of log real GDP

<table>
<thead>
<tr>
<th>Model</th>
<th>const</th>
<th>$\ln y_{it}$</th>
<th>$\ln \text{Pop}_{t-1}$</th>
<th>$\ln (\text{Imp}_{t-1})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FE (1)</td>
<td>$6.88E - 05$</td>
<td>$0.136**$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>FE(2)</td>
<td>$7.1E - 05$</td>
<td>$0.136**$</td>
<td>$0.035$</td>
<td>-</td>
</tr>
<tr>
<td>FE(3)</td>
<td>$6.9E - 05$</td>
<td>$0.145**$</td>
<td>$0.026$</td>
<td>$-0.003$</td>
</tr>
</tbody>
</table>

Note: ** means significant at the 1% level.

Specifications which adjust for the bias using alternative sets of instruments. The GMM estimation follows the Arellano-Bover (1995) procedure of running a regression with orthogonal deviation to adjust for the country fixed effects. The GMM specifications 1 through 3 report the results for best sets of instruments. First two specifications predict that the rate of country convergence is in line with Barro and Sala-i-Martin (2004) estimate for $\beta$-convergence. The third specification which adds lagged population as a regressor has about a 10% rate of convergence. This convergence is higher than Barro and Sala-i-Martin which could be due to the fact that they do not run FE regression.

Table 2: Dynamic GMM Panel Regression of log real GDP

<table>
<thead>
<tr>
<th>Model</th>
<th>Instruments used</th>
<th>const</th>
<th>$\ln y_{it-3}$</th>
<th>$\ln \text{Inv ratio}$</th>
<th>$\ln \text{Imp}_{t-1}$</th>
<th>J Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>GMM (1)</td>
<td>$\ln y_{it-3}$, $\ln (\text{Imp})$</td>
<td>-</td>
<td>$0.982**$</td>
<td>-</td>
<td>-</td>
<td>1.11(0.29)</td>
</tr>
<tr>
<td>GMM (2)</td>
<td>$\ln y_{it-3}$, $\ln (\text{Exp})$</td>
<td>-</td>
<td>$0.952**$</td>
<td>-</td>
<td>-</td>
<td>1.33(0.24)</td>
</tr>
<tr>
<td>GMM (3)</td>
<td>$\ln y_{it-3}$, $\ln (\text{Inv ratio})$, $\ln (\text{Exp})$</td>
<td>-</td>
<td>$0.894**$</td>
<td>$-3.43$</td>
<td>-</td>
<td>1.88(0.17)</td>
</tr>
</tbody>
</table>

Note: Same as Table 1 and numbers in parenthesis in the last column represent prob values. See also Footnote 14 for details about the instruments.

13 A 2SLS weighting matrix is used for covariance calculation. Using alternative weighting matrices do not change the convergence coefficient remarkably.
14 The instruments used are the third lagged log CF filtered GDP, investment:GDP ratio ($\text{Inv ratio}$), log of import ($\ln (\text{Imp})$) and log of export ($\ln (\text{Exp})$) which produce the best J statistics. Except GDP, other instruments are not CF filtered.
3.1. Inferring the adjustment cost from the FE bias

One can infer the extent of capital adjustment cost based on the FE bias. The actual size of the FE bias can be ascertained by subtracting the FE estimate from the GMM estimate. Based on alternative specifications for GMM the size of the bias ranges from 0.75 to 0.84. According to corollary 1, the bias is given by $\Phi$. Using the conventional estimate of 0.36 as the capital share $\omega$ and 14% of import share in GDP ($\varphi$) in the USA based on the OECD facebook, one can compute an estimate for $\alpha (= \omega / (1 - \varphi))$ equal to 0.42. Given the observed range of bias between 0.75 to 0.84, the value of $\theta$ based on (19) ranges from 0.23 to 0.15. This suggests that the steady-state Tobin’s $q$ (a measure of capital adjustment cost) ranges from 4.35 to 6.66.\textsuperscript{15} Although this bias and the implied adjustment cost seem quite steep, it is not necessarily out of line with the reality. Caballero and Engel (2003) point out that even after aggregating the investment of US manufacturing firms, the speed of adjustment to macroeconomic shocks could be overestimated by 400% in linear models.

4. Conclusion

The economic fundamentals that could generate bias in estimation models have rarely received any attention in macroeconometrics literature. Such a bias could arise due to several economic fundamentals. We identify one such fundamental in income based dynamic panel models which is the long-run capital adjustment cost.

\textsuperscript{15}Based on the adjustment cost function (9), the marginal $q$ is given by $\left( \frac{1}{\theta} \right) \left( \frac{k_{i+1}}{k_i} \right)^{(1-\theta)/\theta}$. In a deterministic steady-state, it is $\frac{1}{\theta}$. See Basu et al. (2012) for the derivation of $q$. 
Using a parametric form for such an adjustment cost technology in a standard Ramsey growth model, we have demonstrated that the fixed effects estimator of income based dynamic panel models could be downward biased even for an infinite time horizon when the adjustment cost of capital is present and not properly accounted for. The size of this bias is larger in economies with a higher adjustment cost of capital. The implication of this bias is that the fixed effects estimate of the "conditional convergence" of countries\regions could be seriously overestimated unless the adjustment cost of capital is taken into account. Our cross country dynamic panel regressions suggest that this bias could be of a serious magnitude which could reflect a very steep adjustment cost.

Although our analysis is based on a specific functional form for the capital adjustment cost technology, the key point is that such an adjustment cost has long-run effects on the economy which translates into an FE bias for a sufficiently large time dimension. This particular conclusion is unlikely to be altered in alternative adjustment cost specifications. The future extension of our work would be to take a more general adjustment cost specification which includes short run investment adjustment cost such as Christiano et al., (2005) and explore the implications for small and large sample biases in FE regressions.
Appendix

A. Optimal capital accumulation

The proof mimics Basu (1987). Write the value function for this problem as:

\[
v(k_{it}, \epsilon_{it}, g_{i1t}, \ldots, g_{iJt}) = \max_{k_{it+1}} \left[ \ln \left\{ \epsilon_{it} \left( \prod_{j=1}^{J} (g_{ijt})^{\lambda_j} \right) (k_{it})^\alpha - (k_{it+1}/k_{it})^{1/\theta} k_{it} \right\} \right. \\
+ \rho_i E_t v(k_{it+1}, \epsilon_{it+1}, g_{i1t+1}, \ldots, g_{iJt+1})
\]

where \( E_t \) is the conditional expectation operator.

Conjecture that the value function is loglinear in state variables as follows:

\[
v(k_{it}, \epsilon_{it}) = \pi_0 + \pi_1 \ln k_{it} + \pi_2 \ln \epsilon_{it} + \pi_3 \sum_{j=1}^{J} \lambda_j \ln g_{ijt}
\]

which after plugging into the value function

\[
\pi_0 + \pi_1 \ln k_{it} + \pi_2 \ln \epsilon_{it} + \pi_3 \sum_{j=1}^{J} \lambda_j \ln g_{ijt} = \max_{k_{it+1}} \left[ \ln \left\{ \epsilon_{it} \left( \prod_{j=1}^{J} (g_{ijt})^{\lambda_j} \right) (k_{it})^\alpha - (k_{it+1}/k_{it})^{1/\theta} k_{it} \right\} \right. \\
+ \rho_i E_t \left\{ \pi_0 + \pi_1 \ln k_{it+1} + \pi_2 \ln \epsilon_{it+1} + \pi_3 \sum_{j=1}^{J} \lambda_j \ln g_{ijt+1} \right\}
\]

Differentiating with respect to \( k_{it+1} \) and rearranging terms one gets:

\[
k_{it+1} = \left[ (\pi_1 \rho_i \theta / (1 + \pi_1 \rho_i \theta)) \right]^{\theta} (\epsilon_{it})^\theta \left( \prod_{j=1}^{J} (g_{ijt})^{\lambda_j} \right) (k_{it})^{\alpha \theta + 1 - \theta}
\]

which after plugging into (A.1) and comparing left hand and right side coefficients of \( \ln k_{it} \) uniquely solves:
\[ \pi_1 = \alpha / (1 - \rho_1 (\alpha \theta + 1 - \theta)) \]

which after plugging into (A.2) we get:

\[ k_{it+1} = \left\{ \frac{\alpha \rho_1 \theta}{(1 - \rho_1 (1 - \theta))} \right\} \theta \epsilon_{it} \theta \left( \prod_{j=1}^{J} (g_{ijit})^{\lambda_j} \right)^\theta (k_{it})^{\alpha \theta + 1 - \theta} \quad (A.3) \]

Note that the decision rule for the capital stock depends only on \( \pi_1 \). The remaining coefficients, \( \pi_0, \pi_2 \) and \( \pi_3 \) can also be solved by using the same method of undetermined coefficients and one can check that they are also uniquely determined by \( \pi_1 \).

**B. Proof of proposition 1**

First rewrite (17), when \( \forall j \lambda_j = 0 \), as:

\[ \ln y_{it} = \gamma \ln y_{it-1} + \eta_i + u_{it} \quad (B.4) \]

where \( u_{it} \) is given by (16b). Then, rewrite (B.4) in a deviation (from individual steady-state mean) form as follows to eliminate the unobserved individual heterogeneity \( (\eta_i) \):

\[ a_{it} = \gamma a_{it-1} + v_{it} \quad (B.5) \]
where

\[ a_{it} \equiv \ln y_{it} - \ln y_i \quad \text{and} \quad a_{it-1} \equiv \ln y_{it-1} - \ln y_{i-1} \]  
(B.6)

\[ v_{it} \equiv (\theta - 1) (\ln \epsilon_{it-1} - \ln \epsilon_{i-1}) + (\ln \epsilon_{it} - \ln \epsilon_i) \]  
(B.7)

For any \( z_i \equiv (T-1)^{-1} \sum_{t=1}^{T-1} \ln z_{it} \) and \( \ln z_{i-1} \equiv T^{-1} \sum_{t=1}^{T} \ln z_{it-1} \).

The FE (the within) estimator of \( \gamma \) is the pooled OLS estimator of the model (B.5),

\[ \hat{\gamma} = \frac{\sum_{i=1}^{N} \sum_{t=1}^{T} (a_{it} a_{it-1})}{\sum_{i=1}^{N} \sum_{t=1}^{T} a_{it}^2} = \gamma + \frac{\sum_{i=1}^{N} T^{-1} \sum_{t=1}^{T} (v_{it} a_{it-1})}{\sum_{i=1}^{N} T^{-1} \sum_{t=1}^{T} a_{it}^2} \]  
(B.8)

When \( T \to \infty \), the terms in the right side of (B.8) can be rewritten as,

\[ \hat{\gamma} = \gamma + \sum_{i=1}^{N} \text{E} [v_{it} a_{it-1}] / \sum_{i=1}^{N} \text{E} [a_{it-1}^2] \]  
(B.9)

where \( \text{E}(.) \) stands for the time expectation operator.

Substituting back (B.6) into (B.9), we obtain,

\[ \hat{\gamma} = \gamma + \frac{\sum_{i=1}^{N} \text{E} [(\ln y_{it-1} - \text{E} [\ln y_{it-1}]) ((\theta - 1) \ln \epsilon_{it-1} + \ln \epsilon_{it})]}{\sum_{i=1}^{N} \text{E} [(\ln y_{it-1} - \text{E} [\ln y_{it-1}]) (\ln y_{it-1} - \text{E} [\ln y_{it-1}])]} \]

\[ = \gamma + \sum_{i=1}^{N} \text{cov} ((\ln y_{it-1}, (\theta - 1) \ln \epsilon_{it-1} + \ln \epsilon_{it}) / \sum_{i=1}^{N} \text{var} [\ln y_{it-1}] \]  
(B.10)

Note that from (6), \( \text{cov}(\ln y_{it-1}, \ln \epsilon_{it}) = 0 \).\(^{16}\) Thus, (B.10) becomes

\(^{16}\)This is also refereed as sequential exogeneity (see Wooldridge, 2010, Ch. 10 & 11).
\[ \hat{\gamma} = \gamma + (\theta - 1) \sum_{i=1}^{N} \text{cov}(\ln y_{it-1}, \ln \epsilon_{it-1})/\sum_{i=1}^{N} \text{var}(\ln y_{it-1}) \]  \hspace{1cm} (B.11)

Then, substitute (6) into (B.11) to obtain,

\[ \hat{\gamma} = \gamma + (\theta - 1) \text{cov}(\alpha \ln k_{it-1} + \ln \epsilon_{it-1}, \ln \epsilon_{it-1})/\text{var}(\ln y_{it-1}) \]

\[ = \gamma + (\theta - 1) N^{-1} \sum_{i=1}^{N} \text{var}(\ln \epsilon_{it-1})/N^{-1} \sum_{i=1}^{N} \text{var}(\ln y_{it-1}) \]  \hspace{1cm} (B.12)

since \( k_{it-1} \) is predetermined and, hence, uncorrelated with \( \epsilon_{it-1} \) (see (6)).

Taking \( N \to \infty \), (B.12) can be rewritten as:

\[ \hat{\gamma} = \gamma + (\theta - 1) E_{i} \text{var}(\ln \epsilon_{it-1})/ E_{i} \text{var}(\ln y_{it-1}) \]  \hspace{1cm} (B.13)

where \( E_{i}(.) \) represents the cross sectional expectation. Since \( \text{var}(\ln \epsilon_{it-1}) = \nu^{2} \) is the same for all \( i \), \( E_{i} \text{var}(\ln \epsilon_{it-1}) = \nu^{2} \).

C. The multivariate case

For the case \( \exists j \lambda_{j} \neq 0 \), first rewrite (17) as:

\[ \ln y_{it} = \tilde{x}_{it} \tilde{\lambda} + \eta_{i} + u_{it} \]  \hspace{1cm} (C.14)

where \( \tilde{x}_{it} \equiv (\ln y_{it-1}, x_{it}) \) is a \( 1 \times (J + 1) \) and \( \tilde{\lambda} \equiv (\gamma, \lambda')' \) is a \( (J + 1) \times 1 \) vector of parameters.
Then, transform the equation in (C.14) to eliminate the fixed effects ($\eta_i$):

$$a_{it} = b_{it}\tilde{\lambda} + v_{it} \tag{C.15}$$

where $b_{it} \equiv \tilde{x}_{it} - \bar{x}_i$.

Recall that:

$$a_{it} \equiv \ln y_{it} - \ln y_i \tag{C.16a}$$

$$v_{it} \equiv (\theta - 1) (\ln \epsilon_{it-1} - \ln \epsilon_{i-1}) + (\ln \epsilon_{it} - \ln \epsilon_i) \tag{C.16b}$$

$$\tilde{x}_{it} \equiv (\ln y_{it-1}, x_{it}) \tag{C.16c}$$

$$x_{it} \equiv (x_{i1t}, x_{i2t}, ..., x_{iJt})$$

$$= (\theta - 1) g_{it-1} + g_{it} \tag{C.16d}$$

$$x_{ijt} \equiv (\theta - 1) \ln g_{ijt-1} + \ln g_{ijt} \tag{C.16e}$$

$$g_{it} \equiv (\ln g_{i1t}, \ln g_{i2t}, ..., \ln g_{iJt}) \tag{C.16f}$$

$$\tilde{\lambda} \equiv (\gamma, \lambda_1, \lambda_2, ..., \lambda_J)' \tag{C.16g}$$

$$b_{it} = (\ln y_{it-1} - \ln y_{i-1}, x_{i1t} - x_{i1}, ..., x_{iJt} - x_{iJ})$$

$$\equiv (b_{i0t}, b_{i1t}, ..., b_{iJt}) \tag{C.16h}$$

The FE estimator of $\tilde{\lambda}$ is the pooled OLS estimator of the model (C.15):
\[
\hat{\lambda} = \left( \sum_{i=1}^{N} \sum_{t=1}^{T} (b'_{it} b_{it}) \right)^{-1} \sum_{i=1}^{N} \sum_{t=1}^{T} (b'_{it} a_{it}) \\
= \bar{\lambda} + \left( \sum_{i=1}^{N} T^{-1} \sum_{t=1}^{T} (b'_{it} b_{it}) \right)^{-1} \sum_{i=1}^{N} T^{-1} \sum_{t=1}^{T} (b'_{it} v_{it}) \quad (C.17)
\]

When \( T \to \infty \), the terms in the right hand side of (C.17) can be rewritten as,

\[
\hat{\lambda} = \bar{\lambda} + \left( \sum_{i=1}^{N} E [b'_{it} b_{it}] \right)^{-1} \sum_{i=1}^{N} E [b'_{it} v_{it}] \quad (C.18)
\]

Note that, the variance and covariance matrix is given by,

\[
b'_{it} b_{it} = \begin{bmatrix}
    b_{i0t}^2 & b_{i0t} b_{i1t} & \ldots & b_{i0t} b_{iJt} \\
    b_{i1t} b_{i0t} & b_{i1t}^2 & \ldots & b_{i1t} b_{iJt} \\
    \vdots & \vdots & \ddots & \vdots \\
    b_{iJt} b_{i0t} & b_{iJt} b_{i1t} & \ldots & b_{iJt}^2
\end{bmatrix}
\]

and, considering that \( x_{ijt} \) are exogenous and thus \( E [x_{ijt} v_{it}] = 0 \), we can simplify the last term in (C.18) as,

\[
E [b'_{it} v_{it}] = (\theta - 1) \text{cov} (\ln y_{it-1}, \ln \epsilon_{it-1}) p \quad (C.19)
\]

where \( p \equiv (1, 0)' \) and \( 0 \) is \( 1 \times J \) zero vector.

Substituting (C.19) into (C.18), we obtain:

\[
\hat{\lambda} = \bar{\lambda} + (E_t E [b'_{it} b_{it}])^{-1} p(\theta - 1)v^2 \quad (C.20)
\]

since, from Appendix B, \( \text{cov} (\ln y_{it-1}, \ln \epsilon_{it-1}) = v^2 \).
D. Countries, variables and data source

List of Countries: Countries samples included in this study (arranged according to the panel ID numbers) are Australia, Austria, Belgium, Canada, Denmark, France, Germany, Italy, Japan, Korea, Rep., Mexico, Netherlands, Norway, Portugal, Spain, Sweden, United Kingdom and the United States.


Manipulation and estimation: First the log of GDP was taken and Christiano-Fitzgerald filter of GDP (cfzz) was computed using the time series (panel) routine in STATA. Panel was constructed with time series of above 18 countries. Then, Eviews routines for panel OLS and GMM were used for estimation of parameters. Eviews commands used include GMM(CX=F) CFZ CFZ(-1) @ CFZ(-3) INV_RATIO; CFZ = C(1) + C(2)*CFZ(-1) + [CX=F]; CFZ = C(1) + C(2)*CFZ(-1); GMM CFZ CFZ(-1) LNPOP(-1) @ CFZ(-3) INV_RATIO; GMM CFZ C CFZ(-1) LNPOP(-1) @ CFZ(-3) LNPOP(-3) LIMP. System GMM was run in STATA [xtdpdsys cfz L(0/2).lgpercapy, lags(1) twostep].

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References


