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Abderrahim Taamouti*

Stock market's reaction to money supply: a nonparametric analysis

Abstract: We empirically investigate the link between monetary policy measures and stock market prices. We document the following stylized facts about stock market's reaction to money supply and examine the effect across the entire distribution of stock returns. Using a *nonparametric* Granger causality in *mean* test, we find that money supply has no impact on stock prices, which confirms many of the existing results that were based on *linear mean regression*. By contrast, when a *nonparametric* causality in *distribution* (hereafter *general* Granger causality) test and *quantile* regression based test were used, the effect of money becomes apparent and statistically very significant. Interestingly, money supply affects the left and right tails of stock return distribution but not its center. This might indicate that the monetary policy measure money supply is effective only during recessions and expansions. We have also investigated the extent to which the impact of money supply on stock returns detected by the nonparametric and quantile regression based tests can be attributed to a time-varying conditional variance of stock returns. After controlling for volatility persistence in stock returns, we continue to find evidence for the reaction of conditional distribution of stock market returns to money supply growth rate.

Keywords: Granger causality in quantile; money supply; nonparametric Granger causality in distribution; nonparametric Granger causality in mean; stock prices.

JEL classification: E44; E52; G12.

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1 Introduction

We have seen remarkable efforts to overcome the recent financial crisis. Many of them were based on monetary policy measures. The objectives were to stimulate the economic activity and guarantee the stability of financial markets, which in turn affects economic growth. Thus, understanding the policy transmission mechanism requires understanding the causal links between monetary policy measures and asset prices. The present paper aims to examine the reaction of stock market prices to changes in monetary policy measure *money supply*. Contrary to the previous work, which only focuses on causality in the mean, here we investigate the effect of changes in money supply on the entire conditional *distribution* of stock market returns. Moreover, we consider a *quantile* regression analysis to measure the effect of money supply across quantiles of stock returns, that gives a broader picture of the effect in various scenarios (recessions and expansions).

Given the importance of the above issues for policy makers, several papers have been written to examine the link between stock market prices and money supply. In the literature, there is no academic consensus about the relation between the two variables. In the early 1970s many empirical studies found that the past of money supply affects stock returns; see for example Homa and Jaffee (1971), Hamburger and Kochin (1972). However, these findings were disputed by subsequent research, which argued that the past changes in money have no impact on stock returns, but that there could be a reverse Granger causality from stock returns to changes in money; see Cooper (1974), Pesando (1974), Rozeff (1974), Rogalski and Vinso (1977) among others.

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Rozeff (1974) found that lagged money supply does not predict future stock returns. Rogalski and Vinso (1977) after improving Rozeff's (1974) analysis wrote: "*causality does not appear to go from money supply to stock prices but rather from stock prices to money supply, Rogalski and Vinso (1977, page 1029)*".

Later on, Lastrapes (1998) has estimated the short-run responses of interest rates and equity prices to money supply shocks in the G-7 countries and the Netherlands using a vector autoregression (VAR) model. In his model, money supply shocks were identified by imposing long-run monetary neutrality. He found that money supply shocks have a positive and significant effect on real equity prices for almost all the countries. Chan, Foresi, and Lang (1996) developed and tested a money-based Capital Asset Pricing Model (M-CAPM). Inside money was used as a proxy for consumption. They found that the pricing errors of the M-CAPM are smaller than those of the consumption based CAPM, which means that money affects (improves) the pricing of equity. Thorbecke (1997) using VAR model and several measures of monetary policy (including nonborrowed reserves) showed that monetary policy exerts large effect on ex-ante and ex-post stock returns. He also found that monetary shocks have larger effects on small firms than on large firms which supports the hypothesis that monetary policy matters partly because it affects firms' access to credit. Patelis (1997) examined the part of stock return predictability which can be attributed to monetary policy. Using different monetary variables (including nonborrowed reserves), long-horizon regressions and short-horizon vector autoregressions, he concluded that monetary policy variables are significant predictors of future returns, though they cannot fully account for observed stock return predictability. Finally, we should also mention that the relationship between monetary policy measures and stock market prices can be studied based on a Taylor rule that includes the stock prices; see for example Belke and Klose (2011) and references therein.

The previous empirical studies are almost entirely on Granger causality in *mean*, using *linear* mean regressions. In the latter the dependence is only due to the mean dependence, thus these studies have ignored the dependence described by high-order moments and quantiles. Consequently, the traditional Granger causality in mean tests might overlook a significant relationship between money supply and stock market returns.

In the present paper, we investigate the stock market's reaction to money supply using two different nonparametric tests. The first one allows to test for the Granger non-causality in mean and the second one looks at the general Granger non-causality in distribution. Both tests do not require to specify the model that might link the two variables of interest, and thus they can help to avoid misleading results due to model misspecification. Moreover, the two tests are able to detect both linear and nonlinear causalities. Finally, we examine the impact across different quantiles of stock market returns.

Several papers have looked at the reaction of conditional distribution and quantiles of some economic and financial variables to other variables. For example, Lee and Yang (2006) have examined whether forecasting the conditional quantile of output growth may be improved using money, Chuang, Kuan, and Lin (2009) have investigated the causal relations between stock return and volume based on quantile regressions, and Barnes and Hughes (2002) have tested whether the conditional CAPM holds at other points of the distribution using quantile regressions. However, to our knowledge the present paper is the first to study the reaction of both *conditional distribution* and *quantiles* of stock market returns to *money supply*. We also believe it is the first to use *nonparametric tests* for testing the Granger non-causality in mean and in distribution from money supply to stock returns. Thus, it can be viewed as an extension of the previous work.

To test for the Granger non-causality in mean we use the nonparametric test that have been recently proposed by Nishiyama et al. (2011) [hereafter NHKJ(2011)]. The test statistic is constructed based on moment conditions for causality in mean. It is also a test for omitted variables in time series regression. To apply this test, a Nadaraya-Watson [see Nadaraya (1964) and Watson (1964)] nonparametric estimator for conditional moments is needed. Using weekly data for the period 1990–2009 on Dow Jones Industrial Average stock index and M2 Money Stock, we find that money supply has no impact on stock market prices. This confirms many of the existing results that were based on *linear mean regression*.

The test of the reaction of *conditional distribution* of stock market returns to money supply is also based on a recent *nonparametric* general Granger causality in distribution test statistic proposed by Bouezmarni and Taamouti (2014). This test can detect dependence in low and high-order moments and quantiles. It is based on a comparison of the estimators of conditional distribution functions using an L_2 metric, where the

distribution functions are estimated using the Nadaraya-Watson approach. Using weekly data, we find convincing evidence of a time-lagged impact of money supply on conditional distribution of stock market returns.

The nonparametric Granger causality test discussed in the previous paragraph helps to detect the impact of money supply on the *entire* distribution of stock returns. However, the rejection of *general Granger non-causality in distribution* hypothesis does not inform us about level(s) of return distribution where the causality exists. To overcome this problem, we consider conditional quantile regression-based tests to identify the effect of money supply at each quantile of stock market returns, which gives a broader picture of the effect in various scenarios. Using the same data as before, we provide new evidence on the reaction of stock market prices. Interestingly, we find that money supply affects the left and right tails of the stock return distribution but not its center. This might indicate that the monetary policy measure money supply matters only during the recessions and expansions. Moreover, we find that the causal effects are usually heterogeneous across stock return quantiles.

The fact that *only* lower and upper quantiles of stock market returns are affected by money supply growth rate may be explained by a time-varying conditional variance of returns, while the conditional mean remains constant. This suggests that money supply may simply affect the variance of stock returns. Thus, we have investigated the extent to which the impact of money supply on stock returns detected by the nonparametric and quantile regression based tests can be attributed to an impact of money supply on stock return volatility. After filtering the stock returns series with an exponential generalized ARCH (EGARCH) model to control for volatility persistence, the nonparametric and quantile regression based tests continue to show evidence of statistically significant impact of money supply on stock returns.

For more robustness check and since in the quantile analysis we have assumed linear dependence at each quantile of stock return distribution, we also consider three nonlinear functional forms for money supply growth rate: squared money supply growth rate, third power of money supply growth rate, and the absolute value of money supply growth rate. These functional forms allow for some types of nonlinearities in the response of quantiles of stock returns to money supply. However, our results show that none of these nonlinearities make the impact of money supply significant. This might indicate that the true form of dependence in quantiles is linear, which can help avoid spurious rejection of the null hypothesis of Granger non-causality in quantiles. Of course, doing so, we are not pretending that this is enough for checking for the presence of spurious rejection, and thus other forms of nonlinearity could characterize the impact of money supply on quantiles of stock market returns. Having said that, the benefits of using linear functional form is that the model is easily interpretable and the sign of the impact can be easily identified. Finally, we have also used other indices such as S&P 500 and the results are still largely similar to the previous ones.

As you can understand from the previous paragraphs, the purpose of this study is to document the stylized facts that characterize the impact of money supply on stock returns. The empirical evidence presented here suggests several directions of future research. One direction that we discuss in the last section of this paper is how to investigate the theoretical justification behind the established stylized facts. We point out the possibility of extending the Money-based Capital Asset Pricing Model (M-CAPM) [see Chan, Foresi, and Lang (1996) and Balvers and Huang (2009)], which is a statement about the conditional mean of asset returns, to a quantile-based M-CAPM using the quantile utility functions developed in Manski (1988) and Rostek (2010). Another direction would be to shed light on the channels through which the monetary policy money supply affects stock returns. One possible explanation that holds during the recessions could be inflation. During a recession and to stimulate the performance of the economy, Federal Reserve injects money into circulation by reducing the reserve requirements. This pushes banks to keep less in reserve and lend out more money to consumers and investors. Thereafter, an increase in money supply will cause an increase in inflation. At short-term, the inflation tends to cause stock prices to go down, this is because the effective rate of return from current dividends and earnings must increase for investors to be interested, since part of the return is now “amortized” by inflation.

The paper is organized as follows. In Section 2, we consider a nonparametric approach to test for statistical significance of stock market's response to changes in the monetary policy measure money supply. In Section 3, we examine the quantile Granger non-causality from money supply to stock market returns.

In Section 4, we perform various robustness checks by considering additional control variables, other functional forms and other indices such as S&P 500 Index. Section 5 contains a discussion about how to investigate the theoretical justification behind the stylized facts that we found. We conclude in Section 6. Additional empirical results can be found in Appendix A.

2 Causality in mean versus in distribution: nonparametric approach

We begin our analysis by testing whether stock market returns react to money supply in a broader framework that allows us to leave free the specification of the underlying models. Nonparametric tests are well suited for that. They do not impose any restriction on the model linking the dependent variable to the independent variables.

Most of the existing empirical studies on the stock market prices-money supply relationship focus exclusively on the traditional *linear* Granger causality tests which are based on the conditional mean regression analysis; see Homa and Jaffee (1971); Hamburger and Kochin (1972); Cooper (1974); Rozeff (1974) among others. Since the traditional Granger causality in mean tests only detect linear dependence, these studies have ignored the dependence described by high-order moments and quantiles.

Here we start by testing for the Granger non-causality in mean, and then we look at the general Granger non-causality in distribution. The idea is to first investigate the impact of money supply on the conditional *mean* of stock market returns without assuming any parametric model for mean. This will help to check what have been found in the literature that uses “*linear causality in mean*” tests, namely that money supply has no impact on stock returns. The nonparametric Granger non-causality in mean test that we consider is able to detect *nonlinearities*. Thereafter, we test for the general Granger non-causality in distribution, again, without assuming any parametric model for the conditional distribution of stock returns. This second test will help us to check whether money supply affects other levels (other than the mean) of stock return distribution.

2.1 Nonparametric Granger causality in mean

To test for the Granger non-causality in mean, we use the nonparametric test that has been recently proposed by Nishiyama et al. (2011) [hereafter NHKJ(2011)]. The test statistic is constructed based on moment conditions for causality in mean. To apply the test, the Nadaraya-Watson nonparametric estimator of moments is needed. Before we show how the test works, let us first set some notations. We denote the time- t logarithmic price of the stock market by p_t and the continuously compounded stock return from time $t-1$ to t by $r_t = p_t - p_{t-1}$. We also denote the time- t growth rate of money supply by ms_t , where $ms_t = \log(MS_t) - \log(MS_{t-1})$ and MS_t is the time- t level of money supply. Let $\{(r_t, ms_t)\}_{t=1}^T$ be a sample of T observations on dependent random variables in $\mathbb{R} \times \mathbb{R}$, with joint distribution function F . Suppose now we are interested in testing the Granger non-causality in mean from ms_{t-1} to r_t . This is to test the null hypothesis

$$H_0^m: \Pr\{E[u_t | Z_{t-1}] = 0\} = 1$$

against the alternative hypothesis

$$H_1^m: \Pr\{E[u_t | Z_{t-1}] = 0\} < 1,$$

where $u_t = r_t - E[r_t | r_{t-1}]$ and $Z_{t-1} = (r_{t-1}, ms_{t-1})' \in \mathbb{R}^2$. If the null hypothesis H_0^m is true, then the past changes in money supply cannot affect the conditional mean of stock market return. NHKJ(2011) have showed that the above null and alternative hypotheses can be rewritten in terms of unconditional moment restrictions:

$$H_0^m: \Pr\{E[u_t f(r_{t-1}) h(Z_{t-1})] = 0\} = 1, \text{ for } \forall h(z) \in S_r^\perp \quad (1)$$

against the alternative hypothesis

$$H_1^m: \Pr\{E[u_t f(r_{t-1})h(Z_{t-1})] = 0\} < 1, \text{ for some } h(z) \in s_r^\perp, \quad (2)$$

where $h(z)$ is any function in the Hilbert space $s_r^\perp$ that is orthogonal to the Hilbert L_2 space

$$s_r = \{s(\cdot) | E[s(r_{t-1})^2] < \infty\}.$$

Since $E[u_t f(r_{t-1})h(Z_{t-1})]$ is unknown, we use a nonparametric approach to estimate it. We follow NHKJ(2011) and use the Nadaraya-Watson method to estimate this conditional mean. To test the null hypothesis (1) against the alternative hypothesis (2), we follow NHKJ(2011) to use the test statistic:

$$\hat{S}_T = \sum_{i=1}^{k_T} w_i \hat{a}_i^2, \quad (3)$$

where $\hat{a}_i = \frac{1}{\sqrt{T}} \sum_{t=2}^T \widehat{u}_t f(r_{t-1}) \hat{h}_i(Z_{t-1})$ and w_i is a nonnegative weighting function, such as $w_i = 0.9^i$. To avoid the technicalities and save space, we refer the reader to Nishiyama et al. (2011) for details concerning the nonparametric estimation of $u_t f(r_{t-1})$ and $h_i(Z_{t-1})$ and on how to choose k_T .

The test statistic $\hat{S}_T$ depends obviously on the sample size. NHKJ(2011) have showed that, under the null hypothesis, $\hat{S}_T$ converges in distribution to $\sum_{i=1}^{\infty} w_i \varepsilon_i^2$, as $T \rightarrow \infty$, where ε_i are i.i.d. $N(0,1)$. Thus, for a given summable positive sequence of weights $\{w_i\}$, the test statistic $\hat{S}_T$ is pivotal and it is asymptotically distributed as an infinite sum of weighted chi-squares. To compute the critical values, NHKJ(2011) truncate the infinite sum to $\sum_{i=1}^L w_i \varepsilon_i^2$ and simulate its distribution using the $N(0,1)$ random variables. One advantage of this test is that the simulation is very simple and the critical values do not dependent on the data.

NHKJ(2011) also show that their test has nontrivial power against $\sqrt{T}$ -local alternatives. Furthermore, they argue that the previously proposed tests [see Bierens (1982, 1990), Bierens and Ploberger (1997), Chen and Fan (1999), Fan and Li (1996) and Robinson (1989)] can be rewritten as special cases of their test statistic, and that the latter has an advantage over the previous ones in that it can control the power properties easily and directly. Finally, in their simulation section they use the following weighting function $w_i = 0.9^i$ and they show that their test has quite good empirical size and power for a variety of linear and nonlinear models. They also discuss, in the section on power of the test, how one can choose the sequence of $\{w_i\}$ such that the power is maximized.

2.2 Nonparametric general Granger causality in distribution

Now we test whether the past changes in money supply can affect the *conditional distribution* of stock market returns. The null hypothesis is defined when the distribution of stock returns conditional on its own past and past changes in money supply is equal to the distribution of stock returns conditional on its own past only, almost everywhere. This is similar to test the *conditional independence* between stock returns and past changes in money supply conditionally on the past stock returns. According to Florens and Mouchart (1982) and Florens and Fougère (1996), this is also a test of Granger non-causality in *distribution*, as opposed to the tests of Granger non-causality in *mean*. Working with the conditional distributions will allow us to capture the dependence due to both low and high-order moments and quantiles. Further, Granger causality tests will provide useful information on whether knowledge of past changes in money supply can improve short-run forecast of movements in the stock market returns.

We consider a new nonparametric test statistic proposed recently by Bouezmarni and Taamouti (2014) [hereafter BT(2014)]. The test is based on a comparison of conditional distribution functions using an L_2 metric. Suppose we are interested in testing the Granger non-causality in *distribution* from ms_{t-1} to r_t . This is to test the null hypothesis

$$H_0^D: \Pr\{F(r_t|r_{t-1}, ms_{t-1}) = F(r_t|r_{t-1})\} = 1 \quad (4)$$

against the alternative hypothesis

$$H_1^D: \Pr\{F(r_t|r_{t-1}, ms_{t-1}) = F(r_t|r_{t-1})\} < 1, \quad (5)$$

where $F(r_t|r_{t-1}, ms_{t-1})$ is the conditional distribution function of r_t given r_{t-1} and ms_{t-1} , and $F(r_t|r_{t-1})$ is the conditional distribution function of r_t given only r_{t-1} . If the null hypothesis H_0^D is true, then the past changes in money supply cannot affect the conditional distribution of stock market returns. Since $F(r_t|r_{t-1}, ms_{t-1})$ and $F(r_t|r_{t-1})$ are unknown, we use a nonparametric approach to estimate them. If we denote $\bar{V}_{t-1} = (r_{t-1}, ms_{t-1})' \in \mathbb{R}^2$ and $\bar{v} = (r, ms)'$, then the Nadaraya-Watson estimator of the conditional distribution of r_t given r_{t-1} and ms_{t-1} is defined by

$$\hat{F}_{h_1}(r_t|\bar{v}) = \frac{\sum_{t=2}^T K_{h_1}(\bar{v} - \bar{V}_{t-1}) I_{A_{h_1}}(r_t)}{\sum_{t=2}^T K_{h_1}(\bar{v} - \bar{V}_{t-1})}, \quad (6)$$

where $K_{h_1}(\cdot) = h_1^{-2} K(\cdot/h_1)$, for $K(\cdot)$ a kernel function, $h_1 = h_{1,T}$ is a bandwidth parameter, and $I_{A_{h_1}}(\cdot)$ is an indicator function which is defined on the set $A_{h_1} = [r_t + \infty)$. Similarly, the Nadaraya-Watson estimator of the conditional distribution of r_t given r_{t-1} is defined by:

$$\hat{F}_{h_2}(r_t|r) = \frac{\sum_{t=2}^T K_{h_2}^*(r - r_{t-1}) I_{A_{h_2}}(r_t)}{\sum_{t=2}^T K_{h_2}^*(r - r_{t-1})}, \quad (7)$$

where $K_{h_2}^*(\cdot) = h_2^{-1} K^*(\cdot/h_2)$, for $K^*(\cdot)$ a different kernel function, and $h_2 = h_{2,T}$ is a different bandwidth parameter. Observe that the Nadaraya-Watson estimators of the conditional distribution functions are positive and monotone.

To test the null hypothesis (4) against the alternative hypothesis (5), we follow BT(2014) to use the test statistic:

$$\hat{\Gamma} = \frac{1}{T} \sum_{t=2}^T \{ \hat{F}_{h_1}(r_t|\bar{V}_{t-1}) - \hat{F}_{h_2}(r_t|r_{t-1}) \}^2 w(\bar{V}_{t-1}), \quad (8)$$

where $w(\cdot)$ is a nonnegative weighting function of the data $\bar{V}_{t-1}$, for $2 \leq t \leq T$. The test statistic $\hat{\Gamma}$ is close to zero if conditionally on r_{t-1} , the variables r_t and ms_{t-1} are independent and it diverges in the opposite case. BT(2014) have established the asymptotic distribution of the nonparametric test statistic in (8). They show that the test is asymptotically pivotal under the null hypothesis and follows a normal distribution. Since the distribution of the test statistic is valid only asymptotically, for finite samples they suggest to use a local bootstrap version of the test statistic in (8). The simple resampling from the empirical distribution will not conserve the conditional dependence structure in the data, hence it is important to use the local smoothed bootstrap suggested by Paparoditis and Politis (2000). The latter improves quite a lot the finite sample properties (size and power) of the test.

BT(2014) report the results of a Monte Carlo experiment to illustrate the size and power of their test using reasonable sample sizes. In the simulation study they considered two groups of data generating processes (DGPs) that correspond to linear and nonlinear regression models with different forms of heteroscedasticity. They used four DGPs to evaluate the empirical size and five DGPs to evaluate the power. They also considered two different sample sizes, $T=200$ and $T=300$. For each DGP and for each sample size, they have generated 500 independent realizations and for each realization 500 bootstrapped samples were obtained. Since optimal bandwidths are not available, they have considered the bandwidths $h_1 = c_1 T^{-1/4.75}$ and $h_2 = c_2 T^{-1/4.25}$ for various values of c_1 and c_2 , which corresponds to the most practical. These bandwidths satisfy the assumptions needed to derive the asymptotic distribution of the test statistic. Based on 500 replications, the standard

error of the frequency of rejection in their simulation study is 0.0097 at the nominal level $\alpha=5\%$ and 0.0134 at $\alpha=10\%$. Globally, the size of their test is fairly well controlled even with series of length $T=200$. At 5%, all rejection frequencies are within 2 standard errors. However, at 10%, three rejection frequencies are between 2 and 3 standard errors (two at $T=200$ and one at $T=300$). There is no strong evidence of over rejection or under rejection. Finally, the empirical power of the test performs quite well.

2.3 Data description

Our data consist of weekly observations on Dow Jones Industrial Average stock index and seasonally adjusted M2 money supply from Federal Reserve Bank-St Louis.¹ The sample runs from January 1990 to February 2014 for a total of 1262 observations. The Federal Reserve (Fed) publishes weekly data on M2 money supply. They are reported at 4:30 p.m. every Thursday by the Fed and appear in some Friday newspapers.

We choose money supply among many other macroeconomic variables because it is considered as an important indicator that the Fed uses to set its short term monetary policy. Moreover, financial market participants devote their attention to the weekly money stock announcement. Urich and Wachtel (1981) provided substantial evidence for the impact of *weekly* money supply announcement on interest rates. They found that the financial markets respond very quickly to the weekly money supply announcement, which is interpreted as a policy anticipation effect. Moreover, Roley (1982) examined the fluctuations in interest rates associated with weekly money supply announcements in order to determine the role of the change in the Fed's operating procedures on interest rate volatility. He found that the money supply announcement has caused a significant increase in the volatility of 3-month Treasury bill yield. Finally, we choose *weekly* observations to ensure that the quantile regression analysis will produce very precise estimates of extreme quantiles.

We have calculated the descriptive statistics (not reported) for weekly Dow Jones stock return and money supply, and we found that the unconditional distribution of stock return shows the expected excess kurtosis and negative skewness. The sample kurtosis is greater than the normal distribution value of three. Money supply growth rate also has very high kurtosis with a positive skewness. The p-values of Jarque-Bera test for normality of stock returns (0.0000) and money supply growth rate (0.0000) show that the two variables cannot be normally distributed. The non-normality of stock market returns might indicate that there is no justification for modeling the conditional mean of stock market returns as linear function of money supply. Moreover, the least squares estimation is optimal only when the errors in the regression model are normally distributed. Finally, we have performed Augmented Dickey-Fuller (ADF) tests (not reported) for non stationarity of weekly Dow Jones stock return and money supply growth rate; see Table 1. Using an ADF-test with only an intercept and with both intercept and trend, we find convincing evidence that the two random variables are stationary.

Table 1 Augmented Dickey-Fuller tests.

Variables	With intercept		With intercept and trend	
	Test statistic	5% critical value	Test statistic	5% critical value
Dow Jones stock return	-10.928	-2.866	-6.693	-3.417
Money growth rate	-10.962	-2.866	-7.297	-3.417

Note: This table reports the results of Augmented Dickey-Fuller tests (ADF-test) for testing for the non-stationarity of Dow Jones stock return and Money supply growth rate.

¹ M2 money stock includes a broader set of financial assets held principally by households. It consists of M1 plus: (1) savings deposits (which include money market deposit accounts); (2) small-denomination time deposits (time deposits in amounts of less than \$100,000); and (3) balances in retail money market mutual funds.

2.4 Causality results

We have applied the nonparametric test statistic in (3) to test for the Granger non-causality in mean from money supply growth rate to stock market returns. We followed NHKJ(2011) to choose as a weighting function $w_i=0.9^i$. We have also considered many other weighting functions ($w_i=0.5^i, 0.6^i, 0.7^i, 0.8^i$) and the results are largely the same. For all the weighting functions that we considered, we found that there is a negligible change in the critical values obtained from simulating the distribution of $\sum_{i=1}^L w_i \varepsilon_i^2$ when the truncation L is bigger than 300. We have also followed NHKJ(2011) to choose the bandwidth $bT^{-0.3}$ for various values of b : $b=1, 2.5, 5$, and 7.3 . More details on how the test can be implemented can be found in Nishiyama et al. (2011).

The results for testing the time-lagged Granger non-causality in mean are presented in Table 2. The latter reports the test statistics and the corresponding 5% critical value. From this, we see that the values of the test statistic are between 3.954 and 5.815, and all of them are smaller than the 5% critical value 14.38. Consequently, the time-lagged effect of changes in money supply on conditional mean of stock market returns is statistically insignificant.

Thus, the Granger causality in mean analysis shows that there is *no relationship* between stock market returns and money supply growth rate. This raises the question of whether the dependence between these two variables exists at other levels (other than mean) of the conditional distribution of stock market returns. To answer this question, in what follows we apply the general Granger causality in distribution test statistics described above.

We now test for the Granger non-causality in distribution from money supply growth rate to stock market returns. To do so, we test the null hypothesis (4) against the alternative hypothesis (5) using the nonparametric test statistic in (8). We have considered a Gaussian kernel function (second-order kernel) to estimate the conditional distribution functions that are used to compute the test statistic. We have also chosen the bandwidths $h_1=c_1T^{-1/4.75}$ and $h_2=c_2T^{-1/4.25}$ for various values of c_1 and c_2 that corresponds to the most practical: $(c_1=c_2=2)$, $(c_1=c_2=1.5)$, $(c_1=c_2=1)$, $(c_1=1.2, c_2=0.7)$, $(c_1=1.5, c_2=0.7)$. To generate the bootstrap replications, we have used a Gaussian kernel with a different bandwidth than the previous ones, the one provided by the rule of thumb proposed in Silverman (1986). Finally, since the data are standardized, the weighting function $w(\cdot)$ is given by the indicator function defined on the set $A=\{(r, m), -2 \leq r, m \leq 2\}$.

The results for testing the time-lagged general Granger non-causality in distribution are presented in Table 3. The latter reports the *p-values* computed using the local smoothed bootstrap method. Contrary to the results on Granger non-causality in mean test, at 5% significance level, we find *very convincing* evidence that the *time-lagged* changes in money supply Granger cause the conditional distribution of stock market returns. Thus, money supply clearly affects the conditional distribution of stock market returns.

On the one hand, the above results show that considering only mean regression analysis can lead to wrong conclusions about the existence of a relationship between market returns and money supply. On the other hand, the rejection of general Granger non-causality in distribution hypothesis does not inform us about the level(s) of stock return distribution where the causality(ies) exist(s). To overcome this problem, in the next section we use quantile regression analysis to identify the effect of money supply at each quantile of stock return distribution.

Table 2 Test-statistics for time-lagged Granger non-causality in mean.

Test statistic/ H_0	Time-lagged non-causality
	5% critical value=14.38
NHKJ(2011), $b=1$	5.754
NHKJ(2011), $b=2.5$	5.815
NHKJ(2011), $b=5$	5.345
NHKJ(2011), $b=7.3$	3.954

Note: This table reports the test statistics and the corresponding 5% critical values of time-lagged Granger non-causality in mean (NHKJ(2011)) from money supply to stock market returns.

Table 3 P-values for time-lagged Granger non-causality in distribution.

Test statistic/ H_0	Time-lagged non-causality
BT(2014), $c_1=c_2=2$	0.002
BT(2014), $c_1=c_2=1.5$	0.009
BT(2014), $c_1=2, c_2=1$	0.019
BT(2014), $c_1=1.2, c_2=0.7$	0.027
BT(2014), $c_1=1.5, c_2=0.7$	0.021

Note: This table reports the P-values for testing time-lagged Granger non-causality in distribution (BT(2014)) from money supply growth rate to stock market returns.

3 Quantile analysis

While the big majority of regression models are concerned with examining the conditional mean of a dependent variable, there is an increasing interest in methods of modeling other aspects of the conditional distribution. One important and popular approach, quantile regression, models the quantiles of the dependent variable given a set of conditioning variables. As originally developed by Koenker and Bassett (1978), quantile regression model provides estimates of relationship between a set of covariates and a specified quantile of the dependent variable. It offers a more complete description of the conditional distribution than conditional mean analysis. For example, it can describe how the median, or the 10th or 90th quantiles of the response variable, are affected by regressor variables. Moreover, quantile regression does not require strong distributional assumptions and it is robust compared to mean regression against outliers, and can thus be estimated with greater precision than conventional moments [Harvey and Siddique (2000)].

Hereafter we denote the α th quantile of the conditional distribution of stock market returns by $Q_\alpha(r_t|I_{t-1})$, where I_{t-1} is an information set containing the past values of the independent variables. Observe that the null hypothesis in (4) is equivalent to

$$H_0^Q: Q_\alpha(r_t|r_{t-1}, ms_{t-1}) = Q_\alpha(r_t|r_{t-1}), \forall \alpha \in (0,1), \text{ a.s.} \quad (9)$$

If the null hypothesis in (9) holds for all α in $(0,1)$, then changes in money supply do not Granger cause the conditional distribution of stock returns. Testing the null hypothesis in (9), for all α in $(0,1)$, instead of (4) will help to identify the level(s) of the conditional distribution of stock returns at which the Granger causality from money supply to stock returns exists. The null hypothesis for testing the Granger non-causality at a given α th quantile of stock return is given by:

$$H_0^Q: Q_\alpha(r_t|r_{t-1}, ms_{t-1}) = Q_\alpha(r_t|r_{t-1}), \text{ for a given } \alpha \in (0,1). \quad (10)$$

If (10) holds, then this means that changes in money supply do not Granger cause the α th quantile of stock market return.

Now to examine the impact of time-lagged money supply growth rate on the quantiles of stock market returns, we consider the following linear quantile regression specifications:

$$r_t = \theta(\alpha)' w_{t-1} + \varepsilon_t^{(\alpha)}, \text{ for } \alpha \in (0,1), \quad (11)$$

where $w_{t-1} = (1, ms_{t-1}, r_{t-1})'$, $\theta(\alpha) = (\mu^{(\alpha)}, \beta_{ms}^{(\alpha)}, \beta_r^{(\alpha)})'$ is an unknown vector of parameters associated with the α th quantile, and $\varepsilon_t^{(\alpha)}$ is an unknown error term also associated with the α th quantile and satisfies the unique condition:

$$Q_\alpha(\varepsilon_t^{(\alpha)}|r_{t-1}, ms_{t-1}) = 0, \text{ for } \alpha \in (0,1), \quad (12)$$

that is, the conditional α th quantile of the error term is equal to zero. For the purposes of estimation and inference, the i.i.d. (independently and identically distributed) assumption of the error terms $\varepsilon_t^{(\alpha)}$ is not needed.

The null hypothesis in (10) is a general hypothesis, since it does not specify the functional form (linear or nonlinear) of the conditional quantiles. However, in equation (11) we assume that this functional form is linear, thus we implicitly assume that the dependence at each quantile is linear. In Section 4.2 we consider other nonlinear specifications: squared money supply growth rate, third power of money supply growth rate, and the absolute value of money supply growth rate. As we will see later, the estimation results show that none of these nonlinearities makes the impact of money supply statistically significant. This might indicate that the true form of dependence in quantiles is linear, which can help avoid spurious rejection of the null hypothesis. Of course, doing so, we are not pretending that this is enough for verifying the presence of spurious rejection, and thus other forms of nonlinearity could characterize the impact of money supply on quantiles of stock market returns. Having said that, the benefits of using linear functional form is that the model is easily interpretable and the sign of the impact can be easily identified. Thus, under Assumption (12), the α th conditional quantile of r_t given ms_{t-1} and r_{t-1} can be written as follows:

$$Q_\alpha(r_t | r_{t-1}, ms_{t-1}) = \theta(\alpha)' w_{t-1}, \text{ for } \alpha \in (0, 1).$$

Based on the above equation, the time-lagged money supply growth rate does not Granger cause the α th quantile of stock market return if the null hypothesis $H_{\lim, 0}^{Q_\alpha} : \beta_{ms}^{(\alpha)} = 0$ holds.

Using Koenker and Bassett (1978), the quantile regression estimator of the vector of parameters $\theta(\alpha)$ is the solution to the following minimization problem:

$$\hat{\theta}(\alpha) = \underset{\theta(\alpha)}{\operatorname{argmin}} \left(\sum_{t: r_t > \theta(\alpha)' w_{t-1}} \alpha |r_t - \theta(\alpha)' w_{t-1}| + \sum_{t: r_t < \theta(\alpha)' w_{t-1}} (1-\alpha) |r_t - \theta(\alpha)' w_{t-1}| \right). \quad (13)$$

The estimator $\hat{\theta}(\alpha)$ minimizes a weighted sum of the absolute errors $\varepsilon_t^{(\alpha)}$, where the weights α and $(1-\alpha)$ are symmetric and equal to $\frac{1}{2}$ for the median regression case and asymmetric otherwise. This estimator can be obtained as the solution to a linear programming problem. Several algorithms for obtaining a solution to this problem have been proposed in the literature [see Koenker and D'Orey (1987), Barrodale and Roberts (1974), Koenker and Hallock (2001) and Portnoy and Koenker (1997)]. Moreover, under some regularity conditions, the estimator $\hat{\theta}(\alpha)$ is asymptotically normally distributed [see Koenker (2005)]:

$$\sqrt{T}(\hat{\theta}(\alpha) - \theta(\alpha)) \xrightarrow{d} \mathcal{N}(0, \Sigma_\alpha), \text{ for } T \rightarrow \infty, \quad (14)$$

where “ $\xrightarrow{d}$ ” denotes the convergence in distribution, Σ_α is the covariance matrix of $\hat{\theta}(\alpha)$, and T is the sample size. Thus, tests for statistical significance of parameter estimates can be constructed using critical values from Normal distribution.

The computation of an estimator of covariance matrix Σ_α is important in quantile regression analysis. Generally speaking, we distinguish between three classes of estimators: **(1)** methods for estimating Σ_α in i.i.d. settings; **(2)** methods for estimating Σ_α for independent but not-identical distributed settings; **(3)** bootstrap resampling methods for both i.i.d. and independent and non identically distributed settings [see Koenker (2005)]. The estimator most commonly used and the more efficient in small samples is based on the design matrix bootstrap [see Buchinsky (1995)]. The design matrix bootstrap estimator of Σ_α suggested initially by Efron (1979, 1982) is:

$$\hat{\Sigma}_\alpha^* = \frac{T}{B} \sum_{j=1}^B (\hat{\theta}_j^*(\alpha) - \hat{\theta}(\alpha)) (\hat{\theta}_j^*(\alpha) - \hat{\theta}(\alpha))', \quad (15)$$

where $\hat{\theta}_j^*(\alpha)$ is the quantile regression estimator of $\theta(\alpha)$ based on the j th bootstrap sample, for $j=1, \dots, B$. The bootstrap samples $\{(r_t^*, ms_t^*)'\}_{t=1}^T$ are drawn from the empirical joint distribution of r and ms . The design matrix bootstrap is the most natural form of bootstrap resampling, and is valid in settings where the error

terms $\varepsilon_t^{(\alpha)}$ and regressors $(ms_{t-1}, r_{t-1})'$ are not independent. Buchinsky (1995) examined, via Monte Carlo simulations, six different estimation procedures of the asymptotic covariance matrix Σ_α : design matrix bootstrap; error bootstrapping; order statistic; sigma bootstrap; homoskedastic kernel and heteroskedastic kernel. In his study, Monte Carlo samples are drawn from real data sets and the estimators are evaluated under various realistic scenarios. His results favor the design bootstrap estimation of Σ_α for the general case.

3.1 Empirical results

To investigate the time-lagged effect of money supply growth rate on quantiles of stock market returns, we consider the following quantile regression specifications:

$$r_t = \mu_r^{(\alpha)} + \beta_{ms}^{(\alpha)} ms_{t-1} + \beta_r^{(\alpha)} r_{t-1} + \varepsilon_t^{(\alpha)}, \text{ for } \alpha \in (0, 1), \quad (16)$$

where $Q_\alpha(\varepsilon_t^{(\alpha)} | ms_{t-1}, r_{t-1}) = 0$. The estimation of $\mu_r^{(\alpha)}$, $\beta_{ms}^{(\alpha)}$ and $\beta_r^{(\alpha)}$ and the corresponding tests for their statistical significance are performed using the techniques discussed at the beginning of Section 3. To estimate the covariance matrix Σ_α in (14), we use the design matrix bootstrap estimator defined in equation (15) for $B=5000$ replications.

The estimation results are presented in Table 4. In this table we report the results for the 10%, 50%, and 80% quantiles. There is no particular reason for the choice of these quantiles, except that we want to illustrate the time-lagged effect of money supply on the center and the tails of the conditional distribution of stock market returns. The results for the rest of the quantiles are presented in Figure 1 of Appendix A. Table 4 shows that money supply growth rate does not affect the center of the conditional distribution of stock market returns. The time-lagged impact on the conditional median of stock market returns is positive, but *statistically insignificant*. This effect becomes negative and statistically *very significant* (even at 0.02% significance level) in the lower 10% quantile. Moreover, the changes in money supply have an important positive impact on the upper 80% quantile of stock returns, with a p-value equal to 0.0058 meaning that the effect is statistically significant even at 0.6% significance level.

Furthermore, Figure 1 illustrates clearly that changes in money supply affect the left and right tails of stock market return distribution, but not its center. This might indicate that monetary policy measure money supply is effective only during recessions and expansions. This provides empirical evidence that more can be

Table 4 Time-lagged Granger causality in quantiles.

	Coefficient	t-Statistic	Prob.
10th Quantile			
Const.	-0.0227	-19.9234	0.0000
ms_{t-1}	-2.2825	-3.7368	0.0002
r_{t-1}	0.0720	1.1454	0.2523
$R^2(\%)$	1.7105	—	—
50th Quantile			
Const.	0.0033	5.5309	0.0000
ms_{t-1}	0.1211	0.3367	0.7364
r_{t-1}	-0.0483	-1.0040	0.3156
$R^2(\%)$	0.1098	—	—
80th Quantile			
Const.	0.0157	20.7622	0.0000
ms_{t-1}	0.9895	2.7610	0.0058
r_{t-1}	-0.1439	-4.0857	0.0000
$R^2(\%)$	1.9409	—	—

Note: This table reports the results of time-lagged Granger causality in the 10%, 50%, and 80% quantiles from money supply growth rate to stock market returns. These results correspond to the regression equation in (16).

learnt about stock markets through studying the joint dynamics of stock prices and money supply. Quantile analysis offers new stylized facts about how money supply and aggregate stock prices are intertemporally related.

An important issue that needs to be clarified in the future is to shed light on the channels through which the monetary policy money supply affects stock returns. One possible explanation that holds during the recessions could be inflation. During a recession and to stimulate the performance of the economy, Federal Reserve injects money into circulation by reducing the reserve requirements. This pushes banks to keep less in reserve and lend out more money to consumers and investors. Thereafter, an increase in money supply will cause an increase in inflation. At short-term, the inflation tends to cause stock prices to go down, this is because the effective rate of return from current dividends and earnings must increase for investors to be interested, since part of the return is now “amortized” by inflation.

4 Robustness check

4.1 Additional control variables: stock market volatility

The fact that the conditional median of stock market returns is unaffected by money supply, and the lower and upper quantiles are, may possibly be explained by a time-varying conditional variance of returns, while the conditional mean remains constant. This suggests that money supply may simply affect the variance of stock returns.

To investigate the above observation, in this section we retest for the Granger non-causality in distribution and quantiles after controlling for the conditional variance of stock market returns. A similar approach has been proposed by Hiemstra and Jones (1994) who examined whether the Granger causality from volume to stock returns can be explained by volume serving as a proxy for information flow in the stochastic process generating stock return variance as suggested by Clark's (1973) latent common-factor model. Recently, Bekiros and Diks (2008) and De Gooijer and Sivarajasingham (2008) have also used the idea of controlling for conditional variance in order to reexamine the dynamic relationships between exchange rates and international stock markets, respectively. Bekiros and Diks (2008) found that the Granger causality between exchange rates persist after controlling for variance, whereas De Gooijer and Sivarajasingham (2008) found that the relationships between international stock markets disappear after filtering returns with multivariate GARCH models.

The conditional variance of stock market returns, say $\sigma_{r,t}^2$, is not observable. In what follows we estimate it using two Generalized Autoregressive Conditional Heteroskedasticity (GARCH)-type models. The first one is the symmetric GARCH model introduced by Bollerslev (1986):

$$\begin{aligned} r_t &= \phi_0 + \phi_1 ms_{t-1} + \phi_2 r_{t-1} + e_t, \\ e_t &= \sigma_{r,t-1} \eta_t, \end{aligned}$$

where $\eta_t \sim iid.D(0,1)$ and

$$\sigma_{r,t}^2 = \omega + \alpha_1 e_{t-1}^2 + \beta_1 \sigma_{r,t-1}^2.$$

The second volatility model that we consider is given by the Exponential GARCH (EGARCH) model introduced by Nelson (1991). The specification for the conditional variance is

$$\log(\sigma_{r,t}^2) = \omega + \alpha_1 \left| \frac{e_{t-1}}{\sigma_{r,t-1}} \right| + \beta_1 \log(\sigma_{r,t-1}^2) + \gamma_1 \frac{e_{t-1}}{\sigma_{r,t-1}},$$

where the left-hand side of the above equation is given by the logarithm of the conditional variance. The latter implies that the leverage effect is exponential, rather than linear, and that forecasts of the conditional

variance are guaranteed to be nonnegative. The presence of leverage effects can be tested by the hypothesis that $\gamma_1 < 0$, and the impact is asymmetric if $\gamma_1 \neq 0$.

The estimation of GARCH and EGARCH volatility models has been done using two different distributions for the error term η_t : Normal and Student-t distributions. We have estimated the following four GARCH-type models: **(1)** GARCH model with Normal errors η_t [hereafter GARCH-N]; **(2)** GARCH model with Student-t errors η_t [hereafter GARCH-T]; **(3)** EGARCH model with Normal errors η_t [hereafter EGARCH-N]; and **(4)** EGARCH model with Student-t errors η_t [hereafter EGARCH-T]. Since the distribution of stock market returns is fat-tailed relative to the normal distribution, using Student-t distribution could be more appropriate for the estimation of the conditional market volatility.

The estimation results (not reported, but available from the author upon request) show that the estimate of the intercept ϕ_0 is positive and statistically significant at 1% significance level. However, the coefficients ϕ_1 and ϕ_2 of the time-lagged returns and money supply growth rate are negative and *statistically insignificant* at 5% significance level. Moreover, the estimate of the coefficient β_1 is positive, close to one and statistically very significant, which reveals the presence of the well-known volatility clustering phenomenon. We also find a negative and statistically significant estimate for the coefficient γ_1 . The latter captures the well-known asymmetric volatility phenomenon, and its negative value indicates that the response of stock market volatility to positive and negative return shocks of the same magnitude is asymmetric: negative shocks have more impact on volatility than positive shocks. The estimated numbers of degrees of freedom for Student-t distribution in GARCH and EGARCH models are 7.7 and 11.2, respectively. The latter are statistically significant and indicate the presence of fat tails in the distribution of stock market returns. The comparison of log-likelihood function estimates suggests that the best model for stock market volatility is the **EGARCH** model with Student-t distribution errors. Consequently, our Granger causality analysis will be based on the **EGARCH-T** stock market volatility model.

We now reexamine the time-lagged impact of changes in money supply on stock market returns after controlling for stock market volatility. We first use the *nonparametric* general Granger causality in distribution test. The *p-values* of the test are presented in Table 5. From this, we see that money supply growth rate still has statistically very significant impact on the conditional distribution of stock market returns. The Granger causality in distribution from time-lagged money supply to stock market return *persist* after controlling for the variance of stock returns. Consequently, this rules out the hypothesis that money supply may simply affect the variance of stock market returns.

We now use the following quantile regression specifications to identify the effect at each quantile of stock market returns after controlling for stock market volatility

$$r_t = \mu_r^{(\alpha)} + \beta_{r,ms}^{(\alpha)} ms_{t-1} + \beta_r^{(\alpha)} r_{t-1} + \beta_{r,\hat{\sigma}}^{(\alpha)} \hat{\sigma}_{r,t-1}^2 + \varepsilon_t^{(\alpha)}, \text{ for } \alpha \in (0, 1), \quad (17)$$

where $\hat{\sigma}_{r,t-1}^2$ is the estimated **EGARCH-T** volatility at time $t-1$ and $\varepsilon_t^{(\alpha)}$ is an error term that satisfies $Q_\alpha(\varepsilon_t^{(\alpha)} | ms_{t-1}, r_{t-1}, \hat{\sigma}_{r,t-1}^2) = 0$. The estimation results are presented in Table 6 [see also Figure 2 of Appendix A]. This table reports the estimation results for the 10%, 50%, and 80% quantiles. The results are quite similar to the ones obtained in Section 3.1 where stock market volatility is not taken into account. Thus, after controlling for stock market volatility, we still find that the time-lagged changes in money supply affect the left and right tails of stock market return distribution, but not its center.

Table 5 P-values for time-lagged Granger non-causality in distribution, after controlling for stock market volatility.

Test statistic/ H_0	Time-lagged non-causality
BT(2014), $c=2$	0.025
BT(2014), $c=1.5$	0.018
BT(2014), $c_1=2, c_2=1$	0.005
BT(2014), $c_1=1.2, c_2=0.7$	0.003
BT(2014), $c_1=1.5, c_2=0.7$	0.004

Note: This table reports the P-values for testing time-lagged Granger non-causality in distribution (BT(2014)) from money supply growth rate to stock market returns, after controlling for stock market volatility.

Table 6 Time-lagged Granger causality in quantiles, after controlling for stock market volatility.

	Coefficient	t-Statistic	Prob.
10th Quantile			
Const.	-0.0139	-6.5302	0.0000
ms_{t-1}	-2.2471	-3.6468	0.0003
r_{t-1}	0.0467	0.7156	0.4743
$\hat{\sigma}_{r,t-1}^2$	-18.6565	-4.3271	0.0000
$R^2(\%)$	6.4911	—	—
50th Quantile			
Const.	0.0027	2.0483	0.0407
ms_{t-1}	0.1422	0.4108	0.6813
r_{t-1}	-0.0542	-1.1217	0.2622
$\hat{\sigma}_{r,t-1}^2$	1.5816	0.4212	0.6736
$R^2(\%)$	0.1384	—	—
80th Quantile			
Const.	0.0095	8.1515	0.0000
ms_{t-1}	0.6709	2.0363	0.0419
r_{t-1}	-0.1639	-3.5342	0.0004
$\hat{\sigma}_{r,t-1}^2$	17.1297	5.9377	0.0000
$R^2(\%)$	6.3479	—	—

Note: This table reports the results of time-lagged Granger causality in the 10%, 50%, and 80% quantiles from money supply growth rate to stock market returns after controlling for stock market volatility. These results correspond to the regression equation in (17).

4.2 Other functional forms for money supply

Here, we consider other functional forms for the quantile regression of stock market return on money supply growth rate. After controlling for stock market volatility, we compare the results from the following conditional quantile regressions: **(i)** regression of stock returns on the *squared* time-lagged money supply growth rate and time-lagged stock volatility and stock returns (**Model 1**); **(ii)** regression of stock returns on the *third power* of time-lagged money supply growth rate and time-lagged stock volatility and stock returns (**Model 2**); and **(iii)** regression of stock returns on the *absolute value* of time-lagged money supply growth rate and time-lagged stock volatility and stock returns (**Model 3**). The estimation results for the 10%, 50%, and 80% quantiles of stock market returns are reported in Table 7. From this, we see that the effects of *squared*, *third power*, and *absolute value* of time-lagged money supply growth rate are statistically *insignificant* at all conventional significance levels. This may present some evidence in favor of linearity in quantile regression models. Consequently, we conclude that only level of money supply growth rate matters to explain the lower and upper quantiles of stock market returns.

We also consider additional regressions to investigate the time-lagged Granger causality in quantiles from powers of lagged money supply growth rate simultaneously (ms_{t-1} , ms_{t-1}^2 , ms_{t-1}^3) to stock market returns, after controlling for stock market volatility. In particular, we use the following quantile regressions:

$$r_t = \mu_r^{(\alpha)} + \beta_{r,ms}^{(\alpha)} ms_{t-1} + \beta_{r,ms^2}^{(\alpha)} ms_{t-1}^2 + \beta_{r,ms^3}^{(\alpha)} ms_{t-1}^3 + \beta_r^{(\alpha)} r_{t-1} + \beta_{r,\hat{\sigma}}^{(\alpha)} \hat{\sigma}_{r,t-1}^2 + \varepsilon_t^{(\alpha)}, \text{ for } \alpha \in (0, 1), \quad (18)$$

where $\hat{\sigma}_{r,t-1}^2$ is the estimated EGARCH-T volatility defined in Section 4.1. We next consider the following joint hypothesis:

$$\begin{cases} IH_0: \beta_{r,ms^2}^{(\alpha)} = \beta_{r,ms^3}^{(\alpha)} = 0 \\ H_1: \text{No } H_0, \end{cases} \quad (19)$$

to test whether or not lagged money supply *nonlinearly* cause the quantiles of stock returns.

The results of estimating the quantile regressions (18) and of testing the joint hypothesis (19) for the 10%, 50%, and 80% quantiles of stock market returns are reported in Tables 8 and 9. From these, we confirm that

Table 7 Time-lagged Granger causality in the quantiles, other functional forms.

	Model (1)	Model (2)	Model (3)
10th Quantile			
ms_{t-1}^2	-96.464 (0.398)	—	—
ms_{t-1}^3	—	-4195.93 (0.713)	—
$ ms_{t-1} $	—	—	-2.273 (0.051)
$R^2(\%)$	5.810	5.717	5.494
50th Quantile			
ms_{t-1}^2	60.165 (0.526)	—	—
ms_{t-1}^3	—	1877.37 (0.781)	—
$ ms_{t-1} $	—	—	0.785 (0.180)
$R^2(\%)$	0.362	0.264	0.275
80th Quantile			
ms_{t-1}^2	21.623 (0.747)	—	—
ms_{t-1}^3	—	738.435 (0.906)	—
$ ms_{t-1} $	—	—	0.768 (0.097)
$R^2(\%)$	6.187	6.215	6.204

Note: This table reports the results of time-lagged Granger causality in the 10%, 50%, and 80% quantiles from money supply growth rate to stock market returns, after controlling for stock market volatility. The results correspond to the estimation of the following quantile regressions: **(Model (1))** regression of stock returns on the squared time-lagged money supply growth rate and time-lagged stock volatility and stock returns; **(Model (2))** regression of stock returns on the third power of time-lagged money supply growth rate and time-lagged stock volatility and stock returns; and **(Model (3))** regression of stock returns on the absolute value of time-lagged money supply growth rate and time-lagged stock volatility and stock returns. The p-values are reported between parentheses.

Table 8 Time-lagged Granger causality in quantiles: powers of lagged money supply.

	Coefficient	t-Statistic	Prob.
10th Quantile			
Const.	-0.0143	-7.6147	0.0000
ms_{t-1}	-2.0076	-3.1646	0.0016
ms_{t-1}^2	93.9643	0.5075	0.6119
ms_{t-1}^3	-4501.49	-0.3011	0.7633
r_{t-1}	0.0665	1.0929	0.2746
$\hat{\sigma}_{r,t-1}^2$	-19.040	-4.5938	0.0000
$R^2(\%)$	6.5134	—	—
50th Quantile			
Const.	0.0027	2.1120	0.0349
ms_{t-1}	-0.0290	-0.0582	0.9535
ms_{t-1}^2	103.050	0.8090	0.4186
ms_{t-1}^3	-1324.46	-0.2852	0.7755
r_{t-1}	-0.0480	-1.0479	0.2948
$\hat{\sigma}_{r,t-1}^2$	1.1475	0.4220	0.6730
$R^2(\%)$	0.4223	—	—
80th Quantile			
Const.	0.0095	8.4212	0.0000
ms_{t-1}	0.7282	2.2582	0.0241
ms_{t-1}^2	-86.948	-1.4857	0.1376
ms_{t-1}^3	2749.34	1.3872	0.1656
r_{t-1}	-0.1473	-3.8656	0.0001
$\hat{\sigma}_{r,t-1}^2$	17.675	7.7831	0.0000
$R^2(\%)$	6.3956	—	—

Note: This table reports the results of time-lagged Granger causality in the 10%, 50%, and 80% quantiles from powers of lagged money supply growth rate to stock market returns, after controlling for stock market volatility. These results correspond to the estimation of the quantile regressions in (18).

Table 9 Time-lagged Granger causality in quantiles: F-test.

Quantiles	F-statistic	Prob.
10th Quantile	0.1009	0.9040
50th Quantile	0.4370	0.6460
80th Quantile	0.4307	0.6501

Note: This table reports the results of testing the null hypothesis that the squared money supply growth rate, third power of money supply growth rate (simultaneously) do not cause the 10%, 50%, and 80% quantiles of returns, after controlling for lagged money supply growth rate and stock market volatility. These results correspond to testing the joint hypothesis in (19).

only lagged money supply growth rate causes the 10% and 80% quantiles of returns. In other words, the results show that the nonlinear terms of lagged money supply growth rate have no impact on quantiles of stock returns.

4.3 Other indices: S&P 500 Index

Here, we repeat the above analyses using S&P 500 Index. As for Dow Jones index, the sample of the S&P 500 Index runs from January 1990 to February 2014 for a total of 1262 observations.

We first use the nonparametric Granger non-causality in mean and in distribution tests to examine, without assuming any parametric model for the conditional mean and conditional distribution of stock returns, the time-lagged effect of changes in money supply on the S&P 500 stock returns. The results for non-causality in mean are presented in Table 10. The latter confirms that the money supply growth rate does not affect the conditional mean of stock market returns. The results for non-causality in distribution test are reported in Table 11. From this, we see that the general Granger causality from time-lagged money supply to stock returns *persist* even after changing the stock market index. The impact of time-lagged money supply growth rate on the conditional distribution of the S&P 500 returns is *statistically very significant*.

Table 10 Test-statistics for Granger non-causality in mean, SP 500 Index.

Test statistic/ H_0	Time-lagged non-causality
	5% critical value=14.38
NHK(2011), $b=1$	6.345
NHK(2011), $b=2.5$	5.862
NHK(2011), $b=5$	6.357
NHK(2011), $b=7.3$	9.954

Note: This table reports the test statistics and the corresponding 5% critical values of time-lagged Granger non-causality in mean (NHK(2011)) from money supply to S&P 500 returns.

Table 11 P-values of Granger non-causality in distribution, S P 500 Index

Test statistic/ H_0	Time-lagged non-causality
BT(2014), $c=2$	0.033
BT(2014), $c=1.5$	0.041
BT(2014), $c_1=2; c_2=1$	0.019
BT(2014), $c_1=1.2; c_2=0.7$	0.035
BT(2014), $c_1=1.5; c_2=0.7$	0.037

Note: This table reports the P-values for testing time-lagged Granger non-causality in distribution (BT(2014)) from money supply growth rate to S&P 500 returns.

Finally, Figure 3 of Appendix A reports the results of quantile regression analysis. From this, we are still finding that time-lagged changes in money supply affect the left and right tails of the conditional distribution of stock market returns, but not its center.

5 Discussion

It is true that the main focus of the present paper is to examine *empirically* the relationship between money and stock market price. However, we also think that it is of great importance to investigate the *theoretical* justification behind this relationship. We believe that the empirical evidences that we found should open the door to theoretical exploration in order to formulate the complex economic problems underlying the relationship between money and stock market prices. There are already several studies attempting to provide considerable insight to the understanding of the theoretical relationships between economic fundamentals and asset prices. Here we review some of them and we discuss how this can help to formulate the relationship between money and stock market return.

In Chapter 7 of the recent Handbook of the equity risk premium edited by Rajnish Mehra, Cochrane (2008) surveys work on the intersection between macroeconomics and finance. He begins by recalling the central idea of modern finance that is summarized by the asset pricing Euler equation in which the present asset price p_t is expressed as conditional expectation of the future payoff of the asset x_{t+1} and stochastic discount factor m_{t+1} :

$$p_t = E_t(m_{t+1}x_{t+1}), \quad (20)$$

with m_{t+1} is equal to the growth in the marginal value of wealth. In the Consumption-based Capital Asset Pricing Model (C-CAPM) and under the power utility form, the stochastic discount factor is given by $m_{t+1} = \delta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma}$, where C_{t+1} is the aggregate consumption, γ is the coefficient of relative risk-aversion, and δ is a subjective time-discount factor.

Using the real risk-free rate $r_f = \frac{1}{E(m)}$, we can rewrite equation (20) in terms of excess return r_{t+1}^e

$$E_t(r_{t+1}^e) = -\text{Cov}_t(r_{t+1}^e, m_{t+1}).$$

The above equation indicates that the expected excess return or what is called *risk premium* is higher for assets that have a large negative covariance with the discount factor. Thus, the risk premium is driven by the covariance of returns with the marginal value of wealth. Consequently, the investors would prefer an asset that does well when they are desperate for a little bit of extra wealth [bad times], and that does badly when they do not particularly value extra wealth [good times]. Thus, investors want assets whose payoffs have a positive covariance with bad times, and they will avoid assets with a negative covariance. Hence, for Cochrane (2008) the main challenge in formulating the complex relationship between macroeconomics and asset prices “is to find the right measure of “bad times,” rises in the marginal value of wealth, so that we can understand high average returns or low prices as compensation for assets’ tendency to pay off poorly in “bad times.”” With respect to the contribution of monetary aggregates to the improvement of the equity pricing, Cochrane (2008) wrote “Having said “macroeconomics,” “risk” and “asset prices,” the reader will quickly spot a missing ingredient: money. In macroeconomics, monetary shocks and monetary frictions are considered by many to be an essential ingredient of business cycles; see page 314 of Cochrane (2008).”

From the above discussion we can understand that the objective now is to find a stochastic discount factor that depends on the aggregate money. As pointed out by Chan, Foresi, and Lang (1996), while the relation between asset prices and aggregate consumption has been extensively investigated, less attention has been devoted to the relation between monetary aggregates and asset prices that arises from models

which incorporate a monetary sector. Further, it is often argued that one possible reason for the rejection of the consumption-oriented CAPM (C-CAPM) is the absence of monetary considerations. For all these reasons, Chan, Foresi, and Lang (1996) developed and tested a Money-based Capital Asset Pricing Model (M-CAPM). In their framework, the money, instead of consumption, was used to define a stochastic discount factor m_{t+1} and the asset return r_{t+1} satisfies the stochastic pricing condition that corresponds to the Euler equation:

$$E_t(\delta(r_{M,t+1})^{-\gamma} r_{t+1}) = 1, \quad (21)$$

where $r_{M,t+1}$ is the real inside money growth rate from time t to time $t+1$. Using this money-based asset pricing model, Chan, Foresi, and Lang (1996) found that the pricing errors of the M-CAPM are smaller than those of the consumption based CAPM. Hence, taking into account the ingredient money has improved the equity pricing.

Moreover, recently Balvers and Huang (2009) have extended the capital asset pricing model (CAPM) and the consumption CAPM (C-CAPM) to show that money growth can be an additional factor determining returns. They explored an improvement to the C-CAPM by arguing that the availability of money as a source of liquidity facilitates transactions and affects the marginal value of wealth in generating consumption. In their environment the marginal value of financial returns is determined by the marginal utility of consumption together with the marginal cost of doing transactions. Accordingly, in their model the stochastic discount factor varies with real consumption growth as well as with real money growth.

Obviously, the above Money-based Capital Asset Pricing Model (M-CAPM) is a statement about the *conditional mean* of asset returns, and consequently it does not say anything about how the relationship should behave at other parts (quantiles) of the conditional distribution of asset returns. Roughly speaking, the tests of interquantile variation that we considered in the previous sections can be interpreted as a test of quantile version of the M-CAPM model. But of course we now need to check whether the Euler equation defined by a money-based stochastic discount factor can also be expressed in terms of *conditional quantile* rather than the conditional mean as in equation (20). Hence, the problem now is to find a quantile-based Euler equation that leads to a quantile version of the M-CAPM model in which the conditional quantiles of stock market returns can be expressed as a function of future payoff of stock market and money-based stochastic discount factor. The investigation of the quantile-based Euler equation is the topic of on-going research. Our preliminary theoretical results indicate that obtaining a quantile version of the Euler equation is very possible by using quantile-type utility functions as in Manski (1988) and Rostek (2010).

6 Conclusion

We studied the reaction of stock market prices to past changes in money supply. We first investigated non-parametrically the impact of money supply growth rate on both conditional mean and conditional distribution of stock market returns. We then examined the impact across the conditional quantiles of the conditional distribution of stock market returns.

We have documented the following stylized facts about stock market's reaction to money supply. Using a *nonparametric* Granger causality in *mean* test, we found that money supply has no impact on stock market prices, which confirms many of the existing results that were based on *linear mean regression*. However, when a *nonparametric* general Granger causality in *distribution* test and *quantile* regression based test were used, we found that money supply has a *very significant* effect on stock market prices. Interestingly, money supply affects the left and right tails of stock return distribution but not its center. This might indicate that the monetary policy measure money supply is effective only during recessions and expansions. We have also

investigated the extent to which the impact of money supply on stock returns detected by the nonparametric and quantile regression based tests can be attributed to a time-varying conditional variance of stock returns. After controlling for volatility persistence in stock returns, we continue to find evidence for the reaction of conditional distribution of stock market returns to money supply growth rate.

The empirical evidence presented here suggests several directions of future research. One direction that we discussed in this paper is how to investigate the theoretical justification behind the established stylized facts. We pointed out the possibility of extending the Money-based Capital Asset Pricing Model (M-CAPM) [see Chan, Foresi, and Lang (1996) and Balvers and Huang (2009)], which is a statement about the conditional mean of asset returns, to a quantile-based M-CAPM using the quantile utility functions developed in Manski (1988) and Rostek (2010). Another direction would be to shed light on the channels through which the monetary policy money supply affects stock returns. One possible explanation that holds during the recessions could be inflation. During a recession and to stimulate the performance of the economy, Federal Reserve injects money into circulation by reducing the reserve requirements. This pushes banks to keep less in reserve and lend out more money to consumers and investors. Thereafter, an increase in money supply will cause an increase in inflation. At short-term, the inflation tends to cause stock prices to go down, this is because the effective rate of return from current dividends and earnings must increase for investors to be interested, since part of the return is now “amortized” by inflation.

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Appendix

A. Additional empirical results

Figure 1 Time-lagged impact of money supply growth rate on Quantiles of stock market returns. The results correspond to the regression equation in (16).

Figure 2 Time-lagged impact of money supply growth rate on Quantiles of stock market returns after controlling for the stock market volatility. The results correspond to the regression equation in (17).

Figure 3 Time-lagged impact of money supply growth rate on Quantiles of S&P 500 returns.

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