Probing the structure of $f_0(980)$ through radiative $\phi$ decays

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Abstract

We consider the radiative transition $\phi \to f_0\gamma$, which is a sensitive probe of the nature of the $f_0(980)$ particle. Using the QCD sum-rule technique, we estimate the branching ratio of such decay mode to be: $B(\phi \to f_0\gamma) = (2.7 \pm 1.1) \times 10^{-4}$, in fair agreement with present experimental data. As for the structure of the $f_0$, the result suggests a sizeable $s\bar{s}$ component; however, this result does not exclude the possibility of further components and allows a more complex structure than indicated by the naive quark model.
The quark model provides a rather good description of hadrons, which fit into suitable multiplets reasonably well. In its simplest version the model then interprets mesons as pure $q\bar{q}$ states. Scalar mesons present a remarkable exception to this successful scheme. Indeed, the nature of these mesons is not established yet [1]. There are more scalars than can fit into one quark model multiplet. Consequently, some of these states could be either glueballs or admixtures of quark and gluonic states, or belong to multiquark multiplets. A particular feature of some of these particles is that they appear to be rather wide [2, 3, 4, 5]. They have very short lifetimes and large couplings to hadronic channels, such as $K\bar{K}$ or $\pi\pi$. This might suggest that they can be identified as composite systems of hadrons, or that they spend an appreciable part of their lifetimes as such states. This could be the result of hadronic dressing, whereby the strong interaction enriches a $q\bar{q}$ state with other components such as $|K\bar{K}\rangle$, $|\pi\eta\rangle$, etc. Such a viewpoint could also explain why the scalar mesons seem to contradict the OZI rule. Since the two mesons composing the state in which they spend much of their lifetime may readily annihilate to $q\bar{q}$, leading to a subsequent OZI allowed decay.

In this letter, we focus on the structure of the $f_0(980)$ and the possibility of gleaning information about this from radiative $\phi$ decays. According to the quark model, the $f_0(980)$ should be an $s\bar{s}$ state, an interpretation supported in Refs. [6, 7, 8]. However, this does not explain its mass degeneracy with the $a_0(980)$, that should be a $(u\bar{u} - d\bar{d})/\sqrt{2}$ state. There are also suggestions that the $f_0(980)$ could be a four quark $qq\bar{q}\bar{q}$ state [9]. In this case, it could either be nucleon-like [10], i.e. a bound state of quarks with symbolic quark structure $s\bar{s}(u\bar{u} + d\bar{d})/\sqrt{2}$, or deuteron-like, i.e. a bound state of hadrons, which is usually referred to as a $K\bar{K}$ molecule [2, 11, 12, 13]. In the former of these two possibilities, the mesons are treated as point-like objects, while in the latter they should be considered as extended objects. Some objections have been raised against the $K\bar{K}$ molecular model [2, 14]. In particular, such an interpretation requires a width smaller than the binding energy of the molecule itself, which has been estimated to be $\epsilon \simeq 10 - 20$ MeV [2], in contrast to the measured width lying in the range $40 - 100$ MeV [13].

Various ways have been suggested of clarifying the situation, such as the analysis of the $f_0 \rightarrow \gamma\gamma$ decay [16, 17] or of the ratio $\frac{\Gamma(\phi \rightarrow a_0\gamma)}{\Gamma(\phi \rightarrow f_0\gamma)}$ [11]. In the naive quark model, for example, it is expected that $B(\phi \rightarrow f_0\gamma)$ and $B(\phi \rightarrow a_0\gamma)$ would differ by a factor of 10. Moreover the rate for $\phi \rightarrow f_0\gamma$ may distinguish among the different possibilities [11].

\footnote{Within the same framework the isovector partner $a_0(980)$ is written as $s\bar{s}(u\bar{u} + d\bar{d})/\sqrt{2}$.}
since, according to the existing theoretical estimates, the expected branching ratio would be as high as $10^{-4}$ in the $qq\bar{q}\bar{q}$ case, $\mathcal{O}(10^{-5})$ in the $s\bar{s}$ case. For a $K\bar{K}$ molecule, the branching ratio clearly depends on its size. For a compact state this is $\sim 7 \cdot 10^{-5}$, while for a diffuse, deuteron-like system, it is down below $10^{-5}$ \cite{11}.

From the experimental point of view, the PDG value \cite{15}:

$$B(\phi \to f_0 \gamma) = (3.4 \pm 0.4) \times 10^{-4}$$  \hspace{1cm} (1)

stems from averaging the results of the CMD2 \cite{18} and SND \cite{19} collaborations, analysing $\pi^+\pi^-\gamma$, $\pi^0\pi^0\gamma$ and $5\gamma$ final states. What is more, a significant improvement is expected at the $\phi$ factory DAΦNE \cite{20}, where the first results give:

$$B(\phi \to f_0 \gamma \to \pi^0\pi^0\gamma) = (0.81 \pm 0.09\text{(stat)} \pm 0.06\text{(syst)}) \times 10^{-4}$$  \hspace{1cm} (2)

and

$$B(\phi \to f_0 \gamma \to \pi^-\pi^+\gamma) < 1.64 \times 10^{-4}$$  \hspace{1cm} (3)

at 90\% C.L. \cite{21}.

The present letter is devoted to analysis of the radiative decay $\phi \to f_0 \gamma$ using QCD sum-rules \cite{22}, which we previously applied to the radiative $\phi$ transitions to $\eta, \eta'$ \cite{23}. That the $f_0(980)$ couples significantly through $s\bar{s}$ components has long been known \cite{1} from its appearance as a peak in $J/\psi \to \phi f_0$ \cite{25} and $D_s \to \pi f_0$ \cite{26}, as discussed in Refs. \cite{3}, and in more detail in \cite{27}. Our calculation relies on the assumed coupling of the $f_0$ to the scalar $s\bar{s}$ density. As a preliminary, we evaluate the strength of this coupling using two point QCD sum-rules. The result will then be exploited in the three point QCD sum-rule evaluation of the relevant quantity needed to compute $B(\phi \to f_0 \gamma)$.

The coupling of the $f_0(980)$ to the scalar current $J^s = \bar{s}s$ can be parametrized in terms of a constant $\tilde{f}$:

$$\langle 0 | J^s | f_0(p) \rangle = m_{f_0} \tilde{f}.$$  \hspace{1cm} (4)

In order to compute this parameter by QCD sum-rules, we consider the two-point correlator:

$$T(q^2) = i \int d^4xe^{iq\cdot x} \langle 0 | T[J^s(x)J^{s\dagger}(0)] | 0 \rangle ,$$  \hspace{1cm} (5)

\footnote{And noticed more recently by Delbourgo et al. \cite{24} for $\phi \to f_0 \gamma$.}
which is given by the dispersive representation:

\[ T(q^2) = \frac{1}{\pi} \int_{4m^2}^{\infty} ds \frac{\rho(s)}{s - q^2} + \text{subtractions} . \]  

(6)

In the region of low values of \( s \), the physical spectral density contains a \( \delta \)-function term corresponding, in the small width approximation, to the coupling of the \( f_0 \) to the scalar current. Picking up this contribution and dropping possible subtractions which we discuss later, we can write:

\[ T(q^2) = \frac{m_{f_0}^2}{m_{f_0}^2 - q^2} \tilde{f}^2 + \frac{1}{\pi} \int_{s_0}^{\infty} ds \frac{\rho^{\text{had}}(s)}{s - q^2} , \]  

(7)

assuming that the contribution of higher resonances and continuum of states start from an effective threshold \( s_0 \). On the other hand, the correlator \( T(q^2) \) can be computed in QCD for large Euclidean values of \( q^2 \), by using the Operator Product Expansion (OPE) to expand the \( T \)-product in Eq. (5) as the sum of a perturbative contribution plus non-perturbative terms which are proportional to vacuum expectation values of quark and gluon gauge-invariant operators of increasing dimension, the so called \textit{vacuum condensates}. In practice, only a few condensates are included, the most important contributions coming from the dimension 3 \( \langle \bar{q}q \rangle \) and dimension 5 \( \langle \bar{q}g\sigma Gq \rangle \).

In the QCD expression for the two-point correlator considered, the perturbative term can also be written dispersively, so that:

\[ T^{\text{QCD}}(q^2) = \frac{1}{\pi} \int_{4m^2}^{\infty} ds \frac{\rho^{\text{pert}}(s)}{s - q^2} + d_3 \langle \bar{q}s \rangle + d_5 \langle \bar{q}g\sigma Gs \rangle + \ldots , \]  

(8)

where the spectral function \( \rho^{\text{pert}} \) and the coefficients \( d_3, d_5 \) can be computed in QCD. The next step consists in assuming quark-hadron duality, which amounts to the claim that the physical and the perturbative spectral densities give the same result when integrated appropriately above some \( s_0 \). This leads to the sum-rule:

\[ \frac{m_{f_0}^2}{m_{f_0}^2 - q^2} \tilde{f}^2 = \frac{1}{\pi} \int_{s_0}^{\infty} ds \frac{\rho^{\text{pert}}(s)}{s - q^2} + d_3 \langle \bar{q}s \rangle + d_5 \langle \bar{q}g\sigma Gs \rangle + \ldots \]  

(9)

This expression can be improved by applying to both sides of Eq. (9) a Borel transform, defined as follows:

\[ B[\mathcal{F}(Q^2)] = \lim_{Q^2 \to \infty, n \to \infty} \frac{1}{\pi z^2 - M^2} \frac{1}{(n-1)!} (-Q^2)^n \left( \frac{d}{dQ^2} \right)^n \mathcal{F}(Q^2) , \]  

(10)

\[ B[\mathcal{F}(Q^2)] = \lim_{Q^2 \to \infty, n \to \infty} \frac{1}{\pi z^2 - M^2} \frac{1}{(n-1)!} (-Q^2)^n \left( \frac{d}{dQ^2} \right)^n \mathcal{F}(Q^2) , \]  

(10)
where $\mathcal{F}$ is a generic function of $Q^2 = -q^2$. The application of such a procedure to the sum-rules amounts to exploiting the following result:

$$
\mathcal{B}\left[1/(s+Q^2)^n\right] = \frac{\exp(-s/M^2)}{(M^2)^n(n-1)!},
$$

(11)

where $M^2$ is known as the Borel parameter. This operation improves the convergence of the series in the OPE by factorials in $n$ and, for suitably chosen values of $M^2$, enhances the contribution of low lying states. Moreover, since the Borel transform of a polynomial vanishes, it is correct to neglect subtraction terms in Eq. (10), which are polynomials in $q^2$. The final sum-rule reads:

$$
m^2_{f_0} \tilde{f}^2 \exp\left(-\frac{m^2_{f_0}}{M^2}\right) = \frac{3}{8\pi^2} \int_{4m^2_s}^{s_0} ds \left(1 - \frac{4m^2_s}{s}\right)^{3/2} \exp\left(-\frac{s}{M^2}\right) \\
+ m_s \exp\left(-\frac{m^2_s}{M^2}\right) \left[<\bar{s}s> \left(3 + \frac{m^2_s}{M^2} + \frac{m^4_s}{M^4}\right) + <\bar{s}g\sigma Gs> \frac{1}{M^2}\left(1 - \frac{m^2_s}{2M^2}\right)\right],
$$

(12)

In the numerical evaluation of Eq. (12) we use $<\bar{s}s> = 0.8$, $<\bar{q}q> = (-0.24)^3$ GeV$^3$, $<\bar{s}g\sigma Gs> = 0.8$ GeV$^2$, $<\bar{s}s>$, $m_{f_0} = 0.980$ GeV. The strange quark mass is chosen in the range $m_s = 0.125 - 0.160$ GeV, obtained in the same QCD sum-rule framework [28]. The threshold is chosen below a possible $f_0(1370)$ pole and varied between $s_0 = 1.6 - 1.7$ GeV$^2$. Since the Borel parameter has no physical meaning, we look for a range of its values (“stability window”) where the sum-rule is almost independent on $M^2$. Such a window is usually sought in a restricted interval of values of the Borel parameter chosen by requiring that the perturbative contribution is at least 20% of the continuum and additionally requiring that the perturbative term is greater than the non-perturbative contribution. The stability window for $M^2$ is selected in $[1.2, 2]$ GeV$^2$, as seen in Fig. 1, where, taking into account the uncertainty on $m_s$, we obtain the coupling:

$$
\tilde{f} = (0.180 \pm 0.015) \text{ GeV}.
$$

(13)

This result will be used in the analysis of the decay $\phi \to f_0\gamma$ as we shall see in the following.

The relevant matrix element describing the transition $\phi \to f_0$ induced by a strange vector current $J_\mu = \bar{s}\gamma_\mu s$, can be parameterized as follows:

$$
\langle f_0(q_2)|J_\mu|\phi(q_1, \epsilon_1)\rangle = F_1(q^2) \left(q_1 \cdot q_2\right) \epsilon_{1\mu} + F_2(q^2) \left(\epsilon_1 \cdot q_2\right) \left(q_1 + q_2\right)_{\mu} \\
+ F_3(q^2) \left(\epsilon_1 \cdot q_2\right) q_\mu,
$$

(14)
where \( q = q_1 - q_2 \). In order to consider the radiative decay \( \phi \to f_0 \gamma \), one needs the amplitude

\[
A(\phi(q_1, \epsilon_1) \to f_0(q_2)\gamma(q, \epsilon)) = -\frac{1}{3} \epsilon^{*\mu} [F_1(0) (q_1 \cdot q_2) \epsilon_{1\mu} + F_2(0) (\epsilon_1 \cdot q_2) (q_1 + q_2)_\mu],
\]

where the charge of the strange quark has been explicitly written. Eq. (15) shows that only two of the three form factors appearing in Eq. (14) are actually needed. Furthermore, gauge invariance requires that \( q^\mu \cdot [F_1(0) (q_1 \cdot q_2) \epsilon_{1\mu} + F_2(0) (\epsilon_1 \cdot q_2) (q_1 + q_2)_\mu] = 0 \), which relates the values of \( F_1 \) and \( F_2 \) at \( q^2 = 0 \):

\[
F_2(0) = F_1(0) \frac{m_\phi^2 + m_{f_0}^2}{2(m_\phi^2 - m_{f_0}^2)}.
\]

In terms of \( F_1(0) \), the rate for the process we consider becomes:

\[
\Gamma(\phi \to f_0 \gamma) = \alpha [F_1(0)]^2 \frac{(m_\phi^2 - m_{f_0}^2)(m_\phi^2 + m_{f_0}^2)^2}{216 m_\phi^3}.
\]

Three-point QCD sum-rules can be applied to evaluate the form factor \( F_1(q^2) \). We consider the three-point function:

\[
\Pi_{\mu\nu}(q_1^2, q_2^2, q^2) = i^2 \int d^4x \, d^4y \, e^{-iq_1 \cdot x} e^{iq_2 \cdot y} \langle 0 | T[J^\mu(x)J_\nu(0)J_\mu(x)] | 0 \rangle.
\]
After a double Borel transform in the variables $-\phi D$ where the domain $J$ should now also satisfy the kinematical constraints specified below. After a double Borel transform in the variables $-q_1^2$ and $-q_2^2$, we obtain:

\[
\frac{1}{2} m_{f_0} \tilde{\phi} m_{f_0} \phi (q^2 - m_0^2 - m_{f_0}^2) F_1(q^2) \exp \left( - \frac{m_0^2}{M_1^2} - \frac{m_{f_0}^2}{M_2^2} \right) =
\]

\[
\frac{1}{\pi^2} \int_D ds_1 ds_2 \exp \left( - \frac{s_1^2}{M_1^2} - \frac{s_2^2}{M_2^2} \right) \rho_{\text{pert}}(s_1, s_2)
\]

where $J^\gamma$ has been defined above and $J_\nu = \gamma_\nu s$ is the vector current. The correlator Eq. (18) can be written in terms of invariant structures as follows:

\[
\Pi_{\mu\nu}(q_1^2, q_2^2, q^2) = \Pi(q_1^2, q_2^2, q^2) g_{\mu\nu} + \Pi_1(q_1^2, q_2^2, q^2) q_{\mu} q_{\nu} + \cdots
\]

and a QCD sum-rule can be built up for the structure $\Pi(q_1^2, q_2^2, q^2)$. The method closely follows the one described for the two-point sum-rule. We assume $\Pi(q_1^2, q_2^2, q^2)$ obeys a dispersion relation in both the variables $q_1^2, q_2^2$:

\[
\Pi(q_1^2, q_2^2, q^2) = \frac{1}{\pi^2} \int ds_1 \int ds_2 \frac{\rho(s_1, s_2, q^2)}{(s_1 - q_1^2)(s_2 - q_2^2)} ,
\]

(20)

with possible subtractions. Such a representation is true at each order in perturbation theory and, as is standard in QCD sum rule analyses, it is assumed to hold in general. In this case the spectral function contains, for low values of $s_1, s_2$, a double $\delta-$function corresponding to the transition $\phi \to f_0$. Extracting this contribution, we can write:

\[
\Pi(q_1^2, q_2^2, q^2) = -\frac{m_{f_0} \tilde{\phi} m_\phi f_\phi F_1(q^2)(q_1 \cdot q_2)}{(m_\phi^2 - q_1^2)(m_{f_0}^2 - q_2^2)} + \frac{1}{\pi^2} \int_{s_0}^{\infty} ds_1 \int_{s_0}^{\infty} ds_2 \frac{\rho_{\text{had}}(s_1, s_2, q^2)}{(s_1 - q_1^2)(s_2 - q_2^2)} ,
\]

(21)

where subtractions are neglected as later they will vanish on taking a Borel transform. The parameter $\tilde{\phi}$ appearing in the previous equation is just the coupling of the $f_0$ to the scalar current, computed previously. Deriving an OPE-based QCD expansion for $\Pi$ for large and negative $q_1^2, q_2^2$ and $q^2$, one can write:

\[
\Pi(q_1^2, q_2^2, q^2) = \frac{1}{\pi^2} \int_{4m_0^2}^{\infty} ds_1 \int_{4m_0^2}^{\infty} ds_2 \rho_{\text{pert}}(s_1, s_2, q^2) + c_3 < \bar{s}s > + c_5 < \bar{s}g \sigma Gs > + \ldots .
\]

(22)

Invoking quark-hadron global duality as before, we arrive at the sum-rule:

\[
\frac{m_{f_0} \tilde{\phi} m_\phi f_\phi F_1(q^2)}{2(m_\phi^2 - q_1^2)(m_{f_0}^2 - q_2^2)} = \frac{1}{\pi^2} \int_D ds_1 ds_2 \frac{\rho_{\text{pert}}(s_1, s_2, q^2)}{(s_1 - q_1^2)(s_2 - q_2^2)} + c_3 < \bar{s}s > + c_5 < \bar{s}g \sigma Gs > + \ldots ,
\]

(23)

where the domain $D$ should now also satisfy the kinematical constraints specified below.
+ \exp \left( -\frac{m_s^2}{M_1^2} - \frac{m_s^2}{M_2^2} \right) \left\{ <\bar{s}s> \left[ q^2 + 2m_s^2 - \frac{m_s^2 q^2}{M_1^2} \right. \\
+ \frac{m_s^2 q^2 (2m_s^2 - q^2)}{2 M_1^2 M_2^2} + \frac{m_s^2 q^2}{2} \left( \frac{1}{M_1^4} + \frac{1}{M_2^4} \right) \right] \\
+ \bar{s} g \sigma G(s) \left[ -\frac{1}{3} + \frac{q^2 - m_s^2}{3 M_1^2} + \frac{2 q^2 + m_s^2}{3 M_2^2} - \frac{q^2 (5 m_s^2 - 2 q^2)}{6 M_1^2 M_2^2} - \frac{m_s^2 q^2}{4 M_2^2} - \frac{m_s^2 (3 q^2 + m_s^2)}{12 M_1^4} \right] \right\} ,

where:

\[ \rho_{\text{pert}}(s_1, s_2) = \frac{3m_s}{4} \left( (4m_s^2 + s_1 - s_2 + q^2) \left[ (s_1 + s_2 - q^2)^2 - 4s_1 s_2 \right] + 4 q^2 s_1 s_2 \right) / \left[ (s_1 + s_2 - q^2)^2 - 4s_1 s_2 \right]^{3/2} . \] (25)

The integration domain \( D \) over the variables \( s_1, s_2 \) depends on the value of \( q^2 \). For \( (q^2) > s_0 - 4m_s^2 \), \( D \) is specified by: \( (s_2)_- \leq s_2 \leq s_0 \) \( 4m_s^2 \leq s_1 \leq s_0 \); while, for \( (q^2) < s_0 - 4m_s^2 \), \( D \) is bounded by: \( (s_2)_- \leq s_2 \leq (s_2)_+ \) if \( 4m_s^2 \leq s_1 \leq (s_1)_- \) and \( (s_2)_- \leq s_2 \leq s_0 \) if \( (s_1)_- \leq s_1 \leq (s_0)_0 \), with: \( (s_2)_\pm = \left[ 2m_s^2 q^2 + (2m_s^2 - q^2) s_1 \pm \sqrt{s_1 q^2 (q^2 - 4m_s^2) (s_1 - 4m_s^2)} \right] / 2m_s^2 \) and \( (s_1)_\pm = \left[ 2m_s^2 q^2 + (2m_s^2 - q^2) s_0 \pm \sqrt{s_0 q^2 (q^2 - 4m_s^2) (s_0 - 4m_s^2)} \right] / 2m_s^2 \).

Since we consider the form-factor \( F_1(q^2) \) for arbitrary negative values of \( q^2 \), we could perform a double Borel transform in the two variables \( Q_1^2 = -q_1^2 \) and \( Q_2^2 = -q_2^2 \), which allows us to remove single poles in the \( s_1 \) and \( s_2 \) channels from the sum-rule. Our procedure is therefore to compute the form-factor \( F_1(q^2) \) and then to extrapolate the result to \( q^2 = 0 \). In the numerical analysis we use: \( m_\phi = 1.02 \) GeV, \( f_\phi = 0.234 \) GeV (obtained from the experimental datum on the decay to \( e^+e^- \) [13]). We compute the result for two values of the \( \phi \) threshold: \( s'_0 = 1.8, 1.9 \) GeV\(^2 \). \( s_0 \) coincides with the \( f_0 \) threshold chosen as for the two point function. The extrapolation to \( q^2 = 0 \) shown in Fig. 2 gives:

\[ F_1(0) = 0.34 \pm 0.07 , \] (26)

which, using \( \Gamma(\phi) = 4.458 \) MeV [13] and Eq. (17), gives:

\[ B(\phi \to f_0 \gamma) = (2.7 \pm 1.1) \times 10^{-4} . \] (27)

Both the results Eq. (13) and Eq. (26) have been derived without the inclusion of radiative \( \alpha_s \) corrections, an approximation which is usually believed more accurate for the three point sum rule, where the \( \mathcal{O}(\alpha_s) \) corrections are expected to cancel in the ratio of a three-point and a two-point function.
Our result of Eq. (27) is in reasonable agreement with the outcome of refs. [29, 11], where the decay is supposed to proceed through the chain \( \phi \rightarrow K\bar{K}\gamma \rightarrow f_0\gamma \), and so depends on the coupling \( g_{f_0K\pi\pi} \). Their results are: \( B(\phi \rightarrow f_0\gamma) = 1.9 \times 10^{-4} \) [29] and \( B(\phi \rightarrow f_0\gamma) = 1.35 \times 10^{-4} \) [11]. On the other hand, QCD spectral sum rules are exploited in ref. [31] to predict \( B(\phi \rightarrow f_0\gamma) = 1.3 \times 10^{-4} \).

A different strategy is proposed in [16], where the experimental datum is assumed together with the structure \( f_0(980) = n\pi \cos \theta + s\pi \sin \theta \), where \( n\pi = (u\pi + d\pi)/\sqrt{2} \) and \( \theta \) is a mixing angle. A theoretical prediction is derived describing the particles (\( \phi, f_0 \)) through wave functions depending on the radii of the mesons. Such a prediction is then compared to the experimental datum in order to constrain the mixing angle.

Although our result of Eq. (27) is affected by a rather large uncertainty, it is in agreement with the available data [18, 19]. Since our sum rule analysis is based on the hypothesis that the \( f_0(980) \) couples to the scalar \( \bar{s}s \) current, this agreement leads to the conclusion that an \( \bar{s}s \) component is present in such a state. However, our branching ratio is an order of magnitude larger than the naive quark model gives for a pure \( \bar{s}s \)

\[ \text{Such a coupling is taken from ref. [30].} \]
state. Our result is consequently consistent with the view that the $f_0(980)$ is a meson
with a basic $\bar{q}q$ composition, which spends a sizeable part of its lifetime in a two meson
state, such as $K\bar{K}$. This is in keeping with the analyses of \cite{3, 8, 17, 32} that attribute
such multi-hadron components to dressing. While the effect of $K\bar{K}$ couplings have been
studied phenomenologically in Ref. \cite{10} in a range of hadronic reactions, they have been
dynamically calculated by Marco et al. \cite{33} explicitly for the radiative decay we study
here and found to give $\mathcal{B}(\phi \to f_0\gamma) = 2.4 \times 10^{-4}$, in reassuringly good agreement with
our sum-rule result, Eq. (27).

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