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NEW CONSTRAINTS ON THE EVOLUTION OF THE STELLAR-TO-DARK MATTER CONNECTION: A COMBINED ANALYSIS OF GALAXY–GALAXY LENSING, CLUSTERING, AND STELLAR MASS FUNCTIONS FROM z = 0.2 TO z = 1

ALEX LEAUAHD1,2,3, JEREMY TINKER4, KEVIN BUNDYS, PETE SR. BEHROOZ6, RICHARD MASSEY7, JASON RHODES8,9, MATTHEW R. GEORGE5, JEAN-PAUL KNEHR, ANDREW BENSON8, RISA H. WECHSLER9, MICHAEL T. BUSHA10,11, PETER CAPAK12, MARINA CORTS2, OLIVIER ILBERT10, ANTON M. KOEKEMEIR13, OLIVER LE FÈVRE10, SIMON LILLY14, HENRY J. McCracken15, MÁRA SALVATO16, TIM SCHRAABACK6,17, NICK SCOVILLE8, TRISTAN SMITH3, AND JAMES E. TAYLOR18

1 Institute for the Physics and Mathematics of the Universe, the University of Tokyo, Chiba 277-8582, Japan
2 Lawrence Berkeley National Lab, 1 Cyclotron Road, Berkeley, CA 94720, USA; alexleauthaud@lbl.gov
3 Berkeley Center for Cosmological Physics, University of California, Berkeley, CA 94720, USA
4 Center for Cosmology and Particle Physics, Department of Physics, New York University, NY, USA
5 Department of Astronomy, University of California, Berkeley 94720, USA
6 Kavli Institute for Particle Astrophysics and Cosmology, Physics Department, Stanford University, and SLAC National Accelerator Laboratory, Stanford, CA 94305, USA
7 Institute for Astronomy, Blackford Hill, Edinburgh EH9 3HJ, UK
8 California Institute of Technology, MC 350-17, 1200 East California Boulevard, Pasadena, CA 91125, USA
9 Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA 91109, USA
10 LAM, CNRS-Univ Aix-Marseille, 38 rue F. Joliot-Curie, 13013 Marseille, France
11 Institute for Theoretical Physics, Department of Physics, University of Zurich, CH-8057, Switzerland
12 Spitzer Science Center, 314-6 Caltech, 1201 E. California Blvd, Pasadena, CA 91125, USA
13 Space Telescope Science Institute, 3700 San Martin Drive, Baltimore, MD 21218, USA
14 Institute of Astronomy, Department of Physics, ETH Zurich, CH-8093, Switzerland
15 Institut d’Astrophysique de Paris, UMR 7095, 98 bis Boulevard Arago, 75014 Paris, France
16 SUPA, Institute for Astronomy, The University of Edinburgh, Royal Observatory, Edinburgh EH9 3HJ, UK
17 Leiden Observatory, Leiden University, Niels Bohrweg 2, NL-2333 CA Leiden, The Netherlands
18 Department of Physics and Astronomy, University of Waterloo, 200 University Avenue West, Waterloo, ON N2L 3G1, Canada

ABSTRACT

Using data from the COSMOS survey, we perform the first joint analysis of galaxy–galaxy weak lensing, galaxy spatial clustering, and galaxy number densities. Carefully accounting for sample variance and for scatter between stellar and halo mass, we model all three observables simultaneously using a novel and self-consistent theoretical framework. Our results provide strong constraints on the shape and redshift evolution of the stellar-to-halo mass relation (SHMR) from z = 0.2 to z = 1. At low stellar mass, we find that halo mass scales as M_h ∝ M^0.46∗ and that this scaling does not evolve significantly with redshift from z = 0.2 to z = 1. The slope of the SHMR rises sharply at M_∗ ≥ 5 × 10^10 M⊙ and as a consequence, the stellar mass of a central galaxy becomes a poor tracer of its parent halo mass. We show that the dark-to-stellar ratio, M_h/M_*, varies from low to high masses, reaching a minimum of M_h/M_∗ ≈ 27 at M_∗ = 4.5 × 10^10 M⊙ and M_h = 1.2 × 10^12 M⊙. This minimum is important for models of galaxy formation because it marks the mass at which the accumulated stellar growth of the central galaxy has been the most efficient. We describe the SHMR at this minimum in terms of the “pivot stellar mass,” M_{piv}∗, the “pivot halo mass,” M_{piv}h, and the “pivot ratio,” (M_{piv}h/M_{piv}∗). Thanks to a homogeneous analysis of a single data set spanning a large redshift range, we report the first detection of mass downsizing trends for both M_{piv}h and M_{piv}∗. The pivot stellar mass decreases from M_{piv}∗ = 5.75 ± 0.13 × 10^10 M⊙ at z = 0.88 to M_{piv}∗ = 3.55 ± 0.17 × 10^10 M⊙ at z = 0.37. Intriguingly, however, the corresponding evolution of M_{piv}h leaves the pivot ratio constant with redshift at (M_{piv}h/M_{piv}∗)_{piv} = 27. We use simple arguments to show how this result raises the possibility that star formation quenching may ultimately depend on M_h/M_∗ and not simply on M_h, as is commonly assumed. We show that simple models with such a dependence naturally lead to downsizing in the sites of star formation. Finally, we discuss the implications of our results in the context of popular quenching models, including disk instabilities and active galactic nucleus feedback.

Key words: dark matter – galaxies: evolution – galaxies: formation – galaxies: luminosity function, mass function – galaxies: stellar content – gravitational lensing: weak

Online-only material: color figures
1. INTRODUCTION

A fundamental goal in observational cosmology is to understand the link between the luminous properties of galaxies and the dark matter halos in which they reside. From an astrophysical perspective, measurements of the relationship between dark matter halo mass ($M_h$) and galaxy observables such as luminosity or stellar mass ($M_\ast$) are critical for understanding how galaxy properties and their evolution with time are shaped by the halos that host them. Growing evidence suggests that halos accumulate stellar mass with an efficiency $\eta$ such as luminosity or stellar mass ($M_\ast$) are critical for understanding how galaxy properties and their evolution with time are shaped by the halos that host them. Growing evidence suggests that halos accumulate stellar mass with an efficiency $\eta$ and peaking at high mass, peaking at $M_h \sim 10^{12} M_\odot$ and declining toward lower and higher masses at $z \sim 0$ (e.g., Mandelbaum et al. 2006b; Conroy & Wechsler 2009; Moster et al. 2010; Behroozi et al. 2010; Guo et al. 2010; More et al. 2010). The stellar mass content is determined by the past merging of smaller sub-components but also by processes that regulate the conversion of gas into stars, including the rate at which fresh material is supplied to the halo, feedback mechanisms from supernovae, galactic winds, and active galactic nuclei (AGNs), and environmental effects such as ram pressure stripping, just to name a few. The global relationship between halo mass and average stellar content—the stellar-to-halo mass relation (SHMR)—probes the integrated outcome of these processes and, as such, provides clues to their physical nature and constrains both semi-analytic models (e.g., Bower et al. 2006; Croton et al. 2006; Somerville et al. 2008; Zehavi et al. 2011a) and hydrodynamical simulations (Kereš et al. 2005, 2009; Crain et al. 2009; Brooks et al. 2009; Gabor et al. 2011; Agertz et al. 2011) that aim to disentangle the relative contributions of such mechanisms.

From a cosmological perspective, the SHMR is vital for determining how galaxies trace dark matter. A complete picture of the manner in which galaxies populate dark matter halos enhances the reconstruction of the dark matter power spectrum from redshift surveys (Sánchez & Cole 2008; Yoo et al. 2009) and leads to improved constraints on cosmological parameters (Yoo et al. 2006; Zheng & Weinberg 2007; Cacciato et al. 2009).

There are currently only two observational techniques capable of directly probing the dark matter halos of galaxies out to large radii (above 50 kpc): galaxy–galaxy lensing (e.g., Brainerd et al. 1996; McKay et al. 2001; Hoekstra et al. 2004; Sheldon et al. 2004, 2009; Mandelbaum et al. 2006a, 2006b; Heymans et al. 2006a; Johnston et al. 2007; Leauthaud et al. 2010) and the kinematics of satellite galaxies (McKay et al. 2002; Prada et al. 2003; Brainerd & Specian 2003; van den Bosch et al. 2004; Conroy et al. 2007; Becker et al. 2007; Norberg et al. 2008; More et al. 2009, 2010). The galaxy–galaxy lensing technique (hereafter "g–g lensing") uses weak gravitational lensing to probe the gravitational potential around foreground ("lens") galaxies. The kinematic method uses satellite galaxies as test particles that trace the local velocity field (and hence the local gravitational potential).

Another popular albeit more indirect method to infer the galaxy–dark matter connection is to measure the statistics of galaxy clustering. The results are commonly interpreted using the Halo Occupation Distribution (HOD) model which describes the probability distribution $P(N|M_h)$ that a halo of mass $M_h$ is host to $N$ galaxies above some threshold in luminosity or stellar mass (e.g., Seljak 2000; Peacock & Smith 2000; Scoccimarro et al. 2001; Berlind & Weinberg 2002; Bullock et al. 2002; Zehavi et al. 2002, 2005, 2011b; Zheng et al. 2005, 2007; Tinker et al. 2007; Skibba & Sheth 2009; Brown et al. 2008; Wake et al. 2011; Foucaud et al. 2010; White et al. 2011). Variations on the HOD approach include the conditional luminosity function $\Phi(L|M_h)dL$ which specifies the average number of galaxies of luminosity $L \pm dL/2$ that reside in a halo of mass $M_h$ (e.g., Yang et al. 2003; van den Bosch et al. 2003, 2007; Vale & Ostriker 2004, 2008; Cooray 2006) and the conditional stellar mass function (SMF) $\Phi(M_\ast|M_h)dM_\ast$ which yields the average number of galaxies with stellar masses in the range $M_\ast \pm dM_\ast/2$ as a function of host halo mass $M_h$ (e.g., Yang et al. 2009; Moster et al. 2010; Behroozi et al. 2010).

Finally, constraints on the SHMR have also been derived from the so-called abundance matching technique, which assumes that there is a monotonic correspondence between halo mass (or circular velocity) and galaxy stellar mass (or luminosity) (e.g., Kravtsov et al. 2004; Vale & Ostriker 2004, 2006; Tasitsiomi et al. 2004; Conroy & Wechsler 2009; Drory et al. 2009; Moster et al. 2010; Behroozi et al. 2010; Guo et al. 2010).

While individual applications of the techniques described above have provided important insight, in Leauthaud et al. (2011b, hereafter Paper I) we take a further step by combining separate probes into a self-consistent theoretical framework. Specifically, our method combines measurements of g–g lensing, galaxy clustering, and the galaxy SMF. Beginning with the standard HOD formalism, we make several modifications that enable us to (1) extract the parameters that determine the SHMR and to (2) simultaneously fit data from multiple probes while allowing for independent binning schemes for each probe. The goal of the current paper is to apply the methodology developed in Paper I to observations from the COSMOS survey from $z = 0.2$ to $z = 1.0$. This enables confident measurements of the shape of the SHMR and its evolution with time, with important implications for models of galaxy formation. In Paper III (Leauthaud et al. 2011b), we will use the HOD constraints from this paper to probe the total stellar content of dark matter halos.

While stellar mass estimates are a key galaxy observable in this work, it is important to highlight the uncertainties and sometimes unknown systematic biases that affect them and, if not treated carefully, can muddy attempts to compare results from disparate surveys (see discussion in Behroozi et al. 2010). One of the advantages of the COSMOS data set is that evolutionary trends can be studied within the sample using self-consistent stellar mass estimates, making our conclusions more robust.

The layout of this paper is as follows. The data are described in Section 2 followed by the presentation of the g–g lensing, clustering, and SMF measurements in Section 3. In the interest of brevity, we only give a short and necessarily incomplete review of the theoretical background in Section 4. We strongly encourage the reader to refer to Paper I for a complete description of the theoretical foundations for this work. Our main results are presented in Section 5. Finally, we discuss the results and draw up our conclusions in Sections 6 and 7.

We assume a ΛCDM cosmology with $\Omega_m = 0.258$, $\Omega_\Lambda = 0.742$, $\Omega_b h^2 = 0.02273$, $n_s = 0.963$, $\sigma_8 = 0.796$, $H_0 = 72$ km s$^{-1}$ Mpc$^{-1}$ (Hinshaw et al. 2009). All distances are expressed in physical Mpc units. The letter $M_h$ denotes halo mass in general whereas $M_{200h}$ is explicitly defined as $M_{200h} \equiv M(< r_{200h}) = 200 \pi \bar{\rho}_{200h} r_{200h}^3$, where $r_{200h}$ is the radius at which the mean interior density is equal to 200 times the mean matter density ($\bar{\rho}$). Stellar mass is denoted $M_\ast$, and has been derived using a Chabrier initial mass function (IMF).
Stellar mass scales as $1/H_0^2$. Halo mass scales as $1/H_0$. All magnitudes are given on the AB system.

2. DESCRIPTION OF DATA

The COSMOS survey (Scoville et al. 2007) brings together a broad array of panchromatic observations with imaging data from X-ray to radio wavelengths and a large spectroscopic follow-up program (zCOSMOS) with the Very Large Telescope (VLT; Lilly et al. 2007). In particular, the COSMOS program has imaged the largest contiguous area (1.64 deg$^2$) to date with the Hubble Space Telescope (HST) using the Advanced Camera for Surveys (ACS) Wide Field Channel (WFC; Koekemoer et al. 2007).

2.1. The ACS Lensing Catalog

The general methodology for the construction of the COSMOS ACS weak lensing catalog and our shape measurement procedure are presented in Leauthaud et al. (2007) and Rhodes et al. (2007). In this section, we present several updates to the pipeline that we have implemented since those publications. In Leauthaud et al. (2007), we used a parametric correction for the effects of charge transfer inefficiency (CTI) on galaxy shape measurements. Instead, in this paper, we use a physically motivated CTI correction scheme that operates on the raw data and returns electrons to pixels from which they were unintentionally dragged out during readout. This correction scheme has been shown by Massey et al. (2010) to reduce the CTI trails by a factor of $\sim 10$ everywhere in the CCD and at all flux levels.

Following CTI correction in the raw images, image registration, geometric distortion, sky-subtraction, cosmic ray rejection, and the final combination of the dithered images are performed by the MultiDrizzle algorithm (Koekemoer et al. 2002). As described in Rhodes et al. (2007), a finer pixel scale of 0.03 pixel$^{-1}$ was used for the final co-added images. The lensing source catalog is constructed from 575 ACS/WFC tiles. Defects and diffraction spikes are carefully removed, leaving a total of $1.2 \times 10^6$ objects to a limiting magnitude of $I_{P1640W} = 26.5$.

The next step is to measure the shapes of galaxies and to correct them for the distortion induced by the time varying ACS point-spread function (PSF; see Rhodes et al. 2007). We continue to use a PSF model based on physical parameters rather than arbitrary principal components. In Leauthaud et al. (2007), we modeled the PSF as a multivariate polynomial in x, y, and focus. We now fit the PSF as a function of x, y, focus, and velocity aberration of the pointing (recorded in CALACS headers as “VAFACtor”). Schrabback et al. (2010) found that VAFactor partially correlates with the higher-order PSF variations, motivating our use of this quantity. In the $g$–$g$ lensing analysis presented here, the weak lensing shear is azimuthally averaged. Thus, any effects of PSF anisotropy cancel to leading order. Therefore, our science analysis is insensitive to subtle differences in the PSF modeling. We confirmed this by repeating our analysis with the independently obtained weak lensing catalog by Schrabback et al. (2010), which yields fully consistent results.

Finally, simulated images are used to derive the shear susceptibility factors that are necessary in order to transform shape measurements into unbiased shear estimators (Leauthaud et al. 2007). Representing a number density of 66 galaxies per arcminute$^2$, the final COSMOS weak lensing catalog contains $3.9 \times 10^5$ galaxies with accurate shape measurements.

2.2. Photometric and Spectroscopic Redshifts

We use two updated versions (v1.8 dated from 2010 July 13 and v1.7 dated from 2009 August 1) of the photometric redshifts (hereafter photo-$z$’s) presented in Ilbert et al. (2009) which have been computed with over 30 bands of multi-wavelength data. In particular, deep $K_s$, $J$, and $u^*$ band data allow for a good photo-$z$ estimate at $z > 1$ via the 4000 Å break which is increasingly shifted into the near-infrared (IR). Further details regarding the data and the photometry can be found in Capak et al. (2007).

The photo-$z$ catalog v1.8 has improved redshifts at $z > 1$ compared to v1.7. At $z < 1$, the difference between the two catalogs is minor. We use catalog v1.8 for $g$–$g$ lensing measurements and v1.7 for the SMF and galaxy clustering. Interchanging the two catalogs does not affect our results.

Photo-$z$’s were estimated using a $\chi^2$ template fitting method (Le Phare; Ilbert et al. 2009) and calibrated with large spectroscopic samples from VLT-VIMOS (Lilly et al. 2007) and Keck-DEIMOS. The dispersion in the photo-$z$’s as measured by comparing to the spectroscopic redshifts is $\sigma_{z_{\text{spec}}/z_{\text{phot}}} = 0.007$ at $i_{\text{AB}} < 22.5$, where $\Delta z = z_{\text{spec}} - z_{\text{phot}}$. The deep IR and IRAC (Infrared Array Camera on the Spitzer Space Telescope) data enable the photo-$z$’s to be calculated even at fainter magnitudes with a reasonable accuracy of $\sigma_{z_{\text{spec}}/z_{\text{phot}}} = 0.06$ at $i_{\text{AB}} \sim 24$ mag.

Figure 1 compares the spectroscopic and photometric redshifts of 8812 galaxies that belong to both the lensing source catalog and the zCOSMOS “bright” or “faint” programs. This figure also illustrates the sensitivity of $g$–$g$ lensing signals to photometric redshift errors. As can be seen from Figure 1, $g$–$g$ lensing signals are increasingly insensitive to photometric redshift errors for source galaxies at higher redshifts.

2.3. Stellar Mass Estimates

Stellar masses are estimated using the Bayesian code described in Bundy et al. (2006) assuming a Chabrier IMF. Briefly, an observed galaxy’s spectral energy distribution (SED) and redshift is referenced to a grid of models constructed using the Bruzual & Charlot (2003) synthesis code. The grid includes models that vary in age, star formation history, dust content, and metallicity. The assumed dust model is Charlot & Fall (2000). At each grid point, the probability that the observed SED fits the model is calculated, and the corresponding stellar mass to luminosity distance that results from the photo-$z$ uncertainties is measured by other authors. Since the primary goal of this paper is to study the redshift evolution of the SHMR derived from COSMOS data alone, we do not include these systematic uncertainties in our analysis and refer to Conroy et al. (2009) and Behroozi et al.
that the mass estimates in Bundy et al. (2010) agree with those
background selection (bottom hashed region). Second, photometric errors such
will have no effect on the signal because such objects are not included in the

Effects of photometric redshift errors on galaxy–galaxy lensing

There are three ways in which photometric redshift errors can impact g–g
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emphasize that since the spectroscopic sample does not go as faint as our source
catalog, this figure is necessarily an underestimate of the true redshift errors for
the source catalog—a complete sample of faint galaxies with spectroscopic
redshift would be necessary in order to fully test the photometric redshifts.

Figure 1. Effects of photometric redshift errors on galaxy–galaxy lensing
signals. This figure illustrates the quality of the photometric redshifts for
source galaxies in this paper by comparing them to a combined sample of
8812 spectroscopic redshifts from the zCOSMOS “bright” and “faint” programs
for which we have applied the same selection as for source galaxies. We
emphasize that since the spectroscopic sample does not go as faint as our source
catalog, this figure is necessarily an underestimate of the true redshift errors for
the source catalog—a complete sample of faint galaxies with spectroscopic
redshift would be necessary in order to fully test the photometric redshifts.
There are three ways in which photometric redshift errors can impact g–g
lensing signals. First, any type of photometric error such that \( z_{\text{phot}} < z_{\text{lens}} \)
will have no effect on the signal because such objects are not included in the
background selection (bottom hashed region). Second, photometric errors such
that \( z_{\text{phot}} > z_{\text{lens}} \) and \( z_{\text{spec}} < z_{\text{lens}} \) will lead to a signal dilution (left hashed
region). Finally, photometric errors such that \( z_{\text{phot}} > z_{\text{lens}} \) and \( z_{\text{spec}} > z_{\text{lens}} \)
but \( z_{\text{phot}} \neq z_{\text{spec}} \) will lead to a bias in \( \Delta z \) (the surface mass density contrast)
because \( \Sigma_{\text{eff}} \) will be misestimated when transforming \( \gamma \) (gravitational shear)
into \( \Delta z \). The dotted and dashed lines indicate where photo-z errors lead to a
10% and a 20% error on \( \Delta z \) for a lens located at \( z = 0.2 \). As can be seen, g–g
lensing signals are increasingly insensitive to photometric redshift errors for
source galaxies at higher redshifts.

(A color version of this figure is available in the online journal.)

(2010) for a broad discussion of systematic errors in stellar mass
estimates.

Following the approach in Bundy et al. (2010), we obtained
PSF-matched 3.0 diameter aperture photometry from the
ground-based COSMOS catalogs (filters \( u^\ast, B_J, V_J, g^\ast, r^\ast, i^\ast, z^\ast, K_s \)) described in Capak et al. (2007), Ilbert et al. (2009), and
McCracken et al. (2010), after applying the photometric zero-
point offsets tabulated in Capak et al. (2007). The depth in all
bands reaches at least 25th magnitude (AB) with the \( K_s \)-band
limited to \( K_s < 24 \) mag. Unlike Drory et al. (2009), we require
\( K_s \)-band detections for all galaxies in the sample. We have found
that the mass estimates in Bundy et al. (2010) agree with those
of Drory et al. (2009) within the expected uncertainties (i.e.,
<0.2 dex). The mass estimates used in this work are slightly
different from those in Bundy et al. (2010) in that they are
based on updated redshift information (v1.7 of the photo-z cat-
alog and the latest available spectroscopic redshifts as compiled
by the COSMOS team) and use a slightly different cosmology
(\( H_0 = 72 \) km s\(^{-1}\) Mpc\(^{-1}\) instead of \( H_0 = 70 \) km s\(^{-1}\) Mpc\(^{-1}\)).

The bins in our analysis are defined by using two estimates of
the mass completeness of the sample, as determined by the
magnitude limits \( K_s < 24 \) mag and \( I_{814W} < 25 \) mag. The
first estimate is more conservative and comes from estimating

the observed magnitude of a maximal \( M_*/L \) stellar population
model with solar metallicity, no dust, and a \( \tau = 0.5 \) Gyr burst
of star formation that occurred at \( z_{\text{form}} = 5 \). As a function of
redshift, the stellar mass of such a population at the point where
its observed \( K_s \) - and \( I_{814W} \)-band flux falls below the magnitude
limits defines the mass completeness and roughly matches the
80% completeness limits determined when deeper samples are
available (Bundy et al. 2006). This redshift-dependent limit is
plotted as the dashed line in Figure 2. In practice, low-mass
galaxies exhibit more star formation and therefore have lower
\( M_*/L \) ratios than the passive template described above so we
also define a second, more liberal mass limit (i.e., lower) that
Corresponds to a star-forming population. This is plotted as the
dotted line. The majority of the sample bins lie above the more
conservative limit, but the lowest mass bins are allowed to reach
the star-forming mass limit under the assumption that passive
stellar populations (potentially missed) at such low masses are
extremely rare.

3. MEASUREMENTS

3.1. Sample Selection

We use two COSMOS galaxy catalogs: the Subaru (photo-z)
catalog and the ACS (lensing) catalog. The ACS catalog
Corresponds to the COSMOS area that has been imaged with
HST and covers a slightly smaller area than the Subaru catalog.
The ACS coverage of COSMOS is 1.64 deg\(^2\) and the Subaru
coverage of COSMOS is 2.3 deg\(^2\). The ACS catalog is used for
our g–g lensing measurements and for the SMFs. The Subaru
catalog is used to calculate the galaxy clustering.

Galaxies are selected with \( K_s < 24 \) mag to ensure that
stellar masses can be computed for all galaxies in the sample.
Note that for the g–g lensing analysis, these cuts only apply to
the foreground lens sample. In addition to these cuts, we also
reject galaxies that are in masked areas where the photom-
etry is deemed unreliable. For this, we use the union of four
masks in the \( B_J, V_J, i^\ast, \) and \( z^\ast \) (“COSMOS.B.mask,” “COS-
MOS.V.mask,” “COSMOS.ip.mask,” “COSMOS.zp.mask”).
While the redshift interval, \( j \), is defined by \([z_{\text{min}}, z_{\text{max}}]\) and \( z_{\text{K}0_{\text{lim}}}^i \) and \( z_{\text{lim}}^i \) refer to the redshift (see Table 1 for the redshift limits) at which the galaxy would still be detected below the \( K_\star \) and \( I_{814W} \)-band limits. We use the best-fit SED template as determined by the stellar mass estimator to calculate these values, thereby accounting for the \( k \)-corrections necessary to compute \( V_{\text{max}} \) values (no evolutionary correction is applied).

### 3.3. Galaxy Autocorrelation Function

The galaxy clustering samples are defined by a series of stellar mass thresholds rather than bins. For galaxy clustering, threshold samples are the most straightforward to model in the HOD context. At small scales, the signal-to-noise ratio \((S/N)\) in galaxy clustering is due mostly to Poisson noise and at large scales it is subject to sample ("cosmic") variance. The optimal binning scheme for clustering can be different than that for \( g-g \) lensing. For example, massive galaxies produce the strongest shear signals, thus a relatively small sample is required to produce a robust measurement relative to a simple pair-counting statistic like the two-point correlation function whereas the inverse would be true for low-mass galaxies. The threshold samples we employ for clustering measurements are listed in Table 2. Except at the high-mass end where galaxies become scarce, the binning scheme employed for the clustering is a constant 0.5 dex in \( \log_{10}(M_\star) \).

Since we do not need galaxy shape information to measure the angular correlation function \( w(\theta) \), we are not restricted to the ACS coverage of the COSMOS field. We therefore use the COSMOS Subaru catalog to calculate \( w(\theta) \). For each threshold sample we measure \( w(\theta) \) using the well-known Landy & Szalay (1993) estimator of

\[
 w(\theta) = \frac{DD - 2DR + RR}{RR},
\]

where \( DD \) are the number of data–data pairs in a given bin of angle \( \theta \), \( RR \) are the number of random–random pairs, and \( DR \) are the number of data–random pairs. Data and random pairs are normalized by the total number of galaxies and random points, respectively. In all measurements we use \( 10^5 \) random points. The distribution of the random points is taken from a combination of four COSMOS masks \((B_J, V_I, i^*, \text{and } z^*)\), thus mimicking the angular completeness and geometry of the survey.

Because the volumes probed in each sample are small, our clustering measurements are subject to the effect of the integral

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**Table 1**

| Characteristics of Three Redshift Bins |
|-----------------|-----------------|-----------------|
| \( z_{\text{min}} \) | \( z_{\text{max}} \) | \( z \text{ Median} \) | \( \text{ACS Volume} \) \( (10^6 \text{ Mpc}^3) \) | \( \text{Subaru Volume} \) \( (10^6 \text{ Mpc}^3) \) | \( \text{Min } M_\star \) | \( N \text{ Galaxies ACS}^4 \) | \( N \text{ Galaxies Subaru}^4 \) |
| 0.22 | 0.48 | 0.37 | 0.88 | 1.24 | \( 10^{9.7} \) \( M_\odot \) | 14956 | 20426 |
| 0.48 | 0.74 | 0.66 | 2.03 | 2.84 | \( 10^{9.3} \) \( M_\odot \) | 15103 | 20068 |
| 0.74 | 1.0 | 0.88 | 3.13 | 4.39 | \( 10^{9.8} \) \( M_\odot \) | 14387 | 18853 |

**Note.** Numbers are quoted after stars have been removed, the selection \( K_\star < 24 \) as well as the lower limit stellar mass limit has been applied, and objects in masked areas have been removed from the catalog.

**Table 2**

| Thresholds for the Angular Correlation Function |
|-----------------|-----------------|-----------------|
| \( w(\theta) \) | \( w(\theta) \) | \( w(\theta) \) | \( w(\theta) \) | \( w(\theta) \) | \( w(\theta) \) |
| \( w(\theta) \) | \( w(\theta) \) | \( w(\theta) \) | \( w(\theta) \) | \( w(\theta) \) | \( w(\theta) \) |
| 11.1 | 10.8 | 10.3 | 9.8 | 9.3 | None |
| 11.1 | 10.8 | 10.3 | 9.8 | 9.3 | None |
| 11.1 | 10.8 | 10.3 | 9.8 | 9.3 | None |

---
constraint (IC; Groth & Peebles 1977). Due to spatial fluctuations in the number density of galaxies, the mean correlation function measured from an ensemble of samples will be smaller than the correlation function measured from a single contiguous sample of the same volume as the sum of the ensemble sample. This attenuation of \( w(\theta) \) becomes relevant on angular scales significant with respect to the sample size. We estimate the IC correction to our \( w(\theta) \) measurements through the use of mock galaxy distributions described in Paper I and Section 4. In practice, we do not modify the measurements for the IC but rather adjust the theoretical models to account for the finite sample size.

Finally, in order to compute \( w(\theta) \) from our model, we need to know the normalized redshift distribution of the galaxy sample, \( N(z) \) (see Equation (32) in Paper I). For this we use the probability distribution functions of the photometric redshifts to estimate the true \( N(z) \) for our photo-z slices. However, the COSMOS photo-z errors are accurate enough such that in tests we find a minimal effect when using a flat top-hat \( z \)-bin for \( N(z) \).

3.4. Galaxy–Galaxy Lensing: From Galaxy Shapes to \( \Delta \Sigma \)

In the weak gravitational lensing limit, the observed shape \( \epsilon_{\text{obs}} \) of a source galaxy is directly related to the lensing induced shear \( \gamma \) according to

\[
\epsilon_{\text{obs}} = \epsilon_{\text{int}} + \gamma,
\]

where \( \epsilon_{\text{int}} \) is the source galaxy’s intrinsic shape that would be observed in the absence of gravitational lensing. In our notation, \( \epsilon_{\text{int}}, \epsilon_{\text{obs}}, \) and \( \gamma \) are spin-2 tensors. The above relationship indicates that galaxies would be ideal tracers of the distortions caused by gravitational lensing if the intrinsic shape \( \epsilon_{\text{int}} \) of each source galaxy was known a priori. However, lensing measurements exhibit an intrinsic limitation, encoded in the width of the ellipticity distribution of the galaxy population, denoted here as \( \sigma_{\epsilon_{\text{int}}} \), and often referred to as the “intrinsic shape noise.” Because the intrinsic shape noise (of order \( \sigma_{\epsilon_{\text{int}}} \sim 0.27 \); Leauthaud et al. 2007) is significantly larger than \( \gamma \), shears must be estimated by averaging over a large number of source galaxies.

Throughout this paper, the gravitational shear is noted as \( \gamma \) whereas \( \tilde{\gamma} \) represents our estimator of \( \gamma \). The uncertainty in the shear estimator is a combination of unavoidable intrinsic shape noise, \( \sigma_{\epsilon_{\text{int}}} = \sqrt{\langle \epsilon_{\text{int}}^2 \rangle} \), and of shape measurement error, \( \sigma_{\text{mean}} \),

\[
\sigma_{\tilde{\gamma}}^2 = \sigma_{\epsilon_{\text{int}}}^2 + \sigma_{\text{mean}}^2.
\]

We will refer to \( \sigma_{\epsilon_{\text{int}}} \) as “shape noise” whereas \( \sigma_{\epsilon_{\text{int}}} \) will be called the “intrinsic shape noise.” The former includes shape measurement error and will vary according to each specific data set and shape measurement method. Averaged over the whole COSMOS field, the weak lensing distortions represent a negligible perturbation to Equation (6).

The derivation of our shear estimator is presented in Leauthaud et al. (2007). We employ the RRG method (see Rhodes et al. 2000 for further details) for galaxy shape measurements. Briefly, we form \( \tilde{\gamma} \) from the PSF-corrected ellipticity according to

\[
\tilde{\gamma} = C \times \frac{\epsilon_{\text{obs}}}{G},
\]

where the shear susceptibility factor,\(^{19} \) \( G \), is measured from moments of the global distribution of \( \epsilon_{\text{obs}} \) and other, higher-order shape parameters (see Equation (28) in Rhodes et al. 2000). Using a set of simulated images similar to those of Shear TESting Program (STEP; Heymans et al. 2006b; Massey et al. 2007) but tailored exclusively to this data set, we find that, in order to correctly measure the input shear on COSMOS-like images, the RRG method requires an overall calibration factor of \( C = (0.86^{+0.07}_{-0.05})^{-1} \). Shear calibration factors are a generic feature of all shape measurement algorithms. This is because no shear measurement technique is independent of the observing conditions and the underlying galaxy population (Massey et al. 2007; Bernstein 2010; Zhang & Komatsu 2011). Thus, it is a common practice in weak lensing measurements to derive survey-specific calibration factors using simulations designed to mimic both the observing conditions and the surveyed galaxy population.

The shear signal induced by a given foreground mass distribution on a background source galaxy will depend on the transverse proper distance between the lens and the source and on the redshift configuration of the lens–source system. A lens with a projected surface mass density, \( \Sigma(r) \), will create a shear that is proportional to the surface mass density contrast, \( \Delta \Sigma(r) \):

\[
\Delta \Sigma(r) \equiv \bar{\Sigma}(<r) - \bar{\Sigma}(r) = \Sigma_{\text{crit}} \times \gamma_1(r).
\]

Here, \( \bar{\Sigma}(<r) \) is the mean surface density within proper radius \( r \), \( \bar{\Sigma}(r) \) is the azimuthally averaged surface density at radius \( r \) (e.g., Miralda-Escude 1991; Wilson et al. 2001), and \( \gamma_1 \) is the tangentially projected shear. The geometry of the lens–source system intervenes through the critical surface mass density,\(^{20} \) \( \Sigma_{\text{crit}} \), which depends on the angular diameter distances to the lens (\( D_{\text{OL}} \)), to the source (\( D_{\text{OS}} \)), and between the lens and source (\( D_{\text{LS}} \)):

\[
\Sigma_{\text{crit}} = \frac{c^2}{4\pi G_N} \frac{D_{\text{OS}}}{D_{\text{OL}} D_{\text{LS}}},
\]

where \( G_N \) represents Newton’s constant. Hence, if redshift information is available for every lens–source pair, each estimate of \( \gamma_1 \) can be directly converted to an estimate of \( \Delta \Sigma(r) \) which is a more desirable quantity than \( \gamma_1 \) because it depends only on the mass distribution of the lens.

To measure \( \Delta \Sigma(r) \) with high S/N, the lensing signal must be stacked over many foreground lenses and background sources. For every \( i \)-th lens and \( j \)-th source separated by a proper distance \( r_{ij} \), an estimator of the mean excess projected surface mass density (\( \Delta \Sigma_{ij} \)) at \( r_{ij} \) is computed according to

\[
\Delta \Sigma_{ij}(r_{ij}) = \tilde{\gamma}_{t,ij} \times \Sigma_{\text{crit},ij},
\]

where \( \tilde{\gamma}_{t,ij} \) is the tangential shear of the source relative to the lens. The COSMOS photometric redshifts described in Section 2.2 are used to estimate \( \Sigma_{\text{crit},ij} \) for every lens–source pair. In order to optimize the S/N, an inverse variance weighting scheme is employed when \( \Delta \Sigma_{ij} \) is summed over many lens–source pairs. Each lens–source pair is attributed a weight that is equal to the estimated variance of the measurement:

\[
u_{ij} = \frac{1}{(\Sigma_{\text{crit},ij} \times \sigma_{\tilde{\gamma}_{t,ij}})^2},
\]

\(^{19} \) Not to be confused with Newton’s constant which we have noted \( G_N \).

\(^{20} \) Note that some authors define the comoving critical surface mass density which has an extra factor of \((1+z)^{-2}\) with respect to ours due to the use of comoving instead of physical distances.
density is the weighted sum over all lens–source pairs:

$$\Delta \Sigma = \frac{\sum_{j=1}^{N_{\text{source}}} \sum_{i=1}^{N_{\text{lens}}} \chi_{ij} \times \tilde{y}_{ij} \times \chi_{\text{crit},ij}}{\sum_{j=1}^{N_{\text{lens}}} \sum_{i=1}^{N_{\text{source}}} \chi_{ij}}.$$  \hfill (12)

### 3.5. Galaxy–Galaxy Lensing Measurements

We only give a brief outline of the overall methodology used to compute the g–g lensing signals since this has already been presented in detail in Leauthaud et al. (2010). Foreground lens galaxies are divided into three redshift samples and then are further binned by stellar mass (see Figure 2 and Table 3). For each lens sample, $\Delta \Sigma$ is computed according to Equation (12) from 25 kpc (physical distance) to 1.5 Mpc in logarithmically spaced radial bins of 1.8 dex. In Leauthaud et al. (2010), we used a theoretical estimate of the shape measurement error in order to derive the inverse variance for each source galaxy. Instead, in this paper, the dispersion of each shear component is measured directly from the data in bins of S/N and magnitude. The measured shear dispersion is equal to the quadratic sum of the intrinsic shape noise and of the shape measurement error (Equation (6)). Our empirical derivation of the shear dispersion varies from $\sigma_\gamma \sim 0.25$ for bright galaxies with high S/N to $\sigma_\gamma \sim 0.4$ for faint galaxies with low S/N. Overall, we find that the theoretical and the empirical schemes yield very similar results with the latter method resulting in slightly larger error bars because the theoretical scheme tends to somewhat underestimate the shape measurement error for faint galaxies.

Photometric redshifts are used to derive $\Sigma_{\text{crit}}$ for each lens–source pair. The lower 68% confidence bound on each source redshift is used to select background galaxies. For each lens–source pair, we demand that $z_{\text{source}} > z_{\text{lens}}$ so as to minimize foreground contamination. The g–g lensing signal is most sensitive to redshift errors when $z_{\text{source}}$ is only slightly larger than $z_{\text{lens}}$ (see Figure 1). For this reason, in addition to the previous cut, we also implement a fixed cut so that $z_{\text{source}} - z_{\text{lens}} < 0.1$. Furthermore, in order to minimize the effects of signal dilution caused by catastrophic errors, we also reject all source galaxies with a secondary peak in the redshift probability distribution function (i.e., the parameter $z_{2\gamma}$ is non-zero in the Ilbert et al. 2009 catalog). This cut is aimed to reduce the number of catastrophic errors in the source catalog. After all cuts have been applied, the g–g lensing source catalog that we use represents 35 galaxies per arcmin².

Finally, we re-compute all our g–g lensing signals using the Schrabback et al. (2010) COSMOS shear catalog which has been independently derived from ours. We find identical g–g lensing signals from both shear catalogs, indicating that any relative shear calibration differences between the two shear catalogs has no impact on these results. This test provides an independent validation of our g–g lensing results.

### 4. THEORETICAL FRAMEWORK

Paper I presents the general theoretical foundations that form the backbone of this paper. In this section, we only give a brief, and thus necessarily incomplete, review of the theoretical background and strongly encourage the reader to refer to Paper I for further details. We adopt the same model and notation as in Paper I.

Paper I describes an HOD-based model that can be used to analytically predict the SMF, g–g lensing, and clustering signals. The key component of this model is the SHMR which is modeled as a log-normal probability distribution function with a log-normal scatter denoted $\sigma_{\log M_{h}}$, and with a mean–log relation denoted $M_{h} = f_{\text{SHMR}}(M_{*})$.

For a given parameter set and cosmology, $f_{\text{SHMR}}$ and $\sigma_{\log M_{h}}$ can be used to determine the central and satellite occupation functions, $\langle N_{\text{cen}} \rangle$ and $\langle N_{\text{sat}} \rangle$. These are used in turn to predict the SMF, g–g lensing, and clustering signals.

#### 4.1. The Stellar-to-halo Mass Relation

Following Behroozi et al. (2010, hereafter B10), $f_{\text{SHMR}}(M_{h})$ is mathematically defined via its inverse function:

$$\log_{10} \left( f_{\text{SHMR}}^{-1}(M_{h}) \right) = \log_{10}(M_{h})$$

$$= \log_{10}(M_{1}) + \beta \log_{10} \left( \frac{M_{*}}{M_{1}} \right)$$

$$+ \frac{\left( \frac{M_{*}}{M_{1}} \right)^{\delta} - 1}{\frac{M_{*}}{M_{1}} - 1} \frac{1}{2},$$  \hfill (13)

where $M_{1}$ is a characteristic halo mass, $M_{1,0}$ is a characteristic stellar mass, $\beta$ is the low-mass slope, and $\delta$ and $\gamma$ control the high-mass slope. We refer to B10 for more detailed justification of this functional form. Briefly, we expect that at least four parameters are required to model the SHMR: a normalization, break, a low-mass slope, and a bright end slope. In addition, B10 have found that the SHMR displays sub-exponential behavior at large $M_{*}$. This is modeled by the $\delta$ parameter which leads to a total of five parameters. Figure 3 illustrates the influence of $\delta$

---

Table 3 - Binning Scheme for the g–g Lensing

<table>
<thead>
<tr>
<th>Limits</th>
<th>g–g bin1</th>
<th>g–g bin2</th>
<th>g–g bin3</th>
<th>g–g bin4</th>
<th>g–g bin5</th>
<th>g–g bin6</th>
<th>g–g bin7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_{1} = [0.22, 0.48]$</td>
<td>min log$<em>{10}(M</em>{*})$</td>
<td>11.12</td>
<td>10.89</td>
<td>10.64</td>
<td>10.3</td>
<td>9.82</td>
<td>9.2</td>
</tr>
<tr>
<td></td>
<td>max log$<em>{10}(M</em>{*})$</td>
<td>12.0</td>
<td>11.12</td>
<td>10.89</td>
<td>10.64</td>
<td>10.3</td>
<td>9.8</td>
</tr>
<tr>
<td>$z_{2} = [0.48, 0.74]$</td>
<td>min log$<em>{10}(M</em>{*})$</td>
<td>11.29</td>
<td>11.05</td>
<td>10.88</td>
<td>10.65</td>
<td>10.3</td>
<td>9.8</td>
</tr>
<tr>
<td></td>
<td>max log$<em>{10}(M</em>{*})$</td>
<td>12.0</td>
<td>11.29</td>
<td>11.05</td>
<td>10.88</td>
<td>10.65</td>
<td>10.3</td>
</tr>
<tr>
<td>$z_{3} = [0.74, 1.0]$</td>
<td>min log$<em>{10}(M</em>{*})$</td>
<td>11.35</td>
<td>11.16</td>
<td>10.97</td>
<td>10.74</td>
<td>10.39</td>
<td>9.8</td>
</tr>
<tr>
<td></td>
<td>max log$<em>{10}(M</em>{*})$</td>
<td>12.0</td>
<td>11.35</td>
<td>11.16</td>
<td>10.97</td>
<td>10.74</td>
<td>10.39</td>
</tr>
</tbody>
</table>
each parameter on the shape of the SHMR and further details on the role of each parameter can be found in Section 2.1 of Paper I.

In contrast to B10, we do not parameterize the redshift evolution of this functional form. Rather, we bin the data into three redshift bins and check for redshift evolution in the parameters a posteriori. We also assume that Equation (13) is only relevant for central galaxies. Following the HOD ansatz, satellite galaxies within groups and clusters occupy subhalos—bound, virialized halos that are contained within the radius of a larger halo. Abundance matching models like B10 assume that the halos and subhalos of the same mass (or circular velocity) contain galaxies of the same stellar mass, where the subhalo mass is taken as the mass at the last time the galaxy was a central galaxy. Here we put no such prior on the galaxy–subhalo connection. Rather, the halo occupation of satellite galaxies is constrained by the data.

4.2. Scatter between Stellar and Halo Mass

The measured scatter in stellar mass at fixed halo mass has an intrinsic component (denoted $\sigma^i_m$), but also includes a stellar mass measurement error due to redshift, photometry, and modeling uncertainties (denoted $\sigma^m_{\log M^*}$). Ideally, we would measure both components but unfortunately we can only constrain the quadratic sum of these two sources of scatter. Nonetheless, given a model for $\sigma^m_{\log M^*}$, we could in principle extract $\sigma^i_m$ from $\sigma^m_{\log M^*}$.

Previous work suggests that $\sigma^m_{\log M^*}$ is independent of halo mass. For example, Yang et al. (2009) find that $\sigma^m_{\log M^*} = 0.17$ dex and More et al. (2009) find a scatter in luminosity at fixed halo mass of 0.16 ± 0.04 dex. Both Moster et al. (2010) and B10 are able to fit the Sloan Digital Sky Survey (SDSS) galaxy SMF assuming $\sigma^m_{\log M^*} = 0.15$ dex and $\sigma^m_{\log M^*} = 0.175$ dex, respectively. However, these results are derived with spectroscopic samples of galaxies. In contrast to these surveys, we expect a larger measurement error for the COSMOS stellar masses due to the use of photometric redshifts. In addition, since photo-$z$ errors increase for fainter galaxies, we might also expect that $\sigma^m_{\log M^*}$ (and thus $\sigma^m_{\log M^*}$) will depend on $M^*$.

To test if the assumption that $\sigma^m_{\log M^*}$ is constant with $M^*$ has any impact on our results, we implement two models for $\sigma^m_{\log M^*}$.

In the first case (called “SIG_MOD1”), $\sigma^m_{\log M^*}$ is assumed to be constant (this is our base-line model). In the second case (called “SIG_MOD2”), we explicitly model $\sigma^m_{\log M^*}$ to reflect stellar mass measurement errors. Note that the goal of this exercise is not to perform a careful and thorough error analysis, but simply to build a realistic enough model to asses whether or not an $M^*$-dependent error has any strong impact on our conclusions.

For the SIG_MOD2 model, we consider three contributions to the stellar mass error budget. The first is called “model error”; this is measured by the 68% confidence interval of the mass probability distribution determined for each galaxy by the mass estimator. It represents the range of model templates (each with its own M/L ratio) that provide reasonable fits to the observed SED. This range is determined by the photometric uncertainty in the observed SED, degeneracies in the grid of models used to fit the data, and how well the grid of models represents the true parameter space of observed galaxy populations as well as their colors. The second term is the photo-$z$ error, which derives from the uncertainty in the luminosity distance owing to the error on a given photometric redshift. The final component is the photometric uncertainty from the observed K-band magnitude, which translates into an uncertainty in luminosity and therefore stellar mass. The total measurement error, $\sigma^m_{\log M^*}$, is the sum in quadrature of these three sources of error. The results are shown in Figure 4 for the three redshift bins.

As detailed in Section 5, we find that our results are largely unchanged, regardless of which form we adopt for $\sigma^m_{\log M^*}$. This can be explained as follows. Since the data are binned by $M^*$, the observables are in fact sensitive to the scatter in halo mass at fixed stellar mass, denoted $\sigma^o_{\log M^*}$. Given a model for the SHMR, $\sigma^m_{\log M^*}$ can be mathematically derived from $\sigma^o_{\log M^*}$. Further details on the mathematical connection between $\sigma^o_{\log M^*}$ and $\sigma^m_{\log M^*}$ can be found in Paper I. We find that the slope of the SHMR increases steeply at $M^* > 10^{11} M_\odot$ so that $\sigma^o_{\log M^*}$ becomes quite large at the high-mass end. For example, $\sigma^o_{\log M^*} \sim 0.46$ dex at $M^* = 10^{11} M_\odot$ and $\sigma^o_{\log M^*} \sim 0.7$ dex at $M^* = 10^{11.5} M_\odot$. As a result, the data are particularly
sensitive to $\sigma_{\log M_*}$ at large $M_*$ but not very sensitive to $\sigma_{\log M_*}$ at low $M_*$. Therefore, accounting for the mass dependence of $\sigma_{\log M_*}$ has little impact on the results because the constraints on $\sigma_{\log M_*}$ mainly originate from high-mass galaxies anyway.

Finally, we note that our derived values for the parameters of the SHMR should be independent of $\sigma_{\log M_*}$ (we will show that $\sigma_{\log M_*}$ is not largely degenerate with any of the other nine parameters in Section 5). The observables (e.g., lensing, clustering, and SMF) do themselves depend on $\sigma_{\log M_*}$ but since we account for $\sigma_{\log M_*}$ in the model, the extracted SHMR should reflect the true underlying physical relationship between halo and stellar mass (this however would not be the case if one were to neglect $\sigma_{\log M_*}$). In other terms, our model fully accounts for Eddington bias in all three observables (also see discussion in Paper I).

4.3. Parameters in the Model

To model the central occupation function, we use five free parameters ($M_0$, $\beta$, $\delta$, $\gamma$) to model $f_{\text{SHMR}}$ and we leave $\sigma_{\log M_*}$ as an additional free parameter. As described in Paper I, the central occupation function for a sample of galaxies more massive than $M_1^c$ (where $M_1^c$ represents some threshold in $M_*$) is expressed as

$$\langle N_{\text{cen}}(M_h | M_1^c) \rangle = \frac{1}{2} \left[ 1 - \text{erf} \left( \frac{\log_{10}(M_1^c) - \log_{10}(f_{\text{SHMR}}(M_h))}{\sqrt{2}\sigma_{\log M_*}} \right) \right].$$ \hspace{1cm} (14)

Our model also has an additional five parameters that are necessary to model the satellite occupation function, $\langle N_{\text{sat}} \rangle$. For a set of galaxies more massive than threshold $M_1^c$, $\langle N_{\text{sat}} \rangle$ is

$$\langle N_{\text{sat}}(M_h | M_1^c) \rangle = \langle N_{\text{cen}}(M_h | M_1^c) \rangle \left( \frac{M_h}{M_{\text{sat}}} \right)^{\alpha_{\text{sat}}} \exp \left( -\frac{M_{\text{cut}}}{M_h} \right).$$ \hspace{1cm} (15)

The free parameters that determine satellite occupation as a function of stellar mass are $\beta_{\text{sat}}$, $B_{\text{sat}}$, $\beta_{\text{cut}}$, $B_{\text{cut}}$, and $\alpha_{\text{sat}}$. The first two parameters, $\beta_{\text{sat}}$ and $B_{\text{sat}}$, determine the amplitude of $\langle N_{\text{sat}} \rangle$. The second two parameters, $\beta_{\text{cut}}$ and $B_{\text{cut}}$, set the scale of the exponential cutoff. These parameters enter into $\langle N_{\text{sat}} \rangle$ as

$$\frac{M_{\text{sat}}}{10^{12} M_\odot} = B_{\text{sat}} \left( \frac{f_{\text{SHMR}}(M_1^c)}{10^{12} M_\odot} \right)^{\beta_{\text{sat}}},$$ \hspace{1cm} (16)

and

$$\frac{M_{\text{cut}}}{10^{12} M_\odot} = B_{\text{cut}} \left( \frac{f_{\text{SHMR}}(M_1^c)}{10^{12} M_\odot} \right)^{\beta_{\text{cut}}}.$$ \hspace{1cm} (17)

Finally, $\alpha_{\text{sat}}$ represents the power-law slope of the satellite mean occupation function. We set $\alpha_{\text{sat}} = 1$ for all samples which should be a good choice because the theoretical expectation is that the number of sub-halos above a given mass scales linearly with halo mass (Kravtsov et al. 2004; Conroy et al. 2006; Moster et al. 2010; Tinker et al. 2010). Results from group catalogs also indicate that $\alpha_{\text{sat}} \sim 1$ (Collister & Lahav 2005; Yang et al. 2009). Previous HOD analyses of clustering results at varying redshifts also vary little from a value of unity (Zehavi et al. 2005, 2011b; Zheng et al. 2007; van den Bosch et al. 2007; Tinker et al. 2007).

In total, our model contains ten free parameters and one fixed parameter ($\alpha_{\text{sat}}$). A summary and description of these parameters can be found in Table 4 and also in Paper I.

4.4. Covariance Matrices

The COSMOS survey covers a relatively small volume and therefore sample variance effects must be taken into account. The volumes probed in the three redshift bins are given in Table 1 and vary from $0.88 \times 10^6$ Mpc$^3$ for the ACS region in $z_1$ ($0.22 < z < 0.48$) to $4.39 \times 10^6$ Mpc$^3$ for the Subaru region in $z_3$ ($0.74 < z < 1.0$). The volumes sampled by COSMOS are too small to obtain an accurate estimate of the sample variance from the data itself.

A series of mock catalogs are used to calculate the covariance matrices for all three observables. Details regarding the construction of these mocks can be found in Paper I. Briefly, COSMOS-like mocks are created from a single simulation (named “Consuelo”) $420 h^{-1}$ Mpc on a side, resolved with 1400$^3$ particles, and a particle mass of $1.87 \times 10^9 h^{-1} M_\odot$. This simulation can robustly resolve halos with masses above

23 In this paragraph, numbers are quoted for $H_0 = 100 h$ km s$^{-1}$ Mpc$^{-1}$.
$\sim 10^{11} \ h^{-1} M_\odot$ and is part of the Las Damas suite\textsuperscript{24} (J. McBride et al., in preparation). We create mocks for the three redshift intervals: $z_1 = [0.22, 0.48], z_2 = [0.48, 0.74]$, and $z_3 = [0.74, 1]$. For each redshift interval, we construct a series of mocks created from random lines of sight through the simulation volume that have the same area as COSMOS and the same comoving length for the given redshift slice. This yields 405 independent mocks for the $z_1$ bin, 172 mocks for the $z_2$ bin, and 109 mocks for the $z_3$ bin. For each redshift bin, mocks are created from the simulation output at the median redshift of the bin (see Table 1).

We converge on the mocks used for estimating errors using an iterative method. To begin with, we find an initial best-fit model to the data, without using any covariance matrices. This initial fit is used to populate the mocks and to create a first set of covariance matrices. We then re-fit the data using these covariance matrices and use the best-fit HOD models to create our final covariance matrices. All results presented here use this final set of covariance matrices.

### 5. RESULTS

We now present the results of fitting the model described in Paper I to the observed COSMOS g–g lensing, clustering, and SMFs.

#### 5.1. Constraining the Model

In order to fit the model to the data, we minimize

$$
\chi^2_{\text{tot}} = \chi^2_{\text{SMF}} + \sum_i \chi^2_{\Delta \Sigma, i} + \sum_j \chi^2_{w(\theta), j},
$$

where the sum over $i$ and $j$ indicates summation over the different stellar mass bins and thresholds, respectively. For the SMF and each stellar mass sample in $w(\theta)$ and $\Delta \Sigma$, $\chi^2$ is calculated by

$$
\chi^2 = \sum_{n, i} (x_n - y_n) \ C^{-1} (x_i - y_i),
$$

where $x_n$ is the model calculation of the quantity for data point $n$, $y_n$ is the measurement, and $C^{-1}$ is the inversion of the covariance matrix.

To obtain the posterior probability distributions for the parameter set ($M_1, M_{*0}, \beta, \delta, \gamma, \sigma_{\log M_*}, \beta_{\text{sat}}, B_{\text{sat}}, B_{\text{cat}}, B_{\text{cat}}$), we implement a Markov Chain Monte Carlo (MCMC) algorithm as follows.

1. We sample the region of interest around the best fit by initializing all chains close to the minimum $\chi^2$ solution with varying initial random spreads.
2. We apply conservative limits on all 10 parameters. As shown in Figure 8, the recovered posterior distributions are independent of these limits.
3. The covariance matrix is updated for the first 3000 elements in the chain, and then held fixed for the remainder of the chain.
4. We run six chains per case, with $\sim 40,000$ steps. We discard all elements before the covariance matrix is constant (the first 3000), leaving $\sim 37,000$ elements per chain.
5. We use the GetDist\textsuperscript{25} package provided with CosmoMC (Lewis & Bridle 2002) for computing convergence diagnostics. The chains are tested for convergence and mixing with the Gelman–Rubin criterion (Gelman & Rubin 1992; Gilks et al. 1996). We impose a limit on the worst $R - 1$ statistic of $R - 1 < 0.01$ and observe that for all cases under study the mean and marginalized posterior distributions are well constrained.
6. We use GetDist output statistics to estimate confidence limits on all parameters.

#### 5.2. Fits to the Data

Figures 5–7 show the best-fit models for each of the three redshift bins for the galaxy clustering, the g–g lensing, and the SMF. The upper panels show the angular correlation function $w(\theta)$ for stellar mass threshold samples. The middle right panel shows the COSMOS SMF which is measured for all galaxies with $M_* > 10^{8.7} M_\odot$ for $z_1$, $M_* > 10^{9.3} M_\odot$ for $z_2$, and $M_* > 10^{9.8} M_\odot$ for $z_3$. The dotted blue line shows the SMF of satellite galaxies from our model. The lower panels show the g–g lensing signals for the stellar mass bins defined in Table 3. When looking at Figures 5–7, one must keep in mind that the data points are correlated (see Paper I for the covariance matrices) and so it is difficult to evaluate “by eye” whether or not the fits are adequate. The reduced $\chi^2$ for the fits are 1.7, 1.6, and 1.9 for $z_1$, $z_2$, and $z_3$, respectively.

For our $z_1$ sample, we have compared the COSMOS mass function with previously published mass functions from SDSS (Li & White 2009; Baldry et al. 2008; Panter et al. 2007). Because of the use of photometric redshifts in COSMOS, we expect a larger stellar mass measurement error which should cause Eddington bias and lead to an inflated observed SMF at the high-mass end in COSMOS compared to SDSS. A more in-depth comparison of the mass functions is discussed further in Section 5.6.

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\textsuperscript{24} Further details regarding this simulation can be found at http://lss.phy.vanderbilt.edu/lasdamas/simulations.html

\textsuperscript{25} The authors suggest using the GetDist package for computing convergence diagnostics.

### Table 4: Parameters in Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Description</th>
<th>$\langle N_{\text{cen}} \rangle$ or $\langle N_{\text{sat}} \rangle$</th>
<th>Free/Fixed</th>
</tr>
</thead>
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<tr>
<td>$M_1$</td>
<td>$M_\odot$</td>
<td>Characteristic halo mass in the SHMR</td>
<td>$\langle N_{\text{cen}} \rangle$</td>
<td>Free</td>
</tr>
<tr>
<td>$M_{*0}$</td>
<td>$M_\odot$</td>
<td>Characteristic stellar mass in the SHMR</td>
<td>$\langle N_{\text{cen}} \rangle$</td>
<td>Free</td>
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<tr>
<td>$\beta$</td>
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<td>Low-mass slope in the SHMR</td>
<td>$\langle N_{\text{cen}} \rangle$</td>
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<tr>
<td>$\delta$</td>
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<td>Controls high-mass slope in the SHMR</td>
<td>$\langle N_{\text{cen}} \rangle$</td>
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</tr>
<tr>
<td>$\gamma$</td>
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<td>Controls the transition regime in the SHMR</td>
<td>$\langle N_{\text{cen}} \rangle$</td>
<td>Free</td>
</tr>
<tr>
<td>$\sigma_{\log M_*}$</td>
<td>dex</td>
<td>Log-normal scatter in stellar mass at fixed halo mass</td>
<td>$\langle N_{\text{cen}} \rangle$</td>
<td>Free</td>
</tr>
<tr>
<td>$\beta_{\text{sat}}$</td>
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<td>Slope of the scaling of $M_{\text{sat}}$</td>
<td>$\langle N_{\text{sat}} \rangle$</td>
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<tr>
<td>$B_{\text{sat}}$</td>
<td>None</td>
<td>Normalization of the scaling of $M_{\text{sat}}$</td>
<td>$\langle N_{\text{sat}} \rangle$</td>
<td>Free</td>
</tr>
<tr>
<td>$\beta_{\text{cat}}$</td>
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<td>$\langle N_{\text{cat}} \rangle$</td>
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</tr>
<tr>
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<td>Normalization of the scaling of $M_{\text{cat}}$</td>
<td>$\langle N_{\text{cat}} \rangle$</td>
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</tr>
<tr>
<td>$\alpha_{\text{cat}}$</td>
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<td>Power-law slope of the satellite occupation function</td>
<td>$\langle N_{\text{sat}} \rangle$</td>
<td>Fixed at 1</td>
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</table>
Figure 5. Best-fit model for the \(z_1 (0.22 < z < 0.48)\) redshift bin (blue solid line). Panels (a)–(f): amplitude of the angular correlation function \(w\) as a function of angular separation \(\theta\) (in arcseconds) in stellar mass thresholds. Note that in this redshift bin, the amplitude of \(w\) at large separations is artificially deflated by integral constraint (but this is accounted for in the fitted model). Panel (g): COSMOS SMF for \(M_\star > 10^{8.7} M_\odot\) (completeness limit for this redshift bin). For reference, in panel (f), we also show the SDSS mass functions from Li & White (2009) (triple Schechter fit, black dashed line), Baldry et al. (2008) (green dash-dotted line), and from Panter et al. (2007) (orange, dash-dotted line). The dotted blue line in panel (g) shows the SMF of satellite galaxies for the best-fit model. Panels (h)–(n): galaxy–galaxy lensing signal in stellar mass bins. Note that in panel (m), there is a negative data point represented by a gray square (this can occur due to noise and is not a concern). The lensing signal is decomposed into four components: the baryonic term (red dotted line), the central one-halo term (green dashed line), the satellite one-halo term (orange dash-dotted), and the two-halo term (gray triple-dot-dashed line).

(A color version of this figure is available in the online journal.)

5.3. Parameter Constraints and Redshift Evolution

Table 5 gives the best-fit values from GetDist for all 10 parameters and for the three redshift bins. Figure 8 shows the one-dimensional and two-dimensional joint marginalized constraints on the parameters for the \(z_2\) bin. All of the parameters have well-behaved, uni-modal distributions. In the interest of brevity, we have not included equivalent figures for \(z_1\) and \(z_3\) but they are similar to Figure 8. All parameters are well constrained in the three redshift bins.

Table 5 lists the best-fit values for \(\sigma_{\text{mod1}}\) where we have assumed that \(\sigma_{\log M_\star}\) is constant and for \(\sigma_{\text{mod2}}\) where we have explicitly accounted for stellar mass-dependent errors. We find no strong difference in our results, regardless of which model we
We conclude that our results are robust to the effects of mass-dependent scatter. In the $\text{sig}_\text{mod2}$ case, we model $\sigma_m \log M_*$ and assume that $\sigma_{\log M_*}$ is the sum in quadrature of $\sigma_{i \log M_*}$ (which is assumed to be constant) and $\sigma_{m \log M_*}$. Therefore, the quantity that we actually fit for in the $\text{sig}_\text{mod2}$ case is $\sigma_{i \log M_*}$ (whereas in $\text{sig}_\text{mod1}$ we fit for $\sigma_{\log M_*}$). This is why, as expected, the best-fit scatter in Table 5 is slightly lower for $\text{sig}_\text{mod2}$ compared to $\text{sig}_\text{mod1}$. Note that we are not claiming to actually extract meaningful values for the intrinsic scatter in stellar mass at fixed halo mass. To do so would require a more thorough error analysis which is beyond the scope of this paper. Overall, our conclusions regarding $\sigma_{\log M_*}$ are twofold. First, we can safely assume that $\sigma_{\log M_*}$ is constant and ignore any mass-dependent effects induced by measurement error. This is due to the fact that all three observables are primarily sensitive to the effects of $\sigma_{\log M_*}$ at large $M_*$ where the slope of the SHMR increases sharply. Similar conclusions were reached by B10 who find that the effects of scatter are insignificant below $M_* = 10^{10.5} M_\odot$ where the slope of the SHMR is not steep enough to have a significant impact. Second, we find that $\sigma_{\log M_*} \sim 0.23 \pm 0.03$ dex, in broad agreement with previous results. For example, B10 estimate that $\sigma_{\log M_*} = 0.07$ dex and $\sigma_{\log M_*} = 0.16$ dex, resulting in a total

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6.png}
\caption{Same as Figure 5 but for the second redshift bin, $z_2$. Panels (a)–(e): angular correlation function. The angular correlation function is less affected by integral constraint in this redshift bin since the volume probed is 2.3 times larger than $z_1$. Panel (f): COSMOS SMF for $M_* > 10^{10.3} M_\odot$. The dotted blue line in panel (f) shows the SMF of satellite galaxies for the best-fit model. Panels (g)–(m): galaxy–galaxy lensing signal. (A color version of this figure is available in the online journal.)}
\end{figure}
scatter of $\sigma_{\log M_*} = 0.175$ dex (their total scatter is lower than ours as expected because we have a larger $\sigma_{\log M_*}$ component).

Figure 9 shows the measured redshift evolution for all 10 parameters. Previous work on this topic has been strongly limited by systematic differences in stellar mass estimates between low- and high-$z$ surveys. The main factors that contribute to this systematic uncertainty arise from the choice of an IMF, a stellar population synthesis (SPS) model, a dust model, and a population history model (estimates for the magnitude of each effect can be found in B10). According to B10, the total systematic uncertainty of stellar mass estimates (excluding IMF uncertainties) is of order 0.25 dex. In addition, the choice of an IMF results in another 0.25 dex uncertainty. We stress that the results in this paper have been derived in a homogeneous fashion from high to low redshift. Our conclusions regarding redshift evolution should thus be more robust than those from work that combine results from distinct low- and high-$z$ surveys. While we believe that a homogeneous analysis globally reduces the systematic errors associated with redshift trends, we should note that our results might still be affected at some level by redshift-dependence systematic errors in stellar mass estimators. These could be caused, for example, by a redshift evolution of the IMF. Currently, however, the magnitude of such redshift-dependence systematic errors is very poorly known. For this reason, we do

Figure 7. Same as Figure 5 but for the third redshift bin, $z_3$. Panels (a)–(d): angular correlation function. The angular correlation function is less affected by integral constraint in this redshift bin since the volume probed is five times larger than $z_1$. Panel (e): COSMOS SMF for $M_* > 10^{10.8} M_\odot$. The dotted blue line in panel (e) shows the SMF of satellite galaxies for the best-fit model. Panels (f)–(k): galaxy–galaxy lensing signal. (A color version of this figure is available in the online journal.)
Figure 8. One- and two-dimensional joint-mean and marginalized distributions for all 10 parameters for the \( z_2 \) bin. Dotted lines in the one-dimensional distributions (and shaded contours in the two-dimensional planes) show mean likelihoods of samples and solid lines show marginalized probabilities. In the two-dimensional planes, blue to red denotes the region of lowest to highest likelihood density. Solid contours in the two-dimensional plane represent the 68\% (1\( \sigma \)) and 95\% (2\( \sigma \)) confidence regions.

(A color version of this figure is available in the online journal.)

not discuss such errors further in this paper, however, this is clearly an issue that requires further investigation.

The most striking result from Figure 9 is the redshift evolution in the two parameters \( M_1 \) and \( M_\star,0 \). This is one of the major results of this paper which we will discuss in more detail in the following section. Apart from these two parameters, there is marginal evidence for some evolution in \( \gamma \) and \( B_{\text{sat}} \). No strong evolution is detected in the remaining six parameters. It is interesting to note that \( \beta \) (which controls the low-mass slope of the SHMR) remains constant at \( \beta \sim 0.46 \). We will provide an interpretation of this result in the discussion section.

5.4. The SHMR from \( z = 0 \) to \( z = 1 \)

Figure 10 shows the best-fit SHMR for \( z_1 \) compared to a variety of low-redshift constraints from weak lensing (Mandelbaum et al. 2006b; Leauthaud et al. 2010; Hoekstra 2007), abundance matching (Moster et al. 2010; Behroozi et al. 2010), satellite kinematics (Conroy et al. 2007; More et al. 2010), and the Tully–Fisher relation (Geha et al. 2006; Pizagno 2006; Springob et al. 2005; Blanton et al. 2008) (see Section 5.5 for a more in-depth comparison). Most of the data are in broad agreement and show clear evidence for a variation in the dark-to-stellar mass ratio with a minimum of \( M_h/M_\star \sim 3 \) at \( M_\star \sim 4.5 \times 10^{10} M_\odot \) and \( M_h \sim 1.2 \times 10^{12} M_\odot \). As demonstrated by B10, however, meaningful and detailed comparisons between various data sets are hampered by systematic uncertainties in stellar mass estimates. For this reason, we will mainly focus in what follows on the conclusions that can be drawn by inter-comparing the three COSMOS redshift bins.

At low stellar mass, \( M_h \) scales as \( M_h \propto (M_\star)^{0.46} \) and this scaling does not evolve significantly with redshift from \( z = 0.2 \).
to \( z = 1 \). At high stellar mass, the SHMR rises sharply at \( M_* > 10^{10.5} M_\odot \) as a result of which \( \sigma_{\log M_*} \) (the scatter in halo mass at fixed stellar mass) also increases and \( M_* \) is clearly no longer a good tracer of halo mass. For example, a galaxy with \( M_* \sim 10^{11.3} M_\odot \) may be the central galaxy of group with \( M_h \sim 10^{15} M_\odot \) or may also be the central galaxy of a cluster with \( M_h > 10^{15} M_\odot \).

A quantity that is of particular interest is the mass at which the ratio \( M_h/M_* \) reaches a minimum. This minimum is noteworthy for models of galaxy formation because it marks the mass at which the accumulated stellar growth of the central galaxy has been the most efficient. We describe the SHMR at this minimum in terms of the “pivot stellar mass,” \( M_p^{\text{vir}} \), the “pivot halo mass,” \( M_h^{\text{vir}} \), and the “pivot ratio,” \( (M_h/M_*)^{\text{vir}} \). Note that \( M_p^{\text{vir}} \) and \( M_h^{\text{vir}} \) are not simply equal to \( M_1 \) and \( M_{\ast,0} \). Indeed, the mathematical formulation of the SHMR is such that the pivot masses depend on all five parameters. The three parameters that have the strongest effect on the pivot masses are \( M_1, M_{\ast,0}, \) and \( \gamma \) (see Paper I).

Figure 11 shows the redshift evolution of the SHMR. Three points are of particular interest in this figure. First, we detect no strong redshift evolution in the low-mass slope of the SHMR (\( M_* < 10^{10.2} M_\odot \)). Indeed, as highlighted in the previous section already, \( \beta \) is remarkably constant out to \( z = 1 \). In Paper I, we have shown that \( \beta \) regulates the low-mass slope of the SMF so in other terms, we could also state that the faint end slope of the SMF shows remarkably little redshift evolution. We do however find that the amplitude of the low-mass (\( M_* < 10^{10.2} M_\odot \)) SHMR was higher at earlier times so that galaxies at fixed stellar mass live in more massive halos at earlier epochs. We will return to this result in the discussion section.

Second, we detect a strong redshift evolution in \( M_p^{\text{vir}} \) and \( M_h^{\text{vir}} \) which is shown more explicitly in Figure 12. The detected evolution is such that both the halo mass and the stellar mass for which the accumulated stellar growth of the central galaxy has been the most efficient are smaller at late times than at earlier times. This trend is a manifestation of at least one meaning of the term “downsizing” (Cowie et al. 1996; Brinchmann & Ellis 2000; Juneau et al. 2005). Originally, the term downsizing referred to the notion that the maximum K-band luminosity of galaxies above a specific star formation rate (SSFR) threshold decreases with time (Cowie et al. 1996). Since then, downsizing has been widely employed to more generally describe the behavior in which a certain parameter that regulates galaxy formation decreases with time (for recent discussions on downsizing see Fontanot et al. 2009 and Conroy & Wechsler 2009). Our results show strong evidence for downsizing in both the pivot halo mass and the pivot stellar...
mass. We have already remarked in the previous section that a strong evolution in $M_1$ and $M_{\ast,0}$ is seen in Figure 9. Although these two parameters are not strictly equal to $M_{\rm piv}^{\ast}$ and $M_{\rm piv}^{h}$, they do have a strong impact on the location of the pivot masses. Thus, the evolution seen in $M_1$ and $M_{\ast,0}$ is directly related to the observed downsizing behavior in the pivot masses that is apparent in Figures 11 and 12. The pivot stellar mass evolves from $M_{\ast,0}^{\rm piv} = 5.75 \pm 0.13 \times 10^{10} M_\odot$ at $z = 0.88$ to $M_{\ast,0}^{\rm piv} = 3.55 \pm 0.17 \times 10^{10} M_\odot$ at $z = 0.37$ with an evolution detected at 10$\sigma$. We note that all errors have been derived by marginalizing over all other parameters.

The evolution in $M_{\ast,0}^{\rm piv}$ varies smoothly in the three redshift bins, however the evolution in $M_{\ast,0}^{\rm piv}$ is less smooth, in particular in the $z_2$ bin. We suggest that $M_{\ast,0}^{\rm piv}$ is more sensitive to sample variance than $M_{\ast,0}^{\rm piv}$. Indeed, the first-order effect of sample variance is to change the normalization of the SMF (see Paper I); this will directly affect $M_1$ and thus $M_{\ast,0}^{\rm piv}$. In summary: $M_{\ast,0}^{\rm piv}$ is sensitive to sample variance between redshift bins whereas $M_{\ast,0}^{\rm piv}$ is sensitive to systematic errors in stellar mass measurements between redshift bins.

Finally, at high masses ($M_\ast > 10^{11} M_\odot$) there is an interesting hint that the amplitude of the SHMR is decreasing at higher redshifts, but we lack the statistics for a clear detection, mainly due to the small volume probed by COSMOS.

5.5. Comparison with Previous Work

Figure 10 compares our $z_1$ SHMR to previous work on this topic at low redshift. The general picture that emerges from Figure 10 is one of remarkable broad agreement between various methods on the overall shape of the SHMR. In detail, however, meaningful comparison between various surveys is severely limited by systematic differences in stellar mass estimates ($\sim 0.25$ dex between different surveys). For this reason, we mainly focus on qualitative comparisons in this section. All results have been adjusted to our assumed value of $H_0 = 72 \, \text{km} \, \text{s}^{-1} \, \text{Mpc}^{-1}$ and unless stated otherwise, halo masses are...
Since Mandelbaum et al. (2006a) only present their results as a SDSS to measure halo masses for lens galaxies at $z$ model. Attempt to correct for differences in the assumed cosmological in $M$ are in broad agreement with B10 but a direct comparison would require a more homogeneous analysis between COSMOS and SDSS. Right panel: redshift evolution $\beta$ COSMOS redshift bins. The low-mass slope of the SHMR remains constant with $z$ decreasing at higher redshifts, but we lack the statistics to make a clear detection, mainly due to the small volume probed by COSMOS. We have also plotted the SHMR reported by B10. However, caution must be taken when making a direct comparison between COSMOS and B10 because our stellar masses have been derived under different assumptions. According to B10, the level of systematic uncertainty in stellar masses is of order 0.25 dex. Given this 0.25 dex systematic uncertainty, we are in broad agreement with B10 but a direct comparison would require a more homogeneous analysis between COSMOS and SDSS. Right panel: redshift evolution in $M_{200b}/M_\ast$. Both $M_{200b}$ and $M_\ast$ exhibit downsizing trends, decreasing at later epochs. This effect is shown more explicitly in Figure 12.

(A color version of this figure is available in the online journal.)

corrected to $M_{200b}$ assuming a Navarro–Frenk–White (NFW) profile and a Muñoz-Cuartas et al. (2011) mass–concentration relation for a WMAP5 cosmology when necessary. All results quoted here assume either a Kroupa or a Chabrier IMF. Since the systematic shift in $M_\ast$ between these two IMFs is small ($\sim 0.05$ dex), we do not adjust for this difference. We also do not attempt to correct for differences in the assumed cosmological model.

5.5.1. Comparison with Previous Work: Low Redshift

Mandelbaum et al. (2006a) have used $g$–$g$ lensing in the SDSS to measure halo masses for lens galaxies at $z \sim 0.1$. Since Mandelbaum et al. (2006a) only present their results as a function of both $M_\ast$ and color, the data points in Figure 10 have been re-computed as a function of $M_\ast$ only (Rachel Mandelbaum 2011, private communication). Except perhaps for one data point at low $M_\ast$, these results are in good agreement with ours.

At high masses, an alternative method to probe the central SHMR is to directly compare the halo masses of groups and clusters of galaxies to the masses of their central galaxies. Since we are primarily interested in $\Phi_\ast(M_\ast|M_\ast)$, it is critical, as much as possible, to use halo mass selected samples of groups and clusters for this type of comparison. Using samples selected on the basis of the stellar mass of the central galaxy, for example, would result in biased conclusions. In Leauthaud et al. (2010), we presented a sample of X-ray groups ($M_{200b} \sim 10^{13}$–$10^{14} M_\odot$) in COSMOS for which we have calibrated the relationship between halo mass and X-ray luminosity ($L_X$) using $g$–$g$ lensing. The expected scatter in halo mass at fixed $L_X$ is of order 0.13 dex so the sample presented in Leauthaud et al. (2010) is halo mass selected to a good approximation. In parallel, George et al. 2011 and M. R. George et al. (in preparation) have constructed an algorithm to identify the central galaxies of these groups and have used the weak lensing signal itself to optimize the algorithm by maximizing the weak lensing signal at small radial separations from the central galaxy. The gray squares in Figure 10 report the stellar mass of the central galaxy versus $M_\ast$ for groups at $0.22 < z < 0.48$ and with a high-quality flag. These data points are directly comparable to ours since we have used exactly the same stellar masses and confirm that our results are consistent with Leauthaud et al. (2010).

We present a similar exercise for a sample of X-ray luminous clusters (A68, A209, A267, A383, A963, A1689, A1763, A2218, A2390, A2219) from Hoekstra (2007) with weak lensing masses from Mahdavi et al. (2008). The central galaxies of these clusters have been studied in detail by Bildfell et al. (2008). Using the same stellar mass code and assumptions as in this paper, we have computed stellar masses for the central cluster galaxies using a compilation of optical data provided by Chris Bildfell. The results are shown by the red asterisk points in Figure 10. Unfortunately, these mass estimates are based on just two optical bands ($B$-band and $R$-band) and as such will have larger uncertainties than the COSMOS stellar masses used in this paper which are constrained with many more filters and normalized to a near-IR luminosity. We estimate that an additional 0.25 dex stellar mass uncertainty should be included when interpreting these data points, which may account for their scatter in stellar mass seen in Figure 10. With this cautionary note, plus the additional caveat that this sample is not as homogeneously selected as the groups from Leauthaud et al. (2010), and that COSMOS is too small to probe overdensities of these masses, the results are nonetheless in good agreement with the extrapolation of our $z_{\ast}$ SHMR.

Both Moster et al. (2010) and B10 have presented constraints on the SHMR and its redshift evolution by using the abundance matching technique. A more detailed comparison with their work is presented in Sections 5.5.3 and 5.5.4.
Conroy et al. (2007) have used the kinematics of satellite galaxies in SDSS and DEEP2 (Davis et al. 2003) to probe the SHMR at $z \sim 0.06$ and at $z \sim 0.8$. Their low-redshift results are shown by the blue diamonds in Figure 10. A 30% downward correction to halo masses has been applied due to incompleteness effects as described in their paper. In addition to systematic differences in stellar mass estimates, direct comparisons with our results are further complicated by the fact that our model describes $(\log_{10}(M_*(M_h)))$ whereas the Conroy et al. (2007) results represent $(M_*(M_h))$. The two averaging systems will yield different results: $(M_*(M_h))$ will be increasingly biased low with respect to $(\log_{10}(M_*(M_h)))$ with increasing $\sigma_{\log M_\ast}$ and for steeper values of the slope of the SHMR.

More et al. (2010) have used SDSS data to probe the halo masses of $\sim 3900$ central galaxies in the range $0.02 \leq z \leq 0.072$ using the kinematics of satellite galaxies. In their paper, More et al. (2010) have analyzed red and blue galaxies separately. For Figure 10 we have asked the authors to provide the data for all central galaxies as a function of $M_\ast$, irrespective of color, and also to convert their results to reflect the mean–log relation $(\log_{10}(M_*(M_h)))$ as opposed to $(M_*(M_h))$. Overall, there is some disagreement between our results and More et al. (2010) regarding the general shape of the SHMR. Indeed, our results display a more strongly varying power-law index compared to More et al. (2010). This disagreement is perhaps more apparent in the lower panel of Figure 10. Indeed, the More et al. (2010) results display a fairly broad minimum in $M_{200b}/M_\ast$ whereas our results predict a more strongly varying $M_{200b}/M_\ast$ ratio that reaches a minimum at $M_\ast \sim 4.5 \times 10^{10} M_\odot$. More et al. (2010) have suggested that satellite kinematics may yield halo masses that may be systematically higher by a factor of two to three than other methods at low $M_\ast$. Lowering the More et al. (2010) results at low $M_\ast$ would certainly bring their results into better agreement with ours in terms of the shape of the SHMR. More et al. (2010) also provide estimates for $\sigma_{\log M_\ast}$. They find $\sigma_{\log M_\ast} = 0.19^{+0.03}_{-0.03}$ for red centrals and $\sigma_{\log M_\ast} = 0.15^{+0.12}_{-0.07}$ for blue centrals. Both of these values are in broad agreement with our estimate of $\sigma_{\log M_\ast} \sim 0.23$ dex.

In the $z_1$ redshift bin, the COSMOS results are limited by completeness to $M_\ast > 10^{8.7} M_\odot$. Nonetheless, it is of interest to see how our results extrapolate to galaxies of even lower stellar masses, even though measurements of $M_h$ for such low-mass galaxies are fraught with difficulties and for the most part limited to the local volume. Blanton et al. (2008) have presented an effort to address the very low mass SHMR (see their Figure 12) using measurements of the maximum circular velocities from H$\alpha$ disks around isolated nearby dwarf galaxies. Since Blanton et al. (2008) have applied criteria to specifically select isolated galaxies, their sample should be dominated by central galaxies and so comparable to our Figure 10. Upon request, the authors provided us with the full data set from Figure 12 in Blanton et al. (2008) which we have reproduced in Figure 10, including additional data from Springob et al. (2005) and Pizagno et al. (2007) based on H$\alpha$ and H$\alpha$ rotation curves, respectively. This data compilation is restricted to galaxies that are isolated and with axis ratios $b/a < 0.5$ in order to minimize inclination uncertainties and extinction corrections. Halo masses in Blanton et al. (2008) have been estimated by assuming that the optical circular velocity, $V_{\text{opt}}$, is equal to $V_{\text{max}}$, the maximum circular velocity for an NFW halo. $V_{\text{max}}$ is then converted to the virial velocity, $V_{200}$, using $N$-body calibrations from Bullock et al. (2001). For galaxy mass halos, $V_{\text{max}}/V_{200} \simeq 1.1–1.2$ under the assumption of no adiabatic contraction of the dark matter due to galaxy formation. When incorporating adiabatic contraction into Tully–Fisher analyses, Gnedin et al. (2007) find a factor of $\sim 2.5$ decrease in the inferred halo mass at fixed stellar mass. Such a correction would put the Tully–Fisher constraints into better agreement with our results.

To first order, there is relatively good agreement between our SHMR and the data from Blanton et al. (2008), albeit with a much larger scatter in the Tully–Fisher based SHMR than predicted by our results. In particular, the dwarf galaxy data points from the Geha et al. (2006) sample are in good agreement with the extrapolation of our SHMR to lower masses. At $10^9 M_\odot < M_\ast < 10^{11} M_\odot$, however, there may be some indication that the halo masses inferred by Blanton et al. (2008) are too high on average compared to our results with the possible implication that the $V_{\text{opt}}/V_{200}$ ratio is larger than $1.1–1.2$. The $V_{\text{opt}}/V_{200}$ ratio contains information about the relative importance of baryons versus dark matter on galaxy scales: $V_{\text{opt}}/V_{200} \simeq 1.1–1.2$ would imply that the baryons have modified the dark matter profile in the very inner halo regions. Dutton et al. (2010) have used the Pizagno et al. (2007) data in combination with a compilation of prior work on the SHMR to place constraints on $V_{\text{opt}}/V_{200}$ for late-type galaxies (the requirement that the Pizagno et al. 2007 galaxies have sufficiently extended He$\alpha$ emission to yield a useful rotation curve implies that this is primarily a late-type sample). They find that $V_{\text{opt}} \sim 200$, however, the normalization of the SHMR that they employ is more similar to the More et al. (2010) results than to ours. Therefore, a similar analysis as Dutton et al. (2010) but applied to the Pizagno et al. (2007) data in combination with our results would yield a higher $V_{\text{opt}}/V_{200}$.

The agreement between the extrapolation of our SHMR to lower masses and the Geha et al. (2006) sample is encouraging. In fact, Busha et al. (2010) found that a similar scaling continues to work down to the faintest satellites of the Milky Way. However, Figure 10 clearly reveals a lack of data at $M_\ast < 10^8 M_\odot$ due to the stellar mass completeness limits of current optical and IR surveys. Pushing the SHMR down to $10^6 M_\odot < M_\ast < 10^8 M_\odot$ using techniques such as described in this paper is clearly an exciting avenue to explore and will be facilitated by upcoming very deep optical and IR surveys such at UltraVista and the Hyper Suprime Cam (HSC) survey on the Subaru telescope.25

5.5.2. Comparison with Previous Work: High Redshift

Heymans et al. (2006a) have used $g$–$g$ lensing to estimate halo masses for a sample of 626 galaxies with $M_g > 10^{10.5} M_\odot$ and with $0.2 < z < 0.8$ from the 0.25 deg$^2$ $HST$/GEMS (galaxy evolution from morphology and SEDs) survey (Rix et al. 2004). Heymans et al. (2006a) find $M_{200}/M_\ast = 53^{+13}_{-16}$ at a mean stellar mass of $M_\ast = 7.2 \times 10^{10} M_\odot$. Converting their result to our assumed value of $H_0$ and to $M_{200b}$ yields $M_{200b}/M_\ast \sim 58^{+17}_{-14}$. In a similar mass and redshift range, our results produce $M_{200b}/M_\ast \sim 34$. We note, however, that a careful comparison between our work and Heymans et al. (2006a) is limited by several differences in the way the analyses have been performed. First, Heymans et al. (2006a) fit an NFW profile to the $g$–$g$ lensing signal and so the masses that they measure will reflect the mean halo mass at fixed stellar mass,

25 For example, the UltraVista survey of the COSMOS field will obtain IR imaging to $Y = 26.7$, $J = 26.6$, $H = 26.1$, and $K_s = 25.6$ mag, pushing low-redshift stellar mass completeness limits to below $M_\ast = 10^8 M_\odot$ at $z < 0.3$. The HSC intermediate layer survey will cover 20 deg$^2$ to $g = 28.6$, $r = 28.1$, $i = 27.7$, $z = 27.1$, and $Y = 26.6$ mag.
which is different than our averaging system. Second, Heymans et al. (2006a) do not account for the contribution of satellite galaxies to the g–g lensing signal and so they will tend to overestimate halo masses. The lens sample of Heymans et al. (2006a) is roughly similar to our g–g bin4 in the z2 redshift range: the contribution to the g–g lensing signal from satellites for this sample is shown in panel (j) of Figure 6. Given these two caveats our results are in fairly good agreement.

5.5.3. Comparison with Moster et al.

Moster et al. (2010) derive constraints on the redshift evolution of the SHMR by abundance matching to the SDSS SMF of Panter et al. (2007) at low redshift and to mass functions from the MUNICS survey (detection limit K < 19.5; area 0.28 deg²; Drory et al. 2004) and the GOODS-MUSIC sample (detection limit K < 23.5, area 143.2 arcmin²; Fontana et al. 2006) at high redshift. There are two main differences between Moster et al. (2010) and our work. First, we adopt the functional form advocated by B10 for the SHMR which is sub-exponential at high \( M_{\ast} \). In contrast, the Moster et al. (2010) parameterization asymptotes to a power law at high \( M_{\ast} \). According to B10, such a parameterization may be problematic. Indeed, because the logarithmic slope of the SHMR increases with increasing \( M_{\ast} \), the best-fit power law for high-mass galaxies will depend on the upper limit in the available data for the SMF. Second, the Moster et al. (2010) errors do not reflect possible systematic errors in stellar mass estimates between Panter et al. (2007), Drory et al. (2004), and Fontana et al. (2006).

Although we find the same qualitative behavior as Moster et al. (2010): \( M_{\ast}/M_{h} \) decreases to a minimum at \( M_{h} \sim 10^{12} \ M_{\odot} \) and then rises at higher masses, interestingly, our results differ regarding the evolutionary trends of the SHMR. The two parameters for which our conclusions differ in particular are \( M_{\ast}^{\text{piv}} \) and \( (M_{\ast}/M_{h})^{\text{piv}} \). In Figure 12 (yellow dash-dotted line), we show the evolution of \( M_{\ast}^{\text{piv}}, M_{h}^{\text{piv}}, \) and \( (M_{\ast}/M_{h})^{\text{piv}}, \) as inferred from Table 7 in Moster et al. (2010).

Our results agree with Moster et al. (2010) in terms of the qualitative downsizing trend seen for \( M_{h}^{\text{piv}} \). However, it is interesting to note that our measurements differ with respect to the normalization of \( M_{\ast}^{\text{piv}} \). The exact origin of this discrepancy remains unclear. In light of the results of B10, we hypothesize that this discrepancy may be caused by the difference in the assumed parametric form of the SHMR. In any case, further investigation regarding the source of this discrepancy, though beyond the scope of this paper, is clearly warranted.

Our conclusions differ with respect to \( M_{\ast}^{\text{piv}} \) and \( (M_{\ast}/M_{h})^{\text{piv}} \). Whereas our results suggest that \( M_{\ast}^{\text{piv}} \) increases with redshift and that \( (M_{\ast}/M_{h})^{\text{piv}} \) remains constant, in contrast, the Moster et al. (2010) results imply that \( M_{\ast}^{\text{piv}} \) is constant with redshift and that instead, \( (M_{\ast}/M_{h})^{\text{piv}} \) increases with redshift. We hypothesize that this discrepancy is simply due to the fact that the errors in Moster et al. (2010) are likely to be underestimated. Indeed, accounting for sample variance with mocks as well as for systematic differences in the relative stellar masses between Panter et al. (2007), Drory et al. (2004), Fontana et al. (2006) would lead to similar errors as B10 (Figure 12, green diamonds). Indeed, the evolution that we detect in \( M_{\ast}^{\text{piv}} \) is \( \sim 0.21 \) dex from \( z = 0.37 \) to \( z = 0.88 \) which is similar to the expected error in stellar mass estimates between different surveys. This could perhaps also explain why we reach similar conclusions regarding the evolution of \( M_{\ast} \) (which should be less affected by systematic errors associated with \( M_{h} \)) but not \( M_{\ast}^{\text{piv}} \) or \( (M_{\ast}/M_{h})^{\text{piv}} \).

5.5.4. Comparison with Behroozi et al.

The closest comparison with our work is B10 since we employ the same functional form for the SHMR, the same halo mass function from Tinker et al. (2008), and we both account for the effect of scatter in the SHMR and for sample variance in the data using mock catalogs.

B10 derive constraints on the redshift evolution of the SHMR by abundance matching to the SDSS SMF of Li & White (2009) at low redshift and to mass functions from the FIDEL Legacy Project in the extended Groth strip at higher redshifts (Pérez-González et al. 2008). As a result of the fact that B10 combine data from distinct surveys, their systematic uncertainties on the evolution of the SHMR are fairly large. We also note that there are differences between the stellar mass estimates used in B10 and in this paper (see Section 5.6) which will lead to normalization differences in \( M_{\ast}^{\text{piv}} \) and \( (M_{\ast}/M_{h})^{\text{piv}} \) for example.

Another difference between B10 and our work is the treatment of satellite galaxies. In our model, the SHMR only applies to central galaxies and satellites are modeled via \( (N_{\text{sat}}) \). Indeed, we require a more sophisticated treatment of satellites in order to fit the clustering and the g–g lensing for which the satellite term plays a larger role than in the SMF (the satellite term is subdominant at all scales for the SMF). In contrast, B10 assume that the SHMR applies also to satellite galaxies, on condition that the “halo mass” for satellite galaxies is defined as the halo mass at the epoch when satellites were accreted onto their parent halos (the “infall mass,” \( M_{\text{infall}} \)). Thus, there could be subtle differences between the two methods due to the treatment of satellite galaxies (for example, see discussion in Neistein et al. 2011).

Figure 12 (green diamonds) shows the prediction from B10 for the pivot quantities. Our results are in striking agreement with B10 with respect to the evolution of \( M_{h}^{\text{piv}} \). The errors from B10 are larger for \( M_{\ast}^{\text{piv}} \) and \( (M_{\ast}/M_{h})^{\text{piv}} \). Thus our results agree with B10 in terms of qualitative evolutionary trends for \( M_{\ast}^{\text{piv}} \) and \( (M_{\ast}/M_{h})^{\text{piv}} \). There is a normalization difference for \( M_{\ast}^{\text{piv}} \) between B10 and our results. However, this normalization offset is not unexpected given systematic differences due to varying assumptions for stellar mass estimates. We address this issue further below.

5.5.5. Testing the Redshift Evolution of the Pivot Masses

Given the striking downsizing signal that we detect for the pivot masses, we would like to check that this result is not an artifact of the method we are using. As such we have also re-analyzed the three COSMOS SMFs using the abundance matching method of B10. In this test, the method followed here is identical to that of B10 but with different constraints on the SMF. Specifically, we use the three cosmos SMFs to constrain the redshift evolution of the SHMR and do not include any SDSS data. The results are shown in Figure 13. We find that the downsizing signal for \( M_{h}^{\text{piv}} \) and \( M_{\ast}^{\text{piv}} \) is clearly detected in the COSMOS data using the methods of B10. This provides an independent test on our detected evolution of the pivot quantities and suggests that the detected downsizing signal is robust to the methodology that is employed.

This test also raises the interesting question of how the method used in this paper (an HOD-based model that includes fits to clustering and g–g lensing) compares to the method of B10 (abundance matching using only the SMF). As can be seen in Figure 13, we find that the two methods yield very similar results for the pivot quantities. We do however find subtle differences
in the actual SHMRs between the two methods. Tracking down the cause of the exact differences, although a very interesting question in itself, is beyond the scope of this paper and we defer this study to follow-up work. For the purposes of this paper, we will simply emphasize that the analysis of B10 applied to the COSMOS results fully agrees with our claims concerning the evolution of the pivot quantities. We conclude that the detected downsizing behavior of $M^\text{pivot}$ and $M^\text{pivot}_h$ is robust to the methodology that is employed.

5.6. The Role of the Stellar Mass Function

In our analysis, the errors on the SMF are small compared to the clustering and the lensing. It is always the case that a measurement of a one-point statistic from a given set of data is more precise than a measurement of a two-point (or higher) statistic. Thus, the SMF plays an important role in constraining our parameter set. Therefore, we investigate the SMF in further detail in this section, and in particular, we show a more in-depth comparison with SDSS mass functions.

Figure 14 shows the COSMOS mass functions compared to various SDSS mass functions that have been commonly employed in the literature (Panter et al. 2007; Baldry et al. 2008; Li & White 2009). The main difference that we may expect between the COSMOS mass functions and the SDSS ones (besides sample variance and systematic error) is that the high end of the mass function may be inflated due to a larger value of $\sigma_{\log M_\star}$ in COSMOS. To gauge how much the COSMOS mass functions are affected by Eddington bias compared to SDSS, we use our model to predict the COSMOS mass functions, convolved to the expected scatter for SDSS ($\sigma_{\log M_\star} \sim 0.17$ dex). The results are shown in the right-hand panel of Figure 14. We find that the difference in scatter is not significant enough to explain the differences between the COSMOS and SDSS mass functions. It is more likely that the differences are due, for example, to varying assumptions regarding stellar population and dust models.

The difference between COSMOS and Li & White (2009) corresponds roughly “by eye” to a “left/right” shift along the $X$-axis ($\log_{10}(M^\text{pivot}) \sim \log_{10}(M^\text{pivot}_{\text{cosmos}}) - 0.2$). This difference is within the estimated systematic uncertainties (0.25 dex according to B10). However, this type of systematic shift will be reflected directly in the SHMR (Figures 10 and 11) by a “left/right” shift along the $X$-axis and will also affect the
normalization of $M_*^{\text{piv}}$ and $(M_h/M_*)^{\text{piv}}$. This $\sim 0.2$ dex shift would bring the normalization of B10 into closer agreement with our results. Tracking down the exact source of this systematic shift is beyond the scope of this paper but it could be associated with differences in the assumed dust model\textsuperscript{26} for example.

We conclude that in order to use the low-$z$ SDSS data as a $z \sim 0$ anchor to study the redshift evolution of the SHMR, a homogeneous analysis of both the SDSS and the COSMOS data is critical. This will be the focus of a future paper.

5.7. The Total Galaxy Stellar Content as a Function of Halo Mass

Figure 15 shows the conditional SMFs for various halo masses and redshifts from our best-fit model. We can use these functions to calculate the total amount of stellar material locked up in galaxies as a function of halo mass, noted hereafter $M_*^{\text{tot}}$ (see Equation (16) in Paper I). Investigating $M_*^{\text{tot}}$ is of interest because it reveals the efficiency with which dark matter halos accumulate stellar mass from the combined effects of in situ star formation and accretion via merging.

One might worry that calculating the total amount of stellar material locked up in satellite galaxies requires extrapolating our model beyond the lower and upper stellar mass bounds for which our model has been calibrated. As discussed in Section 5.5.1, the extrapolation of our model is in good agreement with results from Blanton et al. (2008) at low $M_*$ and with Hoekstra (2007) at high $M_*$. Thus, to first order, this extrapolation does not appear unreasonable. Let us therefore make the assumption that the extrapolation of our SHMR is not wildly incorrect. We will now investigate which mass range of satellite galaxies contributes most to $M_*^{\text{tot}}$.

From Figure 15, it is clear that satellite galaxies are a subdominant component of the total stellar mass at $M_h = 10^{12} M_\odot$.

\textsuperscript{26}Our stellar masses used the Charlot & Fall (2000) dust model whereas Li & White (2009) use Blanton & Roweis (2007).
Our present concern is therefore only relevant for \( M_h > 10^{12} M_\odot \). Let us consider the total stellar mass associated with satellite galaxies as a function of \( M_h \) in a fixed stellar mass bin: \( M^\text{tot,sat}(M_h|M^1_s, M^2_s) \). As shown in Paper I, the expression for \( M^\text{tot,sat}(M_h|M^1_s, M^2_s) \) is given by

\[
M^\text{tot,sat}(M_h|M^1_s, M^2_s) = \int_{M^1_s}^{M^2_s} \Phi_s(M_s|M_h)M_s dM_s. \tag{20}
\]

We have tested how \( M^\text{tot,sat}(M_h|M^1_s, M^2_s) \) varies with the integral limits, \( M^1_s \) and \( M^2_s \). We find that at fixed halo mass, most of the stellar mass associated with satellite galaxies arises from a relatively narrow range in stellar mass. In particular, for halos with \( M_h > 10^{11} M_\odot \), the bulk of \( M^\text{tot,sat} \) is built from satellite galaxies in the range \( 10^{10} M_\odot < M_s < 10^{11} M_\odot \). Therefore, provided that the extrapolation of our model is not wildly incorrect, the bulk of \( M^\text{tot,sat} \) arises from satellites that are within the tested limits of our model.

Having underlined this caveat, we have calculated \( M^\text{sat} \) using the best-fit parameters for each of the three redshift bins and the results are shown in Figure 16. This figure will be discussed in detail in the following section.

6. DISCUSSION

Using a self-consistent framework to simultaneously fit the g−g lensing, spatial clustering, and number densities of galaxies in COSMOS, we have obtained a robust characterization of the evolving relationship between stellar mass and halo mass over two orders of magnitude in \( M_h \). The nature of this relationship, shown in Figure 16, is not only a byproduct of cosmic mass assembly but is also shaped by the physical processes that drive galaxy formation, ultimately providing valuable constraints on both. In this section, we begin by discussing various processes that shape the form of \( M^\text{sat}/M_h \) versus \( M_h \). We will then introduce a simple framework for interpreting evolution in this relation by considering the relative growth of stellar mass as compared to the growth of dark matter halos. Finally, we will discuss the observed evolution of the pivot quantities and we will show how a constant pivot ratio may imply that the mechanism responsible for the shutdown of star formation in massive galaxies may have a physical dependence on \( M^\ast/M_h \).

6.1. The Total Stellar Mass Content of Dark Matter Halos

Figure 16 separates the stellar content of the average dark matter halo into a contribution from the central galaxy and a contribution from the sum of satellite galaxies. Central galaxies show an \( M^\ast/M_h \) ratio that rises steeply to a maximum at \( M_h \sim 10^{12} M_\odot \) before decreasing somewhat more gradually in halos of higher mass. The fact that halos above this mass scale (at the redshifts considered) have cooling times longer than their dynamical times has been invoked by modelers for some time to help explain why cooling and star formation shut down at the highest masses, with some refinement due to the presence of so-called cold-mode accretion (Birnboim & Dekel 2003; Kereš et al. 2005; Birnboim et al. 2007; Cattaneo et al. 2006). We will return to the evolution of this mass scale at a later point in the discussion.

Central galaxies strongly dominate the total stellar mass content at \( M_h \lesssim 2 \times 10^{13} M_\odot \) (“the central dominated regime”), including at the peak mass, \( M_h \sim 10^{12} M_\odot \), while the stellar mass in satellites dominates at \( M_h \gtrsim 2 \times 10^{13} M_\odot \) (“the satellite dominated regime”).

The transition between the two regimes is driven by the steep decline in \( M^\text{sat}/M_h \) at \( M_h > 10^{12} M_\odot \). This decline occurs as the contribution from satellites begins to rise. One might then naturally ask if central galaxies in group-scale halos experience stunted growth simply because stellar mass is accumulating within the halo in the form of satellite galaxies, instead of merging onto the central galaxy. Figure 16 reveals that this is not the case. Indeed, the solid line in this figure demonstrates that the total stellar mass fraction of halos declines at \( M_h > 10^{12} M_\odot \).

Thus, even if all satellite galaxies were allowed to rapidly coalesce at the center of the potential well, the central galaxies of group-scale halos would still have lower \( M^\ast/M_h \) ratios than those in halos of \( M_h \sim 10^{12} M_\odot \). Thus, we conclude that dark
matter halos globally decline in the efficiency by which they accumulate stellar mass at \( M_h > 10^{12} M_\odot \).

We note that in massive halos, the intra-cluster light (ICL; not accounted for in this analysis) is estimated to contribute an additional 20%–30% of the total stellar mass (Feldmeier et al. 2004; Zibetti et al. 2005; Gonzalez et al. 2005; Krick et al. 2006). Adding this to the satellite component in Figure 16 does little to bridge the factor of two to four gap in \( M_\ast^{\text{tot}}/M_h \) between the satellite component in high-mass halos and centrals at the peak mass.

6.2. The Role of Galaxy Mergers in Determining the Shape of \( M_\ast^{\text{tot}}/M_h \)

The majority of the total mass in an average dark matter halo is built from halo–halo mergers with mass ratios above 1:10 (e.g., Hopkins et al. 2010a). At halo masses below \( 10^{12} M_\odot \), the steep rise in \( M_\ast^{\text{tot}}/M_h \) with \( M_h \) implies that the typical stellar mass ratio of galaxy mergers will be less than the typical mass ratio of the dark matter halos hosting these galaxies. In other words, major halo mergers are minor galaxy mergers in this regime. Thus, the accumulation of stellar mass through the effects of merging will be limited compared to the growth in total mass of such halos. The steep rise of \( M_\ast^{\text{tot}}/M_h \) must therefore reflect the greater importance of star formation at masses below \( M_h \sim 10^{12} M_\odot \) over assembly from galaxy mergers (Bundy et al. 2009). Similar conclusions have also been reached by Conroy & Wechsler (2009) (see their Figures 2 and 3 in particular).

Simple arguments suggest, however, that once halos grow past the pivot mass and in the absence of significant star formation, \( M_\ast^{\text{tot}}/M_h \) should dip below the peak value since these halos can only grow by merging with halos with lower values of \( M_\ast^{\text{tot}}/M_h \). At slightly higher mass, the decline in \( M_\ast^{\text{tot}}/M_h \) now means that stellar mass ratios are enhanced with respect to halo mass ratios, and the trend must reverse again. This repeating pattern should cause a flattening of \( M_\ast^{\text{tot}}/M_h \) above the pivot mass. While this behavior is certainly apparent in Figure 16, our different redshift bins also reveal that at fixed mass among high-mass halos (\( M_h > 4 \times 10^{13} \)), the total stellar mass content declines at later epochs. We speculate that this trend could arise from the smooth accretion of dark matter, which brings no new stellar mass, and amounts to as much as 40% of the growth of dark matter halos (e.g., Fakhouri & Ma 2010). One way to test this hypothesis would be to populate a \( z = 0.88 \) N-body simulation with our \( z_3 \) HOD and evolve the subhalo and halo populations to \( z = 0 \), assuming no star formation. This would reveal the amount of stars that are acquired through mergers in this redshift range (Zentner et al. 2005). We note that the destruction of satellites and a growing ICL component could also contribute to this trend given that the ICL at \( z = 0 \) could make up 20%–30% of the stellar content of massive halos, roughly the amount by which \( M_\ast^{\text{tot}}/M_h \) declines over our redshift range.

6.3. The Pivot Quantities and the Quenching of Star Formation in Central Galaxies

The location at which halos reach their maximum accumulated stellar mass efficiency is encoded by the pivot mass quantities, \( M_\ast^{\text{tot}}, M_\ast^{\text{crit}}, \) and \( M_h/M_\ast^{\text{crit}} \). While we observe downsizing trends for both \( M_\ast^{\text{tot}} \) and \( M_\ast^{\text{crit}} \), the co-evolution of these two parameters leaves \( M_h/M_\ast^{\text{crit}} \) constant with redshift. This can be seen in the right panel of Figure 16: the pivot ratio evolves very little over our redshift range, while the mass scale of the peak

![Figure 17](https://example.com/figure17.png)

Figure 17. Schematic illustration of our results in terms of galaxy mass assembly vs. halo growth. The magenta line represents our \( z_3 \) result and the dark blue line represents our \( z_1 \) result. From the relative positions of these two curves we can infer that from \( z_1 \) to \( z_3 \) and at \( M_h < 10^{12} M_\odot \), the stellar mass of the central galaxy has experienced a stronger growth in proportion to the growth of the dark matter. On the contrary, at \( M_h > 10^{12} M_\odot \), the stellar mass of the central galaxy has experienced a more mild growth in proportion to the dark matter.

(A color version of this figure is available in the online journal.)

(\( M_h^{\text{crit}} \)) does evolve downward by nearly a factor of two. Given the low satellite content of halos at these masses, merging is not likely to play a dominant role in driving growth in \( M_\ast \), below the pivot peak, arguing instead that the regulation of star formation is key to understanding this behavior. The physical process that sets the pivot peak at \( M_h = 10^{12} M_\odot \), and drives the subsequent decline in \( M_\ast^{\text{crit}}/M_h \) at higher halo masses, must be linked to the shutdown of star formation in central galaxies. In the reminder of this discussion, we will focus on interpreting the downsizing behavior of the pivot quantities in this context.

6.4. A Simple Model for Interpreting Evolution in \( M_\ast/M_h \) versus \( M_h \)

A complete and comprehensive interpretation of our results requires modeling and accounting for dark matter accretion histories, galaxy merger rates, and star formation rates (SFRs) as a function of redshift (for example, see Conroy & Wechsler 2009). Nonetheless, we will introduce some toy models based on simple arguments to provide a first interpretation of our results.

Our goal here is to evaluate our results in the context of other observations and theoretical work on galaxy formation models and to set the stage for a more detailed treatment in subsequent work. We begin with a general treatment of evolution in the SHMR and then will focus on applying this treatment to interpret the evolution we observe in the pivot quantities.

The physical basis for evolution in \( M_\ast/M_h \) versus \( M_h \) must be considered carefully because the stellar mass and associated halo mass of a galaxy can evolve independently, depending on the mass scale involved and on processes including merging, smooth (diffuse) dark matter accretion, star formation, and even tidal stripping. The sum of these processes on the growth of \( M_h \) and \( M_\ast \) shapes the behavior of \( M_\ast/M_h \) versus \( M_h \) in different ways, as illustrated by the schematic diagram in Figure 17. Here we let \( \eta_1 \) represent the ratio \( M_\ast/M_h \) at redshift \( z_{\text{high}} \) and \( \eta_0 \) represent the equivalent ratio at redshift \( z_{\text{low}} \) with \( z_{\text{high}} > z_{\text{low}} \). We further consider a dark matter halo of mass \( M_h \) that has
grown by a relative factor of $\lambda_{M_b}$ from $z_{\text{high}}$ to $z_{\text{low}}$. We can write that $M_b(z_{\text{high}}) = \lambda_{M_b} \times M_b(z_{\text{low}})$. Characterizing growth in the stellar mass of the central galaxy (although similar arguments apply to $M_{\text{tot}}$) by a factor of $\Lambda_{M_*}$, we can simply write that

$$\Lambda_{M_*} = \frac{\eta_0}{\eta_1} \times \lambda_{M_b}. \quad (21)$$

If $\eta_0 > \eta_1$, we infer that $\Lambda_{M_*} > \lambda_{M_b}$ and that the stellar mass has experienced a stronger relative amount of growth compared to that of the dark matter from $z_{\text{high}}$ to $z_{\text{low}}$. If on the contrary, $\eta_0 < \eta_1$ then the relative growth of the stellar mass is less than the dark matter. Note that this schematic view demonstrates that identical curves for $M_* / M_b$ versus $M_b$ at different redshifts do not necessarily indicate a lack of evolution, since $\Lambda_{M_*}$ is always greater than zero.

We now apply this intuitive framework to the evolution observed in Figure 16. Considering values of $\eta_1$ and $\eta_0$ applied to the total stellar mass curves in Figure 16, we see that below $M_b = 10^{12}$, the fractional growth in stellar mass outweighs the growth in halo mass. This reflects the greater importance of star formation at low masses over assembly from galaxy mergers, the same conclusion reached above by simply considering the shape of the SHMR. This evolutionary trend reverses above $M_b = 10^{12} M_\odot$, consistent with the notion that star formation is largely shut down in centrals above this mass.

### 6.5. Understanding the Evolution of the Pivot Quantities

We now apply these simple arguments to the evolution in the pivot quantities. We focus only on central galaxies, neglecting the minor contribution from satellites near the pivot mass. The aim here is to explore several simple models for the quenching of star formation and to investigate which models might reproduce the observed evolution of the pivot quantities, namely, a pivot halo and stellar mass that decrease at later epochs (downsizing) but leave the pivot ratio constant.

Shown schematically in Figure 18, we consider how our high-redshift ($z = 0.88$) SHMR relation would evolve toward lower redshifts under several prescriptions for stellar and halo growth. We begin with no assumptions about the SFR but adopt a halo growth rate ($\lambda_{M_b}$) that is roughly constant over the mass range spanned by the peak. This assumption is well justified by dark matter mass accretion rates derived from $N$-body simulations (Wechsler et al. 2002; McBride et al. 2009; Fakhouri et al. 2010). For example, Fakhouri et al. (2010) find that $M_b / M_h$ is only weakly dependent on halo mass with $M_b / M_h \propto M_h^{-1}$. We now consider several different quenching models and investigate their impact on the redshift evolution of the pivot quantities.

1. **No quenching model.** To begin with, we consider a model with no quenching of star formation and in which the stellar growth rate ($\lambda_{M_*}$) is constant over the halo mass range spanned by the peak (Row A in Figure 18). We consider the redshift evolution of the SHMR for three values of $\lambda_{M_*}$ defined with respect to $\lambda_{M_b}$. In all three cases, this model leads to an increase in $M_{\text{th}}^{piv}$ with time (contrary to what we observe). We can therefore conclude that $\lambda_{M_*}$ must vary with $M_h$, not a surprise given the expectation that the SFR shuts down above $M_h = 10^{12} M_\odot$.

2. **Fixed halo mass for quenching.** We next consider a model in Row B in which star formation is quenched as galaxies cross a fixed halo mass, $M_h$. However, this instantaneous quenching model fails to reproduce (not surprisingly) the downward evolution of $M_{\text{th}}^{piv}$ and also yields evolution in the pivot ratio, which is not detected.

3. **Redshift-dependent halo mass for quenching.** Row C shows a model in which $M_b$ shifts downward with time. This model leads to downsizing in the pivot halo mass if $\lambda_{M_*} > \lambda_{M_b}$ below $M_b$ (Row C, middle panel). However, in order to keep the pivot ratio fixed in this scenario, the growth rate, $\lambda_{M_*}$, must be tuned with respect to the rate at which $M_b$ declines. This would require a fortuitous coincidence, but obviously cannot be dismissed as an explanation.

4. **Critical $M_*/M_b$ ratio for quenching.** We finally consider an alternative scenario in which star formation is limited by a critical mass ratio, $\eta_{\text{crit}} \equiv M_*/M_b \approx 0.04$ (Row E). In this model, we can qualitatively reproduce our main results, including the downsizing trends in the pivot stellar and halo mass, and, by construction, a constant pivot ratio set by $\eta_{\text{crit}}$ (compare the middle panel of Row D to the middle panel of Row E). However, in order to produce downsizing behavior in this model, $\lambda_{M_*}$ must be larger than $\lambda_{M_b}$ below $\eta_{\text{crit}}$. It is important to note that if $\lambda_{M_*} \leq \lambda_{M_b}$ below $\eta_{\text{crit}}$, this model would fail to produce downsizing.

The model explored in Row E seems a promising and simple mechanism that can explain the observed evolution of the pivot quantities. We now test if observations are consistent with the requirement that $\lambda_{M_*} > \lambda_{M_b}$ below $M_b \sim 10^{12} M_\odot$. A halo of mass $M_b \sim 2 \times 10^{11} M_\odot$ grows by a factor of $\lambda_{M_b} \sim 1.4$ from $z = 0.88$ to $z = 0.37$ (Fakhouri et al. 2010). SFRs as a function of $M_b$ and redshift have recently been measured by Noeske et al. (2007), Cowie & Barger (2008), and Gilbank et al. (2010). These measurements indicate that galaxies of mass $M_* \sim 10^{10} M_\odot$ grow by roughly a factor of $\lambda_{M_*} = 2$–3 from $z = 0.88$ to $z = 0.37$. Therefore, at $z < 1$ and for $M_b < M_{\text{th}}^{piv}$, current estimates for halo growth coupled with estimates for stellar growth are indeed consistent with our observation that stellar mass has experienced a stronger relative amount of growth compared to that of the dark matter ($\lambda_{M_*} < \lambda_{M_b}$ below $\eta_{\text{crit}}$).

We further note that the relatively weak dependence of $\lambda_{M_*}$ on $M_b$ required to maintain a constant slope in $M_* / M_b$ versus $M_b$ with redshift (again for $M_b < M_{\text{th}}^{piv}$) is implied by the weak SFR–$M_b$ relation observed for galaxies with $M_* < M_{\text{th}}^{piv}$ (a typical power-law fit gives SSFR $\sim M_b^{0.4}$). This could explain why $\beta$ (the low-mass slope of the SHMR) is observed to remain remarkably constant at $\beta = 0.46$ at $z < 1$.

Given the simple arguments outlined above, we argue that the fact that the pivot ratio remains constant may suggest that star formation is fundamentally limited by a critical mass ratio, $\eta_{\text{crit}} \equiv M_*/M_b \approx 0.04$. A very elegant and compelling consequence of this model is that the observed downsizing trends in $M_{\text{th}}^{piv}$ and $M_h$ can be automatically explained given the observation that $\lambda_{M_*} > \lambda_{M_b}$ below $M_b \sim 10^{12} M_\odot$. Previous work by Bundy et al. (2006) on the evolution of the galaxy SMF has found a similar downsizing trend for the “transition mass,” $M_{\text{tr}}$, which is defined in terms of $M_*$ by the declining fraction of star-forming galaxies at the highest masses. The transition mass from Bundy et al. (2006) evolves from $M_{\text{tr}} \sim 9 \times 10^{10} M_\odot$ at $z \sim 0.9$ to $M_{\text{tr}} \sim 5 \times 10^{10} M_\odot$ at $z \sim 0.6$. This transition mass is similar to $M_{\text{th}}^{piv}$, reinforcing the notion that the pivot mass marks the end of rapid star formation among halos. The obvious difference between Bundy et al. (2006) and this paper is that our work adds a key missing ingredient, which is the evolution of the pivot halo mass and the pivot ratio. If the quenching of star formation depends on a critical $M_*/M_b$ ratio then the fact that
Figure 18. Schematic (and simplistic picture) of how $M_*/M_h$ (for central galaxies) varies with redshift for different quenching models and for various prescriptions for stellar growth and halo growth (parameterized here by $\lambda_M$ and $\lambda_M^*$, respectively). Our $z=0.88$ relation is represented by the solid red line. In this picture, we are not interested in understanding why the high-redshift relation has the particular form that is observed, but simply in predicting roughly what the evolution of this relation should look like given various quenching models. The orange dashed line and the blue dotted line show how we expect the SHMR to evolve with time. In Row A, we consider a model without any quenching and where $\lambda_M$ and $\lambda_M^*$ are constant with redshift and halo mass. In this model, the pivot halo mass will increase at later epochs which is not what we observe. In Row B, we consider a model in which star formation is quenched above a fixed halo mass, $M_q$. This model fails to produce downsizing. In Row C, we consider a model where $M_q$ decreases at lower redshifts. In this case when $\lambda_M > \lambda_M^*$ below $M_q$, our results. We find that (Mq) above which star formation is quenched. The orange dashed line and the blue dotted line show how we expect the SHMR to evolve with time. In Row E, instead of assuming that star formation is quenched at a fixed halo mass, we now assume that star formation is quenched at a fixed critical $M_*/M_h$ ratio. In this scenario, a pivot ratio that is constant with redshift requires a fine tuning between $\eta_{\text{crit}}$ and the rate at which $M_q$ declines. In Row E, instead of assuming that star formation is quenched at a fixed halo mass, we now assume that star formation is quenched at a fixed critical $M_*/M_h$ ratio. Given this assumption and if $\lambda_M > \lambda_M^*$ below $\eta_{\text{crit}}$ (a reasonable assumption given estimates for halo growth and star formation at these scales), we can qualitatively reproduce our main results (compare the middle panel of Row D to the middle panel of Row E).
low-mass galaxies grow more rapidly than dark matter below the pivot scale provides a simple explanation for the observed downsizing in the sites of star formation observed by studies such as Bundy et al. (2006).

6.6. Physical Mechanisms that might Depend on $M_*/M_h$

In the previous section, we demonstrated that a constant pivot ratio provides important clues concerning the physical mechanisms that quench star formation. We now discuss possible mechanisms that might tie quenching to $M_*/M_h$.

The notion of a fixed maximum stellar-to-dark matter ratio, $\eta_{\text{crit}}$, has been relatively unexplored in the literature. Theoretical arguments tend to favor a relatively fixed (if broad) critical halo mass (see Birnboim & Dekel 2003) at $z \lesssim 1$, with significant modifications from cold-mode accretion occurring mostly at higher redshifts. But, quenching at fixed halo mass alone is not sufficient to reproduce the local SMF and red-sequence fraction, and also fails from simple arguments to produce downsizing in the pivot masses, as shown by Row B of Figure 18. As a result, most semi-analytic models include a quenching channel initiated by mergers or disk instabilities among galaxies in halos below the critical halo mass threshold (e.g., Bower et al. 2006; Croton et al. 2006; Cattaneo et al. 2006). Above the critical halo mass, once a quasi-static halo of hot gas has formed, low-luminosity feedback (i.e., radio-mode AGN feedback) is often invoked to prevent cooling and star formation in massive halos at late times. Using simple arguments, we have shown that mergers do not provide a likely explanation for determining the scale of the pivot mass. We therefore focus on AGN feedback and disk instabilities as possible mechanisms.

It is common practice to consider low-luminosity AGN feedback only in halos above a fixed halo mass. However, in practice, efficient AGN feedback is more complex and requires at least two ingredients. First, a quasi-static halo of hot gas must exist to which the AGN jets can couple. However, AGN feedback also requires a sufficiently large black hole to produce jets powerful enough to initiate this coupling. Given that gas cooling rates scale roughly with halo mass and that black hole mass scales roughly with galaxy mass, via the $M_\bullet - \sigma$ relation (Gebhardt et al. 2000; Ferrarese & Merritt 2000), it is reasonable to speculate that AGN feedback efficiency would depend on $M_*/M_h$.

Considering the galaxy population broadly, this scenario requires a sufficiently large bulge component for AGN quenching to be effective. While bulges may be built stochastically in galaxy mergers, a further link may tie secular bulge formation via disk instabilities to the value of $M_*/M_h$, thereby cementing the relationship between quenching and $\eta_{\text{crit}}$. It has been shown that disks become unstable to bar modes if the disk mass dominates the gravitational potential (Efstathiou et al. 1982; Mo et al. 1998). Semi-analytic models typically consider that disk instabilities occur if

$$V_{\text{max}}/(G N M_{\text{disk}}/r_{\text{disk}})^{0.5} \lesssim 1,$$

where $M_{\text{disk}}$ represents disk mass, $r_{\text{disk}}$ is the disk radius, and $V_{\text{max}}$ is the maximum of the rotation curve. Depending on the implementation of this criterion, $V_{\text{max}}$ may be equated either to the halo virial velocity or the disk velocity at its half-mass radius (see discussion in Parry et al. 2009). An instability will cause either a partial or a total collapse of the disk, leading to a burst of star formation at the center, the formation of a spheroid, and also possibly fueling the central black hole. The disk instability criterion in Equation (22) shows a dependence on ($M_h/M_{\text{disk}}$) (relating $V_{\text{max}}$ to $M_h$) which could be reflected in $M_h/M_*$ as the instability converts cold gas into stars. Disk instabilities might therefore play a role in setting the pivot masses and enforcing $\eta_{\text{crit}}$. The coincident fueling of the central black hole during the instability may help initiate AGN quenching and regulate the global decline in $M_{*\text{vir}}/M_h$ beyond the pivot mass.

Assuming the disk instability framework, we can test whether the predicted sizes of stable galactic disks given our critical pivot ratio, $\eta_{\text{crit}} \equiv M_*/M_h \approx 0.04$, are consistent with observations. Approximating the halo virial velocity for $V_{\text{max}}$, we rewrite Equation (22) to derive the maximum size of stable disks:

$$r_{\text{disk}} < R_h \times (M_*/M_h).$$

Applying this criterion to the pivot masses yields the condition that $r_{\text{disk}} < 8$ kpc. Interestingly, this condition is very well satisfied by the observed size distributions of disk galaxies which tend to fall off rapidly just below $r_{\text{disk}} = 8$ kpc at both $z \approx 0$ (see Figure 11 in Shen et al. 2003) and at $z > 0$ (see Figure 10 in Sargent et al. 2007). We conclude that, via the growth of bulges and initialization of AGN feedback, disk instabilities provide a promising link between our observed critical pivot ratio and quenching, although further investigation is clearly needed.

Finally, we note that $\eta_{\text{crit}}$ might also be related to the competition between the cooling and accretion of cold gas in central starbursts and the resulting feedback from either star formation itself or a co-evolving quasar-mode AGN. This Eddington-like limit has been explored in the context of stellar systems by Hopkins et al. (2010b) who derive a maximum stellar surface density from simple arguments. In the context of dark matter halos explored here, similar arguments might naturally yield a fixed value for $r_{\text{crit}}$ similar to that obtained by our analysis.

7. CONCLUSIONS

The aim of this paper is to study the form and evolution of the SHMR from $z = 0.2$ to $z = 1.0$. To achieve this goal, we have performed a joint analysis of galaxy–galaxy lensing, spatial clustering, and number densities of galaxies in COSMOS. As a result, we have obtained a robust characterization of the evolving relationship between stellar mass and halo mass over two to three orders of magnitude in $M_*$. The nature of this relationship is not only a byproduct of cosmic mass assembly but is also shaped by the physical processes that drive galaxy formation, ultimately providing valuable constraints on both. A complete and comprehensive interpretation of our results requires modeling and accounting for dark matter accretion histories, galaxy merger rates, and SFRs as a function of redshift. This will be the focus of future work. Nonetheless, we show how simple evolutionary models can already provide a first interpretation of our results, setting the stage for a more detailed treatment in future work. Using simple arguments, we evaluate our results in the context of other observations and theoretical work on galaxy formation models.

We have defined the pivot quantities ($M_*^{\text{pivot}}$, $M_h^{\text{pivot}}$, and ($M_h/M_*$)pivot) as the location at which halos reach their maximum accumulated stellar mass efficiency. The evolution of the pivot quantities contains key clues about the physical processes that are responsible for the quenching of star formation in halos above $M_h > 10^{12} M_\odot$. While we observe downsizing trends for both $M_*^{\text{pivot}}$ and $M_h^{\text{pivot}}$, the co-evolution of these two parameters leaves ($M_h/M_*$)pivot roughly constant with redshift. We argue that
this result raises the intriguing possibility that the quenching of star formation may have a physical dependence on $M_h/M_\star$ and not simply on $M_\star$ as is commonly assumed. If the quenching of star formation indeed depends on a critical $M_\star/M_h$ ratio then the fact that low-mass galaxies grow more rapidly than dark matter below the pivot scale provides a simple explanation for observations of downsizing in the sites of star formation. Additional and more precise measurements of the pivot quantities would be highly interesting in order to confirm whether or not the pivot ratio remains constant, or if instead it evolves mildly with redshift. Interestingly, there are hints from the results of Behroozi et al. (2010) that the pivot mass might in fact remain constant back to $z = 4$.

We highlight four avenues of exploration that would be interesting to pursue in the future and that would improve this analysis. First, we note that the comparison of our results with SDSS abundance matching results (e.g., Behroozi et al. 2010) is limited by systematic differences between stellar mass estimates. In order to use the SDSS data set as a low-$z$ anchor point, a homogeneous analysis of SDSS and COSMOS would be necessary. Second, our work in COSMOS is limited at the high-mass end by sample variance. Larger data sets at higher redshifts than SDSS such as the Baryon Oscillation Spectroscopic Survey (BOSS)27 and the Canada–France– Hawai‘i Telescope Legacy Survey (CFHTLS)28 should provide interesting constraints on the evolution of the high-mass end of the SHMR. Third, pushing the SHMR down to $10^8 M_\odot < M_\star < 10^9 M_\odot$ using techniques such as described in this paper is clearly an exciting avenue to explore and will be facilitated by upcoming very deep optical and IR surveys such as UltraVista and the HSC survey on the Subaru telescope. Finally, we note that the SMF is a powerful tool for placing constraints on the SHMR. However, the derivation of SMFs is clearly currently limited by systematic uncertainties in stellar mass estimates. Working toward an improved understanding of stellar mass estimates and toward reducing systematic errors in the SMF will be the single most important avenue for improving the type of analysis presented in this paper.

Finally, while our analysis demonstrates that the combination of multiple and complementary dark matter probes is a powerful tool with which to elucidate the galaxy–dark matter connection, we emphasize that such probe combinations also hold great potential to constrain fundamental physics, including the cosmological model and the nature of gravity. Exploring the sensitivity of the combination of $g$–$g$ lensing, clustering, and the SMF to cosmological parameters will be the focus of a follow-up paper.

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REFERENCES


27 http://cosmology.lbl.gov/BOSS/
28 http://www.cfht.hawaii.edu/Science/CFHTLS/