L-index sensitivity based voltage stability enhancement

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***Abstract*—Voltage stability is a long standing issue in power systems. Due to the requirements of on-line monitoring and high computation efficiency, L-index is used as voltage stability metric in this paper. We propose a novel L-index sensitivity based control algorithm for voltage stability enhancement. The proposed method uses both outputs of wind generators and additional reactive power compensators as control variables. The sensitivities between L-index and control variables are introduced. Based on these sensitivities, the control algorithm can minimise all the control efforts, while satisfying the pre-determined L-index value. This paper then verifies the proposed voltage stability enhancement method using real load and wind generation data in the IEEE 14 bus system. The simulation results prove the effectiveness of proposed methodology in enhancement of voltage stability.**

***Index Terms*—voltage stability, L-index sensitivities, reactive power**

I. Introduction

NOWADAYS, the load demand is constantly increasing. This tendency coupled with the high penetration of renewable energy will not only significantly affect behaviours of power system, but also the way to control. Voltage stability is one of the most critical issues as it may cause power system collapse, e.g. the blackout in Italy 2003. The objective of this paper is to enhance voltage stability by coordinating all the reactive power sources.

The voltage stability issues have been investigated by many researchers in a large body of literature. Monitoring is the first step to enhance voltage stability regarding the current operating status. Thevenin's Theorem based metric is widely used for voltage stability assessment by using local measurements. The principle of the proposed method in [1] is that the system thevenin impedance and the load apparent impedance have the same magnitude at the point of collapse. A powerful analysis method for evaluating voltage stability is to estimate the distance of load flow Jacobian to singularity [2].

Continuation power flow is an alternative approach. PV curve is plotted from continuation power flow that makes collapse point visible and shows the maximum loadability of the power system. A trust region frame work was presented for voltage stability enhancement by coordinating reactive power sources in [3]. Sode-Yome [4] introduced an approach to form an equation which representing load margin as a function of generation directions. Bedoya et al. proposed a fast computation method to find the minimum voltage stability margin focuses on the load parameter space feasibility and infeasibility regions [5]. All of these approaches foucus on enhanceing voltage stability margin by enlarging loadability margin.

On the other hand, there is a trend of high penetration of renewable energy in the traditional grid. Therefore, distributed generations (DGs) should be taken into account when voltage stability problems are considered. The impacts of integrating renewable energy into distribution network were investigated in [6]. Doubly Fed Induction Generators (DFIGs) are widely used in variable speed wind power plants. DFIGs offer many merits, such as low cost, high efficiency with the available wind resource and reactive power control capabilities. In [7], the impacts of induction generators were analyzed under both small and large disturbances. Further, the flexibility of reactive power output of DFIGs was utilized to reduce system losses.

In this paper, a novel approach using L-index sensitivity based voltage control is proposed to enhance voltage stability. The algorithm utilizes L-index, which is a voltage stability metric. The method is expected to be implemented in situations where real data of wind generations and demands are introduced. L-index sensitivities will be used to determine the control actions. This paper is directly targeted at utilizing reactive power output of all reactive sources for voltage stability enhancement. The main contributions of this paper are:

- It presents a novel voltage stability enhancement method using L-index sensitivities. The proposed method has two main features: fast computation and enhance voltage stability with the most sensitive control actions.
- Voltage stability can be enhanced by coordinating the reactive power sources. Both wind generators and additional installed Static Var Compensators (SVCs) are taken into account based on IEEE 14 bus system.

The rest of the paper is organized as follows. In Section II, the comparison of different voltage stability identification methods is presented. In Section III, L-index sensitivities are formulated, and the optimisation control method for voltage stability enhancement is proposed. Case studies presented in Section IV show results of L-index sensitivities based control. Concluding remarks and future research questions are provided in Section V.

II. Voltage Stability Identification

As voltage stability is a long standing power system issue, lots of voltage stability identification methods have been proposed. Their different features make the control performance different. In order to enhance system efficiency and maintain reliability, control actions will need to be quick and effectively decided against rapidly changing of operation status. Therefore, fast computation of methodologies for voltage control will play a crucial role in maintaining safe operation. The comparison of these methods is discussed as follows.

A. Jacobian Matrix-Based Method:

Jacobian matrix is achieved from power-flow computation using Newton-Raphson method [8]. It is introduced for voltage stability identification in diverse ways. In this approach, the voltage stability α_{ij} is calculated as the attenuation of voltage variations between two nodes i and j given by

$$\alpha_{ij} = \partial V_i / \partial V_j = \frac{\partial V_i}{\partial Q_j} / \frac{\partial V_j}{\partial Q_j} \tag{1}$$

where the sensitivity matrix $\partial V/\partial Q$ is the inverse of the matrix $\partial Q/\partial V$, which is the part of the Jacobian matrix. In [7], singular value of a matrix J_R, which is transformed from Jacobian matrix, is proposed. It can be formulated as follows:

$$J_R = \frac{\partial Q}{\partial V} - \frac{\partial Q}{\partial \theta} \frac{\partial P}{\partial \theta}^{-1} \frac{\partial P}{\partial V} \tag{2}$$

where, $\frac{\partial Q}{\partial V}, \frac{\partial Q}{\partial \theta}, \frac{\partial P}{\partial \theta}, \frac{\partial P}{\partial V}$ are the elements in Jacobian matrix.

B. Bus Admittance Matrix-Based Method:

In [9], a control approach against power system voltage instability is proposed. It is building on an admittance matrix-based method called electric distance, which can be easily derived from the absolute value of the inverse of the system admittance matrix. The metric is formulated as:

$$[D] = \left| [Y_{\text{bus}}]^{-1} \right| \tag{3}$$

where Y_{bus} is the admittance matrix of the power system. The elements d_{ij} in the distance matrix $[D]$ indicate active and reactive power sensitivities with voltage changes between buses i and j. It is worth highlighting that smaller the electrical distance means the higher impacts on the voltage change as a result of the change in active and reactive powers.

C. Power-Voltage (PV) curves:

P-V curve analysis is used to determine voltage stability of a network. The curve shows the distance between current operating point and collapse point. For this analysis, power of the load at a particular bus is increased in steps and voltage is observed. Then curves for those particular buses will be plotted based on the continues power flow to determine the voltage stability of a system by static analysis approach. Continuation method, also called continuation power flow, is widely employed in power systems to determine steady state stability limits. The limit is determined from a nose curve where the nose represents the maximum power transfer that the system can handle given a power transfer schedule.

It is reported that voltage stability index based on the minimum singular value would probably be used for long-term planning and 'off line' operational planning studies [10]. By contrast, the elements of Y_{BUS} are usually readily available. They will not necessary to update until the change of network has been made. In this way, the voltage stability can be identified quickly from an earlier known admittance matrix. During the emergency condition, no global knowledge or the new Y_{BUS} of the system is required. Therefore, bus admittance matrix-Based method have better performance in computation speed that can meet the requirements of power system with rapid change in load.

III. Proposed Approach

The basic idea in this paper is to enhance voltage stability with the most sensitive control actions. This makes optimal control in an effectiveness way and voltage stability can be enhanced directly. The objective function is to determine the minimal compulsory control efforts to improve voltage stability. Moreover, computation speed is a critical aspect in real time voltage stability enhancement. Based on the comparison of different voltage stability identification methods in Section II, L-index sensitivity based voltage control approach is proposed. Two sets of control variables are considered: reactive power output of wind generators and compensators. The L-index sensitivity formulations with respect to SVCs and wind generators are introduced.

A. L-index Sensitivities

L-index in [11] has been adopted in this paper for on-line voltage stability enhancement. It computed as

$$L_j = \left| 1 - \sum_{i=1}^{g} F_{ji} \frac{V_i}{V_j} \right| \tag{4}$$

where V indicates the voltages at the bus and subscripts j and i are used to differentiate between load and generator bus numbers. $1, ..., g$ are the generators and n is the total number of buses in the system. F_{ji} in (4) is the element of the F_{lg} matrix which is obtained by admittance matrix calculation. The relationship between voltage and current is stated below,

$$\begin{bmatrix} V_l \\ I_g \end{bmatrix} = \begin{bmatrix} Z_{ll} & F_{lg} \\ K_{gl} & Y_{gg} \end{bmatrix} \begin{bmatrix} I_l \\ V_g \end{bmatrix} \tag{5}$$

where F_{lg} is computed as $[F_{lg}] = -[Y_{ll}]^{-1}[Y_{lg}]$. Y_{ll} is the self-admittance at the node l and Y_{lg} is the mutual admittances between the nodes l and g.

L-index sensitivity based control depends on L-index value L_j and two injection vectors Q_{svc} and Q_{g}. The reactive power outputs of SVCs, Q_{svc} and wind generators, Q_{g} can be controlled independently. The changing of these control variables will affect the L-index values of all load buses. The relationship between the control variables and the L-index value is highly non-linear and depends on the system operating point.

Small changes in the control variables are used to find the corresponding sensitivities. A small change at operating point k, $\overline{\delta}_{Q_{\text{svc}}}$, in the injection vector Q_{svc} is applied and the sensitivity to this small change is found via (6):

$$a^k_{\Delta Q_{\text{svc}}} = \frac{L^k_c(Q_{\text{svc}} + \overline{\delta}_{Q_{\text{svc}}}, Q_{\text{g}}) - L^k_c(Q_{\text{svc}}, Q_{\text{g}})}{\overline{\delta}_{Q_{\text{svc}}}} \tag{6}$$

In the same manner as (6), these resulting critical L-index values can then be utilized to find the corresponding sensitivities as in (7):

$$a^k_{\Delta Q_{\mathrm{g}}} = \frac{L^k_c(Q_{\mathrm{svc}}, Q_{\mathrm{g}} + \bar{\delta}_{Q_{\mathrm{g}}}) - L^k_c(Q_{\mathrm{svc}}, Q_{\mathrm{g}})}{\bar{\delta}_{Q_{\mathrm{g}}}} \tag{7}$$

It is the fact that the power system is non-linear as well as the relationship of dependent control variables and L-index value. However, the linear equation can be formulated at a system operating point. L-index sensitivities can be utilized with dependent control variables that will achieve the predetermined L-index step size, Λ_L:

$$\sum_{i \in T} a^k_{\Delta Q_{\mathrm{svc}}} \Delta Q_{\mathrm{svc}} + \sum_{i \in T} a^k_{\Delta Q_{\mathrm{g}}} \Delta Q_{\mathrm{g}} = \Lambda_L \tag{8}$$

L-index indicates the level of voltage stability. Its value should be between 0 and 1. When a load bus has no load, L-index value of this load bus is equal to 0. When L-index value is 1, it means voltage stability collapse. Therefore, the lower the L-index value the more stable the voltage stability. It can be seen that, both the reactive power injections of SVC and wind generator contributes to decrease in L-index value.

B. Optimal Control of Reactive Power Resources Outputs

The objective function is formulated as a minimization of the control efforts which include two independent control variables for each iteration as below:

$$\mathrm{Min} \sum_{i \in T} (Q^2_{\mathrm{svc,i}} + Q^2_{\mathrm{g,i}}) \tag{9}$$

s.t.

$$G(x) = 0, g(x) = 0, \forall i \in T \tag{10}$$

$$H(x) <= 0, h(x) <= 0, \forall i \in T \tag{11}$$

The decision variable vector x is defined as $x = [\Delta Q_{\mathrm{svc,i}}, \Delta Q_{\mathrm{g,i}}]$. The linear and nonlinear equality constraints are $G(x)$ and $g(x)$, respectively. The linear and non linear inequality constraints are $H(x)$ and $h(x)$, respectively. The most basic linear equality constraints are (12)-(13) which relate to the objective function:

$$Q^{k+1}_{\mathrm{svc,i}} = Q^k_{\mathrm{svc,i}} + \delta_{Q_{\mathrm{svc}}}, \forall i \in T \tag{12}$$

$$Q^{k+1}_{\mathrm{g,i}} = Q^k_{\mathrm{g,i}} + \delta_{Q_{\mathrm{g}}}, \forall i \in T \tag{13}$$

The linear inequality constraints (14)-(15) maintain the control variables within operating limits:

$$Q_{\mathrm{svc,i,min}} \leq Q_{\mathrm{svc,i}} \leq Q_{\mathrm{svc,i,max}} \tag{14}$$

$$Q_{\mathrm{g,i,min}} \leq Q_{\mathrm{g,i}} \leq Q_{\mathrm{g,i,max}} \tag{15}$$

L-index sensitivities with respect to the reactive power injection from SVCs and wind generators are introduced. Control constraints are incorporated with L-index based linear sensitivities. The most sensitive control actions can be achieved by solving the optimisation problem. Solution of (9) will determine the minimal amount of control necessary to enhance voltage stability which is based on the pre-determined L-index value. This makes the control action is more effective.

There are two main tasks in the proposed method. Initially, calculate L-index values for all load buses which shows the voltage stability level and the desire L-index values can be set based on the power system requirement. Then the control efforts can be minimised by the objective function. Additionally, in order to reach higher voltage stability level, control step sized can be enlarged to achieve lower L-index value.

Genetic algorithm is used as optimisation algorithm in this paper. It begins with a large set of randomly generated individuals μ solutions. In each iteration, λ new generation are created. These μ parents and together with these λ generations will be modified to form the next μ parents. This approach hopefully shields against the so-called premature convergence to local optima, if the population is sufficiently diverse [12].

IV. Simulation Results

In this section, we present a case study of the L-index sensitivities based voltage control in a modified 14 bus. The proposed control algorithm is implemented in Matlab. A modified IEEE 14 bus system and Matpower are utilized to verify the effectiveness of the proposed method.

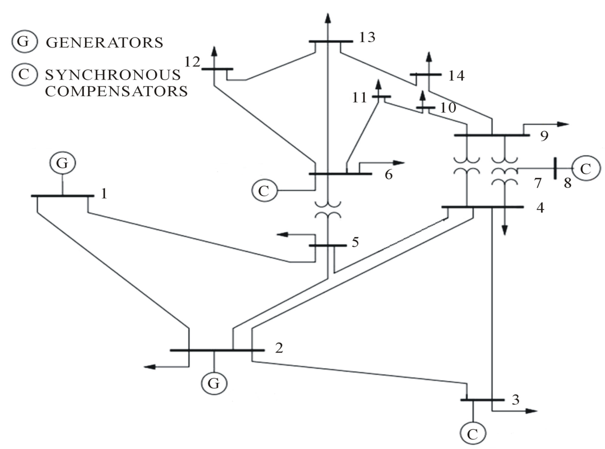

Fig. 1. IEEE 14 bus system

Fig.1 shows the IEEE 14 bus system. It is assumed that generators installed on bus 2 are DFIGs. The wind farm consists of 100 generators each rated at 2 MW. This IEEE 14 bus system contains one wind farm on bus 2. Reactive power compensation is better if done locally, due to the fact that reactive power travel long distance may cause high consumption. Therefore, each load bus has a SVC. Generation and load profiles are obtained from Gridwatch provided by BM reports [13]. The data are for the whole year of 2015. To simplify the calculations, hourly data are extracted from raw generation and load profiles.

In Fig.2, two fold lines are obtained by using a set of load and wind generation data at a time. Red line is drawn under normal condition. After optimisation by using coordination control, the new voltage profile is drawn with red colour. The main generators are installed on bus 1, hence its voltage

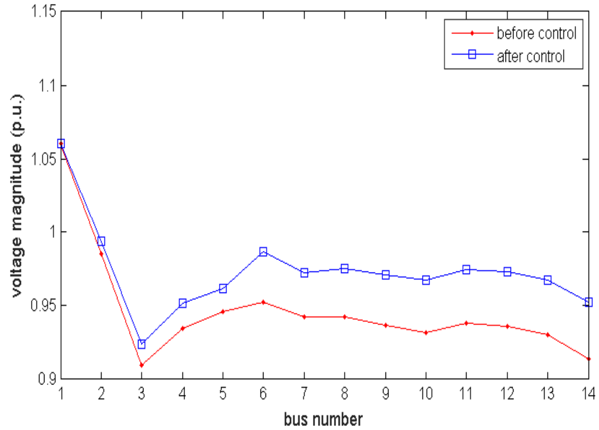

Fig. 2. voltage profile enhancement on buses

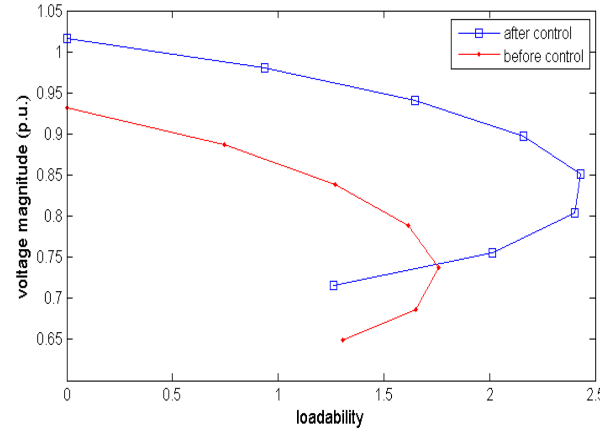

Fig. 4. loadability enhancement

magnitude maintains at 1.06. It can be seen that the voltage level is improved significantly when the proposed method is applied. Due to the heavier load on bus 3 and 14, their voltage magnitudes are relative lower.

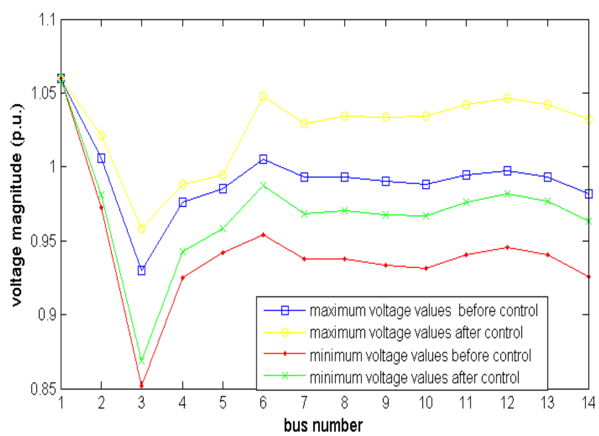

Fig. 3. maximum and minimum voltage profiles

One hundred sets of real wind generations and loads data are utilized to verify the effectiveness of the proposed method. Under normal conditions, the voltage profiles are drawn with blue and red for the highest and lowest values respectively in Fig.3. The yellow line shows the highest voltage profile when the proposed method is applied. While the green line indicates the lowest voltage profile after optimal control. It is noted that the voltage on bus 3 is much lower than others, because of the load on bus 3 is much heavier.

The PV curves in Fig.4 indicate the loadability of normal condition and optimised performance as well as the benefit of the proposed method. It is also important to know the growth rate of loadability between normal condition and after optimisation, which reaches 37%. The red curve shows the largest loadability at around 1.75 without optimal control. In contrast, blue curve shows that there is a significantly increasing in loadability which reaches 2.4.

The optimised control performances are shown as PV curves

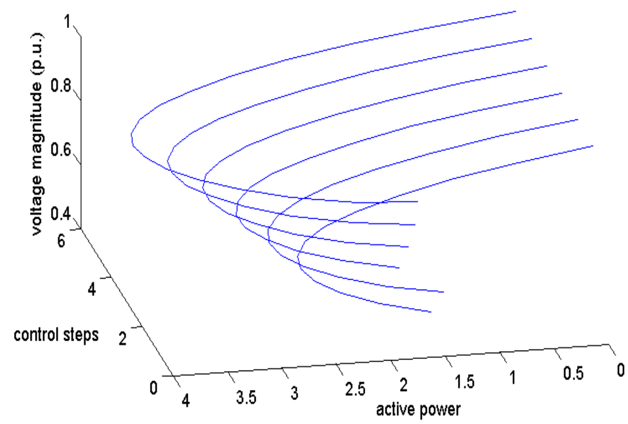

Fig. 5. loadability enhancement with different control steps

in Fig. 5. The PV curves are plotted in 3-dimension with different control steps. Every control step interval means the pre-defined difference value of L-index need to be decreased. It can be seen that with the lower L-index value, the loadability of power system is larger. It is easy to achieve target loadability based on the system requirement by setting a pre-determined L-index value. Moreover, voltage stability can reach the required level with the most sensitive control actions.

V. Conclusion

A voltage stability enhancement method based on L-index sensitivities is presented. Both wind generators and compensators are taken into account. It has been proved that it is feasible to enhance the voltage stability with the proposed method, which provides fast computation and self-determined L-index value. The L-index based voltage stability control method is applied in a modified IEEE 14 bus system. The voltage profiles are improved by using the proposed method. Additionally, one hundred sets of loads and wind generators are used to verify the method. Meanwhile, both maximum and minimum voltage profiles are extracted to indicate its benefits. In addition, the loadability enhancements with different control steps are illustrated by using PV curves. It is worth note that voltage stability can be enhanced with most sensitive control actions. Also, this proposed method can be easily extended to other different networks where the power system contains wind farms and compensators.

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