Student interactions and the development of ideas in multi-touch and paper-based collaborative mathematical problem solving

Emma Mercier,
University of Illinois at Urbana Champaign,

Georgia Vourloumi
Steven Higgins
Durham University

Author Notes:
Emma Mercier is Assistant Professor in the Department of Curriculum and Instruction at the University of Illinois at Urbana-Champaign. Her work focuses on the development and implementation of technology for collaborative learning in classrooms. The work represented in this paper was conducted as part of the TEL research group at Durham University.

Georgia Vourloumi is a doctoral student in the School of Education at Durham University. Her research interests are in the development of technological fluency across home and school contexts.

Steve Higgins is a Professor of Education at Durham University. His main research interests are in the pedagogy of educational technology in schools and the use of evidence in education.

Address for correspondence: Department of Curriculum and Instruction, University of Illinois at Urbana-Champaign, 1310 S. 6th St., Champaign, Illinois, 61820.
Email: mercier@illinois.edu
Student interaction and the development of ideas in multi-touch and paper-based collaborative mathematical problem solving

Abstract
Multi-touch technology is increasingly being used to support collaborative learning activities. However, to understand how this technology can be used most effectively, we need to understand if collaborative interactions differ when groups are using multi-touch technology compared with other tools. In this paper, we compare the interactions of groups of 10-11 year old students working collaboratively to solve three maths problems in either a multi-touch or paper condition. The number of ideas raised, who proposed them and whether they were responded to, were coded to identify differences in idea development and interactions across conditions. Responses by students, to ideas proposed by other students, were coded. Results indicate that similar numbers of ideas were raised across conditions; student responses to ideas raised by other students were more likely to elaborate on the idea or combine it with other ideas in the multi-touch condition than in the paper condition. These results reinforce prior findings that show higher levels of collaborative engagement around ideas when using multi-touch than paper and extend our understanding of how this occurs.

Keywords: Collaborative learning; CSCL; Multi-touch technology; primary education; mathematics
Student interaction and the development of ideas in multi-touch or paper-based collaborative mathematical problem solving

Multi-touch surfaces, that allow multiple simultaneous touches on a screen, are increasingly being used to support collaborative learning. This technology has the potential to alter co-located computer supported collaborative learning activities, by changing the nature of interaction with the technology, and therefore, the interaction among group members (Dillenbourg & Evans, 2011; Higgins et al., 2011). However, there is still limited research on how collaborative interactions change when using this technology, particularly in comparison to collaboration using other tools such as paper or traditional computers. In this paper, we report on the second part of a study of students collaborating on either multi-touch tables or using traditional paper-based tasks. Video analysis of groups is used to examine differences in interactions in groups. In the first part of this study, published in this journal, volume 43, issue 6, results indicated that students working on a history task in the multi-touch condition engaged in more interactive talk, when compared to students using the paper materials (Higgins, Mercier, Burd, & Joyce-Gibbons, 2012). In this paper, we present data of the same students working on mathematics tasks in the opposite condition (e.g. if they worked on a multi-touch table for the history task, they worked with paper for the mathematics task). Due to the differences in the task demands, in this paper we focus on the differences in idea development between the two conditions, in order to further add to our understanding of how interactions differ when using this technology.

Collaborative interactions and mathematical problem solving
Research on collaborative learning indicates that collaboration can be a useful tool for learning and problem solving. In particular, research in mathematics indicates that collaborative learning and CSCL can be an important pedagogic tool (Esmonde, 2009; Mercier & Higgins, 2013). However, there is often considerable variability in group outcomes, with some groups solving the problem and showing learning gains, while others struggle to complete the task or learn from the process (O’Donnell, 2006). Some of this variability can be explained in terms of the way group members interact over the ideas that they are working on. In a study of students working on mathematics problem solving, Barron (2003) found that although there was variability in the outcomes and learning of groups in her
study, similar numbers of correct ideas were present in all groups. By examining differences in what happened in groups when ideas were introduced, she found that successful groups discussed proposed ideas, while less successful groups rejected or ignored them. Similarly, Webb, Troper and Fall (1995) reported that success in mathematics when students were working in collaborative groups was associated both with receiving explanations when help was sought and engaging in constructive activities, again indicating the importance of how students interact around ideas. One key concern for researchers and teachers is whether technology can be used to support interactions in such a way as to lead to better learning for all students engaged in collaborative learning.

Multi-touch technology and collaborative learning
Large multi-touch surfaces can change the way we interact with computers, by allowing several users to touch the content directly and interact simultaneously, rather than having to interact through an input device such as a mouse or keyboard. Therefore, when working in a group around a multi-touch surface, there is no longer the need to negotiate who has access to the content through a single interaction point. Drawing on arguments that technology not only supports interaction practices, but has the potential to alter the way in which interaction occurs (e.g. Cole & Griffen, 1980; Lindgren & Pea, 2012) we argue that it is important to understand not only how this technology can be used for learning, but also how it changes interaction processes, by comparing it with activities which use more traditional tools. As research in this field is still developing, we are yet to have the full picture of how this technology influences collaborative interactions. This paper adds to this research, and specifically explores the development of ideas between groups using multi-touch and paper versions of the same task.

The existing research that compares multi-touch technology to other tools indicates that it increases the amount of collaborative engagement and on-task behaviour. Findings from a study that compared interactions when using a multi-touch table, or doing the same task on a single-touch surface, indicate that in the multi-touch condition, students engaged in more task-focused and less process-focused conversation (Harris et al, 2009). Research that contrasted pairs working on a multi-touch table and doing a similar task on a traditional personal computer (PC) shows that groups spent more time in shared working practices in the multi-touch condition, while groups in the PC condition spent more time with one person watching the work of the other (Basheri, Burd, Munro & Baghaei, 2013). Finally, our prior
work contrasting interactions with multi-touch tables and paper-based versions of a divergent history task, found higher levels of joint attention at the beginning of the task, and more interactive statements, which developed on the prior statements of their team-mates, in the multi-touch condition than in the paper-based condition (Higgins, Mercier, Burd & Joyce-Gibbons, 2012). We build on this work in the current paper, by drawing on data from the same study, contrasting interactions during mathematical problem solving in multi-touch or paper-based conditions.

The current study
In this study, we examine the way students interact over ideas in a mathematics task that was conducted in either a multi-touch or a paper-based environment. Drawing on qualitative methodology, and building on prior research in the field that identifies idea development as a key feature in successful learning (e.g. Barron, 2003; Roschelle 1992), we focus our analysis on students’ responses to ideas across conditions. The use of a coding scheme to identify different patterns of interaction has been used extensively to develop an understanding of the influence of different tools on collaborative learning (e.g. Gressick & Derry, 2010; Kapur & Kinzer, 2007; Meier, Spada, & Rummel, 2007) and is particularly appropriate for the early stages of research with new technology (e.g. Johnson& Onwuegbuzie, 2004; Strijbos & Fischer, 2007).

Method

Design
As the study was focused on interaction in different conditions (multi-touch or paper), a within-subjects design was used for the complete study, with groups working on the multi-touch table to complete one task in one domain, and then completing a similarly structured task on paper. In order to ensure ecological validity, students worked on a different activity during the first and second tasks. These activities used the same mystery structure, but by using different tasks we are able to explore the authentic problem solving collaboration across conditions (see table 1 for the study design). This paper draws on the data from the maths task; results from the history task are described in our earlier paper in this journal.
(Higgins et al., 2012) and will be compared in this paper to extend the findings presented earlier.

Due to the small sample size, a qualitative approach was taken to analyse the data, with a focus on understanding whether there were differences in the way the students interacted with each other across the two conditions.

---------------------

Insert Table 1 about here

---------------------

**Participants**

Sixteen male and sixteen female students in their final year of primary school (10 to 11 year olds) were recruited from two schools in England. The primary schools which participated tend to score at or just below average on national achievement tests.

Following approval from the School of Education’s ethics committee, members of the research team visited the schools, described the project to the students and led the students through some collaborative activities. Parental consent forms were distributed and teachers selected students from those who returned consent forms (return rates were high in both schools).

Groups of eight students came to the lab together (four male and four female students). Due to research that indicates that mixed gender groups can influence interaction patterns in both traditional and computer-supported collaborative learning environments (Prinsen, Volman, Terwel, & Vandeneeden, 2009; Underwood, Underwood, & Wood, 2000; Webb, 1984), students worked in same-gender groups of four for this investigation. At the start of the session, groups worked on one of two multi-touch tables, doing a range of activities to familiarise them with touching the surface and one history mystery to familiarise them with the structure of the task. One group then completed a history mystery on a multi-touch table, while the other group completed the task on paper in another room. After a short break, groups swapped and completed the maths tasks that are the focus of this paper in the opposite condition to the history task.

For each group, one of the two members of the research team worked with the groups both were former primary school teachers. The teachers intervened only when the groups became stuck, supporting the groups in completing the tasks, when necessary, so every group reached the correct answer. The teachers swapped conditions after each data collection session, completing both the maths and history activities in the same condition during each
session. Thus each group of students worked with both teachers (one in each condition), and each teacher completed two maths sessions in the multi-touch condition, and two maths in the paper condition.

**Maths mystery**
The groups completed three mathematical mysteries which were complex word problems with a single correct solution. Mysteries were designed to engage students in discussion and collaborative problem solving (Leat & Higgins, 2002). Three different maths mysteries were chosen for this study, each of which focused on different mathematical skills, in order to examine interactions across a range of problem solving activities. The first activity, Sneaky Sydney, focused on number knowledge, where the students had to reason through a series of clues about which hotel room a stolen statue had been hidden in. The second activity, Waltzer required the students to conduct a series of calculations to determine how much it would cost a waltzer owner to provide a free gift for every tenth person who used the waltzer in a day. A waltzer is a traditional fairground ride where people share one of multiple cars that spin individually while the platform rotates. The final activity, Dinner Disasters, was a logic problem in which fictional children had received the incorrect school dinner, and by reasoning through the children’s preferences, the groups had to work out what one child, Mike, should have to eat. An image of the clues is shown in Figure 2; the full text of the clues appears in Appendix A.

In the paper-based condition, each clue was presented on a separate piece of paper, while in the multi-touch condition, the clues were on digital ‘paper’, which could be moved and re-sized on the screen.

**Data**
Data was captured using two video cameras in each condition so that all members of the group could be seen. The audio was transcribed using a play-script layout and coding was conducted on the transcripts, while also viewing the video.

**Interaction coding**
Drawing on prior research on collaborative problem solving in maths (e.g. Barron, 2003), an emergent coding scheme was developed to categorise the different types of responses to ideas that were raised in the group. A three-stage coding scheme was created, so that in the first stage, the unit of analysis was identified as an idea and responses to that idea. Ideas could
either be clues that were read aloud to the group, drawing attention to a clue, or proposing a new strategy.

The second stage of coding, shown in table 2, aimed to identify who introduced the idea (student or teacher) and who responded to the idea (student, teacher or no one).

In the third stage of coding, the units of analysis that were identified as an idea that was introduced by a student and responded to by a student were coded for type of response. These were classified as commenting on importance of an idea, combining the idea with a different idea, and elaborating or expanding on an idea (see table 3). The codes were not mutually exclusive, with one to three codes being assigned to each unit of analysis.

Reliability of the three stages of the coding scheme was conducted on transcripts from six activities (one of each of the three activities from both conditions) which were coded by a primary and second coder. The primary coder identified 110 ideas across the six transcripts, while the second coder identified 96. In almost every case, the primary coder had broken ideas and responses into smaller units. As the primary coder was more familiar with the activities, the smaller units were used as the basis for further analysis. The reliability of identification of introducer and responder to ideas indicated good agreement between coders, with 85% agreement (Cohen’s Kappa = .78). Agreement on type of responses students made to student ideas was 90% (Cohen’s Kappa = .86).

Results

Task time
Task time was calculated for the entire length of the three activities, from the beginning of the start of the first activity until the end of the last one (including time to change activities). The task time was longer in the multi-touch condition (M = 16.37; SD = 1.82) than the paper condition (M=14.58, SD = 2.54), although the difference was not statistically significant, t(6), = -1.14, p = .297.

Initial strategies
The first thirty seconds of each activity were coded to determine how the groups dealt with the clues. The moves were classified both in terms of whether the clues were displayed for
the whole group to see, or were just viewed by an individual student, and whether they were read aloud or read silently. These were collapsed to create four categories (shared view, read aloud; shared view, read silently; individual view, read aloud; individual view, read silently). As there were no instances of the second category (individual view, read silently), it was dropped. In figure 3, the number of groups in each condition who used each of the three strategies is displayed.

Interaction Coding

Number of ideas
The total number of ideas proposed was calculated for each condition. Results indicated that there were very similar numbers of ideas proposed in the multi-touch (M = 54.5; SD = 5.06) and paper conditions (M = 54.75; SD = 6.45), indicating that the same number of ideas are raised and are available for discussion regardless of the condition.

Introducer of ideas and responder
Each idea was coded for who introduced the idea (student or teacher), and who responded to the idea (student, teacher, or no response). Across all activities, students introduced a mean of 43.25 of ideas in the multi-touch condition (SD = 8.18) and 41 ideas in the paper-based condition (SD = 1.41). Teachers introduced a mean of 11 ideas in the multi-touch condition (SD = 4.69) and 13.75 ideas in the paper-based condition (SD = 5.91). The mean number of ideas in each introducer and responder category, across all activities, are shown in Figure 4.

Student responses to student ideas
The types of responses that students made to the ideas that were raised by other students were classified as one of three types. The most basic response is the denoting of the importance of an idea or clue. Student could also respond to an idea by combining it with other ideas or clues (either already discussed, or new ideas) and by elaborating or expanding on the idea. These categories were not mutually exclusive, and there were times when a student could respond by noting the importance of another student’s idea, then going on to elaborate the
meaning of the idea or clue, and then combining it with other ideas. Figure 5 shows the differences between conditions, which indicates while students in both conditions responded by denoting importance at a similar frequency, students in the multi-touch condition responded by elaborating and combining ideas more often than students in the paper condition.

--------------------------
Insert Figure 5 about here
--------------------------

**Examples of interaction patterns**

An example of one group, working to solve the Waltzer activity in the multi-touch condition, is used to show the different ways the interaction behaviours support the group in coming to a solution (all names used are pseudonyms). This group was selected as it illustrates common features across many groups in the data set. The group began by making repeated short responses to each idea that is raised, denoting importance or elaborating on the meaning of clues. In the first vignette, Jessica draws attention to the clue she has just read, and then reads it aloud to the group, telling them that five people can sit on each of the waltzer cars. Emily responds by asking the group how many cars there are altogether in the waltzer, referring to a clue that had been read earlier, in a clear attempt to combine the ideas. Megan reads the clue for Emily, and then Emily calculates that there would be 50 people on the Waltzer, and therefore they would need 50 cuddly monkeys.

<table>
<thead>
<tr>
<th></th>
<th>There, that’s important [<em>reads clue aloud</em>] “Five people can sit together in a car”</th>
<th>Introduces idea</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jessica</td>
<td>And how many cars are there altogether</td>
<td>Responding with reference to a clue that has been read earlier (combining clues)</td>
</tr>
<tr>
<td>Emily</td>
<td>“There are ten cars altogether” [<em>reading</em>]</td>
<td>Responding with the details of the clue</td>
</tr>
<tr>
<td>Megan</td>
<td>So you could get fifty cuddly monkeys</td>
<td>Combining the clue that there are ten cars with the clue that five people can sit in a car.</td>
</tr>
</tbody>
</table>
The answer of 50 monkeys is incorrect, and the teacher asks the students if they are sure, which leads to them taking more time looking at the clues. The interactions between group members continue, with the teacher prompting them when necessary. Then, towards the end of the activity, as the group have worked out how many monkeys would be necessary for each hour, Megan draws the group’s attention to the clue that says the fair is open for eight hours a day.

<table>
<thead>
<tr>
<th>Megan</th>
<th>The fair is opened eight hours a day, look! So it’s two hundred and fifty times eight.</th>
<th>Introduces idea, pointing to the clue, and elaborating on the meaning of the idea</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chloe, Jessica &amp; Emily</td>
<td>Two hundred and fifty times eight</td>
<td>Responding by repeating Megan’s</td>
</tr>
<tr>
<td>Chloe</td>
<td>Calculator?</td>
<td>Responding by asking for a calculator (all laugh)</td>
</tr>
<tr>
<td>Jessica</td>
<td>Wey, no, it’s only two thousand, because two hundred and fifty is divided into a thousand four times so it’s two thousand.</td>
<td>Elaborating on the idea to show how she came to the solution.</td>
</tr>
</tbody>
</table>

Megan then goes on to prompt the group to finish the activity, which Chloe responds to, combining their last calculation with the clue about how much the monkeys cost (although her calculation is incorrect and the teacher steps in to help them finish the activity).

<table>
<thead>
<tr>
<th>Megan</th>
<th>So how much will it cost the owner for the cuddly monkeys for a day?</th>
<th>Introduces idea</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chloe</td>
<td>Two hundred… wait…. Two thousand, right, no, one thousand pounds</td>
<td>Responding to Megan by combining the idea of how many monkeys are needed with how much they cost.</td>
</tr>
</tbody>
</table>

**Discussion**

*Overview of findings*

The aim of this study was to examine the types of interaction and idea development in groups solving maths problems in either a multi-touch or paper-based condition, to further understand how using multi-touch technology might influence group interaction. Results indicated that task time was similar in both conditions, indicating a similar length of time to engage in the task across conditions. The initial strategies used by students differed across
conditions, with students in the multi-touch condition spreading the clues out for everyone to see, and then reading them aloud, in all but one activity, and students in the paper-based condition sharing out the clues, and reading them aloud to the group but not having an opportunity for joint reading. This indicates that the constraints of the material across conditions had an immediate influence on the way students interacted, replicating findings from our earlier work (Higgins et al, 2012).

Results also showed that a similar number of ideas were raised across conditions, again indicating that there were similar opportunities to engage in the task across conditions. This may be due to the fact that the mysteries could not be solved without most of the clues, and teachers were present to support students in noticing any clues or ideas to which they had not paid sufficient attention. Students raised slightly more ideas in the multi-touch condition than in the paper condition, while teachers raised slightly more ideas in the paper condition than in the multi-touch condition. As can be seen in figure 4, there were also more student responses to ideas raised by students in the multi-touch condition, and more student responses to teacher ideas in the paper condition. Taken together, they suggest more student-student interaction in the multi-touch condition, and more teacher-student interaction in the paper-based condition. This does indicate a different type of interaction between conditions, perhaps indicating that the joint attention facilitated by the multi-touch table, allowed for higher levels of task engagement and collaboration between students.

The way ideas were developed when students did respond to the ideas raised by other students provides more evidence for differences in interaction between conditions. While students in both conditions denoted importance of ideas equally, students in the multi-touch condition engaged in more combining of ideas and elaboration of ideas proposed by their peers than students in the paper-based condition. This suggests that students in the multi-touch condition were engaged in more complex collaborative engagement that students in the paper condition which prior research indicates is one predictor of group success (e.g. Barron, 2003). This finding is important as it indicates that the use of multi-touch tables that allow for more equitable participation also support the type of collaborative engagement that has been associated with better problem solving and learning. One reason for this increased engagement around the ideas may be the joint attention that is seen in the initial strategies, and continues through the task in the multi-touch condition, with students engaging in discussion about the ideas from the beginning of the activity, as they viewed and read them together, setting them on the path for more complex discussion.
Comparison with prior research

While the findings reported in this paper are from a small data-set, they are consistent with the findings we reported previously (Higgins et al., 2012) that groups in the multi-touch condition had higher levels of joint attention and made more interactive statements than groups in the paper-based condition. It should be noted, that the findings previously reported were drawn from the same sample, with the participants in the opposite condition (i.e. the students who used multi-touch in the data presented in this paper, were in the paper-based condition in Higgins et al., (2012)). Different coding schemes were used to examine the types of interactions due to the nature of the tasks. In the history task, students had to create an argument based on the clues. However there was no single correct answer, and the goal was for the students to engage in complex reasoning. Therefore, we examined how the students responded to ideas raised by their peers, by classifying the statements as independent, quasi-interactive, elaborating on the idea raised, or negotiating about the idea raised. These were then aligned with a coding scheme used to assess levels of reasoning. In contrast, all three activities were solved by each group during the maths tasks, either with or without the teachers’ help. Due to this, no alignment with the success of groups was explored in the maths data. In addition, a different coding scheme was used to assess the types of interactions, which built upon prior maths research (e.g. Barron, 2003), highlighting the importance of uptake and development of proposed ideas. Despite these differences, we see a similar pattern – with groups who completed the history in the multi-touch condition engaging in more elaborating and negotiating than groups in the paper condition, and groups who completed the maths tasks in the multi-touch condition responding to ideas by combining them or elaborating on them than groups in the paper condition. Due to the design of the study, groups completed one task (e.g. history) in one condition (e.g. multi-touch) and the second task (e.g. maths) in the opposite condition (e.g. paper). Thus when we look at the maths and history findings together, we are examining data from the same groups of students. The fact that in both parts of the study we found an increased amount of collaborative engagement in the multi-touch condition (as measured by interactive talk or idea development) suggests that more interactive types of interaction emerge from the use of the technology, rather than the group itself.

One measure that differed across the history and maths tasks was the time on task data. The history task took longer in the paper condition, and the maths tasks taking longer in the multi-touch condition. This may reflect that the particular students in those groups (who because of the study design, were the same students) took longer to read and understand the
tasks than the groups who completed the tasks in the opposite conditions. It may also reflect differences in task demands – with the increased joint attention in the multi-touch condition supporting quicker completion of the longer, more complex history task, an effect that was not evident in the shorter maths tasks.

Limitations and future directions
There are a number of limitations to this study, including the small sample size and use of different tasks across conditions. The small sample size leads us to be cautious about over-interpretation of the results, however, the design of the study and stability of findings across the history and maths activities in this study and the earlier study reported in this journal (Higgins et al. 2012) leads us to believe that the differences in interaction behaviours are associated with the use of the technology. The decision to use different tasks across conditions was made in an effort to increase the ecological validity of the study. While in some studies, students are asked to repeat the same task across conditions to compare different technologies, we believe that the repetition of a specific task, particularly a problem solving task, would lead to lower levels of engagement and therefore, influence the interaction behaviours.

Differences in interaction behaviours could be attributed to something other than the use of multi-touch technology in this study. The software itself, beyond the hardware that was being used, was designed to allow for enlarging and moving of the clues. Although this takes advantage of the affordances of the technology, it also indicates the importance of the design of the human-computer interaction experience to support the human-human interaction necessary for students to engage in this type of activities. Additionally, the fact that students stood in the multi-touch condition and sat in the paper condition, and the angle of the multi-touch table, may have influenced their interactions, and further studies should explore this issue.

This study contributes to our understanding of how multi-touch technology may influence the collaborative learning experience, with further evidence for the way this technology may support interactions early in the group process, leading to better idea development or more interactive statements. These studies did focus on one age group of children, and similarly structured tasks, with multi-touch tables built with specific affordances to support joint attention (e.g. enlarging clues), and further research is needed to explore a wider range of tasks. The small sample size across these studies also suggests caution in over-interpreting the findings, and further work with larger samples are necessary.
Additionally, four students working with a teacher is a very unusual learning environment, and future studies in this project will explore multiple groups of students and mixed gender groups using this technology in a classroom setting with several groups to examine whether the same types of interaction behaviours are associated with the multi-touch technology in a more typical formal learning environment.

Acknowledgements
The research described in this paper was funded through the UK’s Teaching and Learning Research Programme (TLRP) Technology Enhanced Learning (TEL) Phase 5, funded jointly by the ESRC and EPSRC, grant number RES-139-25-0400. Any opinions, findings and conclusions expressed in the paper are those of the authors and do not necessarily reflect the views of the sponsoring agencies. The authors wish to acknowledge the schools and students who participated in this study; all video and photographs were collected and reproduced with consent of the participants and their legal guardians. The authors also acknowledge the contribution of Andrew Joyce-Gibbons, Andrew Hatch and the SynergyNet project team in the development work presented here.

References


doi:10.1080/10508400902797958


doi: 10.1111/j.1467-8535.2011.01259.x

http://dx.doi.org/10.3102/0013189X033007014.


doi:10.1016/j.learninstruc.2012.10.004


Figure 1: Groups working in the multi-touch and paper-based conditions.

Figure 2: Mysteries on a multi-touch table
Figure 3: Initial strategies by condition
Figure 4: Mean number of ideas in each introducer and responder category for all tasks
Figure 5: Student responses to student ideas
Table 1: Study Design

<table>
<thead>
<tr>
<th>Group</th>
<th>First task: History (Higgins et al, 2012)</th>
<th>Second task: Maths (Mercier et al, under review)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1</td>
<td>Multi-touch</td>
<td>Paper</td>
</tr>
<tr>
<td>Group 2</td>
<td>Paper</td>
<td>Multi-touch</td>
</tr>
<tr>
<td>Group 3</td>
<td>Multi-touch</td>
<td>Paper</td>
</tr>
<tr>
<td>Group 4</td>
<td>Paper</td>
<td>Multi-touch</td>
</tr>
<tr>
<td>Group 5</td>
<td>Multi-touch</td>
<td>Paper</td>
</tr>
<tr>
<td>Group 6</td>
<td>Paper</td>
<td>Multi-touch</td>
</tr>
<tr>
<td>Group 7</td>
<td>Multi-touch</td>
<td>Paper</td>
</tr>
<tr>
<td>Group 8</td>
<td>Paper</td>
<td>Multi-touch</td>
</tr>
</tbody>
</table>
**Table 2: Phase two of the coding scheme**

<table>
<thead>
<tr>
<th>Code</th>
<th>Introducer of idea</th>
<th>Responder to idea</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student – Student</td>
<td>Student</td>
<td>One or more students</td>
</tr>
<tr>
<td>Student – Teacher</td>
<td>Student</td>
<td>Teacher</td>
</tr>
<tr>
<td>Student – Not developed</td>
<td>Student</td>
<td>No one</td>
</tr>
<tr>
<td>Teacher – Student</td>
<td>Teacher</td>
<td>One or more students</td>
</tr>
<tr>
<td>Teacher – Not developed</td>
<td>Teacher</td>
<td>No one</td>
</tr>
</tbody>
</table>

**Table 3: Phase three of the coding scheme**

<table>
<thead>
<tr>
<th>Code</th>
<th>Definition</th>
<th>Example</th>
</tr>
</thead>
</table>
| Importance               | Responses comment on importance of a clue, but do not elaborate.            | Jane: *The room where the statue is in is not even* [reading clue]  
Anna: *That’s important.*                                                                                      |
| Combining clues or ideas | Student combines clues or ideas to the introduced idea to build on it.     | Paul: *The room the statue is in is not less than fifty* [reading clue]  
Mark: *So it’s above twenty-five*[drawing on previous clue] and less than fifty [re-iterating the clue Paul read] |
| Elaborating              | Student expands or elaborates on an idea or clue. This includes making sense of the clues in context, or conducting appropriate calculations with the clues. | Susie: *The room the statue is in is a multiple of five* [reading clue]  
Sarah: *A multiple of five*  
Susie: *So the room must end in a five*[elaborating on her own idea]  
Julie: *Or a zero*[elaborating further] |
Appendix A: Mysteries

Sneaky Sydney.
Question: In which room is the statue hidden?
Clues:
- Sneaky Sydney has stolen a special stone statue.
- The room number the statue is in is not less than 25.
- The room number where the statue is hidden is not even.
- He has hidden it in a bedroom in the Grand Hotel.
- The Grand Hotel is next to the station.
- The room number where Sydney has hidden the statue is not 25.
- The Grand Hotel is not as expensive as the Caesar Hotel across the street.
- The room number does not contain the digit 3.
- There are 100 rooms in the Grand Hotel.
- The room number where it is hidden is a multiple of 5.
- The statue is in a room whose number is lower than 50.
- The statue is small, heavy and very valuable.

Waltzer.
Question: How much will it cost the Waltzer owner for enough cuddly monkeys for a day?
Clues:
- At the fair there is a Waltzer
- To get people to go on it the owner offers a prize
- Every 10\textsuperscript{th} person to go on will be given a cuddly toy monkey
- It costs the owner £2 to buy one monkey
- The monkeys look happy
- 3 people can sit together in a car
- There are 15 cars altogether on the Waltzer
- You must be at least 10 years old to ride the Waltzer
- There are 10 rides every hour
- The fair is open for 8 hours a day
- All the spaces are taken for every ride all day
- How much will it cost for the owner to buy one monkey for every 10\textsuperscript{th} person who goes on the Waltzer?

Dinner Disaster.
Question: Can you work out what Mike should have to eat?
Clues:
- The new cook at school, Mrs Baker, has mixed up the trays with the children’s school dinners on.
- "YUCK!" cried Ruby, making a face at the slice of pizza in front of her. "I can't stand pepperoni!"
- "Don't look at me," moaned Jack. "I hate any food with cheese on it." At that, he pushed away his cheeseburger.
- "Hey, anybody want these chicken wings?" asked Grace. "I don't like anything with meat in it."
- Mike scooped up a spoonful of his yogurt and grumbled, "Everybody knows I'm allergic to this stuff."
"Well, yogurt is the only thing I like on the menu," replied Tanya. "And there's no way I'm going to eat THIS!" At that, she poked her salad with a fork.