Abstract—In order to alleviate the effects of greenhouse gas emissions, the environmental and economic dispatch (EED) is formulated as a multiobjective problem (MOP) solved by the multiobjective immune algorithm (MOIA). Building on this model, the virtual power plant (VPP) is proposed involving distributed generation (DG), energy storage (ES), and interruptible load (IL), and energy storage (ES) to participate in joint energy and reserve markets. The uncertainties of load prediction, DG, and IL are treated as interval-based optimization in this study. The static and real-time simulations are conducted to demonstrate the validity of the proposed stochastic EED model through the IEEE 30-bus test system.

Keywords—environmental and economic dispatch (EED), virtual power plant (VPP), distributed generation (DG)

I. INTRODUCTION

Greenhouse gas emissions have detrimental effects on sustainable development. It is particularly for the power generation sector which accounts for around 40% of carbon emissions [1]. Conventionally, the economic dispatch is responsible for allocating the optimal generation with the objective of minimizing total operation costs while being subject to system constraints [2]. In addition to the classic economic dispatch, the environmental dispatch, on the contrary, seeks to minimize the total pollutant emissions irrespective of costs [3]. Nevertheless, costs and emissions do not share the same dimension, which presents a challenge to propose a more dedicated dispatch approach coordinating and optimizing balanced operational points reasonably from economic and environmental aspects.

Existing studies aim to solve environmental and economic dispatch (EED) simultaneously. F. Z. Gherbi et al. [4] considered carbon emissions as an additional constraint to optimize the generation costs. The emissions and costs were also optimized separately as a single objective function, before weighted evaluating for each objective [5], [6]. Nonetheless, it would be more useful to apply multiobjective problem (MOP) into EED for the purpose of fairly and effectively evaluating the interests of both costs and carbon emissions. To solve the EED problem, this paper employs the multiobjective immune algorithm (MOIA) because it is able to obtain the optimal solution without sacrificing the interest of any objective [7]. Therefore, a balanced operating point for each generator can be obtained.

Moreover, the generation dispatch can be classified into deterministic approach and stochastic approach in terms of optimization features. A majority of studies have investigated deterministic dispatch problem [8], [9]. However, due to the system uncertainties caused by distributed energy sources and load predictions, there are opportunities in applying the stochastic approach to cope with system uncertainties. H. Wu et al. [10] proposed stochastic programming methods for security constrained unit commitment to deal with uncertainties in renewable energy. The uncertainties of system intermittency and incidents were investigated by using frequency-constrained stochastic optimization model in [11]. The stochastic approach requires the probability distribution function (PDF) of the stochastic variables, which is difficult to be obtained in practical operations of power systems. By contrast, it is easier to establish the interval-based dispatch model to describe the range of uncertain variables. The interval-based optimization was noted in [12]. This paper adopts the interval-based stochastic approach involving the uncertainties of distributed generation (DG), energy storage (ES), and interruptible load (IL).

Additionally, increasing penetrations of DG promotes replacement of grid structures, which economically and technically attributes to these resources through offering energy and reserve services [13]. Meanwhile, the requirements of demand response, system reliability, and security of electricity supplies during these services enable the virtual power plant (VPP) to be a necessary control infrastructure to coordinate each component inside [14]. The VPP is capable of dispatching and optimizing the DG to support power system regulations through using the rapid and flexible characteristics of distributed resources. The carbon emissions issue and uncertainties of distributed resources in the VPP, however, have barely been studied.

Compared with the existing work, contributions of this paper are: 1) We aim to propose an EED model considering both operation costs and carbon emissions as objectives of MOP; 2) We extend the scope of current research in the field of VPP through considering uncertainties and carbon emissions into the MOP. The uncertainties of load predictions, DG, and IL are evaluated by interval-based approach.

The rest of this paper is organized as follows. Section II introduces the EED model considering the uncertainties of load predictions, DG, and IL. The interval-based stochastic model is subsequently described to cope with the uncertainties.
Section III illustrates the transformation from stochastic model to deterministic optimization problem. Case studies are conducted in Section IV to demonstrate the proposed model. Finally, Section V comes to the conclusion.

II. STOCHASTIC EDD MODEL

This section proposes the EED model through establishing objectives and constraints during power system operations. The uncertainties of load prediction, DG, and IL are considered by an interval-based stochastic approach.

A. Objective Functions

The economic dispatch of Conventional Power Plant (CPP) seeks to minimize the operation costs satisfying the total demand:

\[ C(P_G^i) = a_i P_G^i + b_i P_G^i + c_i \]  

where \( C(P_G^i) \) is the generation cost of \( i \)-th CPP at hour \( t \). Here, \( a_i, b_i \), and \( c_i \) are cost coefficients of generator \( i \).

Similarly, the economic dispatch of VPP is formulated to minimize the costs of each component inside. In this paper, the conventional model of VPP is considered including DG, ES, and IL. The cost objective of DG can be described as [15]:

\[ C(P_{DG}^j) = d_j P_{DG}^j + e_j P_{DG}^j + f_j \]  

where \( C(P_{DG}^j) \) is generation cost function of \( j \)-th DG unit at hour \( t \). Here, \( d_j, e_j \), and \( f_j \) are cost coefficients of \( j \)-th DG unit.

As another fundamental component of VPP, the behaviour of storages in ES can be modelled as [15]:

\[ - (P_{ES}^{\text{min}} - \text{SoC}_{j-1}^{\text{ES}}) \leq P_{ES}^j \leq P_{ES}^{\text{max}} - \text{SoC}_{j-1}^{\text{ES}} \]  

\[ \text{SoC}_{j-1}^{\text{ES}} - \text{SoC}_j \leq R_{sh} \]  

\[ \text{SoC}_j - \text{SoC}_{j-1}^{\text{ES}} \leq R_{ch} \]  

where \( P_{ES}^{\text{min}} \) and \( P_{ES}^{\text{max}} \) are minimum and maximum capacities of \( j \)-th ES, respectively, \( \text{SoC}_j \) is state of charge of \( j \)-th ES, and \( R_{sh} \) and \( R_{ch} \) are maximum charge and discharge rates of \( j \)-th ES. Hence, the cost function of ES can be modelled as [15]:

\[ C(P_{ES}^j) = d_{ES} P_{ES}^j + e_{ES} \]  

where \( C(P_{ES}^j) \) is operation cost function of \( j \)-th ES, and \( d_{ES} \) and \( e_{ES} \) are cost coefficients of \( j \)-th ES. Furthermore, the cost function of IL can be described as [15]:

\[ C(P_{IL}^j) = d_{IL} P_{IL}^j + e_{IL} P_{IL}^j + f_{IL} \]  

where \( C(P_{IL}^j) \) is generation cost function of \( j \)-th IL unit, \( P_{IL}^j \) is power generated by \( j \)-th IL unit at hour \( t \), and \( d_{IL}, e_{IL}, \text{ and } f_{IL} \) are cost coefficients of \( j \)-th IL unit. Therefore, the objective function of economic dispatch is minimization of the total cost in CPP and VPP.

**Cost objective of economic dispatch:**

\[
\min \left\{ \sum_{j=1}^{\text{No. of CPP}} C(P_{G}^j) + \sum_{j=1}^{\text{No. of DG}} r C(P_{DG}^j) + \sum_{j=1}^{\text{No. of ES}} C(P_{ES}^j) + \sum_{j=1}^{\text{No. of IL}} r C(P_{IL}^j) \right\}
\]

where \( R_{DG} \) and \( R_{IL} \) are power generated by DG and IL for reserve market, and \( r \) is probability of reserve delivery.

By contrast, the environmental dispatch minimizes the total carbon emissions, which means that the generator with the lowest carbon emission will be triggered first [16]. The carbon emissions of CPP can be modelled using second order polynomial functions [17].

**Emission objective of environmental dispatch:**

\[
\min \left\{ \sum_{j=1}^{\text{No. of CPP}} a_j P_{G}^j + b_j P_{G}^j + c_j \right\}
\]

where \( a_j, b_j \), and \( c_j \) are coefficients of carbon emissions.

Carbon emissions in VPP are not taken into consideration of MOP problem, because these emissions are irrelevant to the operational process. The carbon emissions in VPP will be evaluated through carbon emissions factors based on the life cycle analysis [18].

B. Constraints

1) **Power Balance Constraint:**

\[
\sum_{j=1}^{\text{No. of CPP}} P_{G}^j + \sum_{j=1}^{\text{No. of DG}} (P_{DG}^j + R_{DG}^j) + \sum_{j=1}^{\text{No. of ES}} (P_{ES}^j + R_{ES}^j) + \sum_{j=1}^{\text{No. of IL}} (P_{IL}^j + R_{IL}^j) = [D_t^L, D_t^U]
\]

2) **Power Output Constraints:**

The power output constraint for the CPP is:

\[
P_{G}^{\text{min}} \leq P_{G}^j \leq P_{G}^{\text{max}}
\]

where \( P_{G}^{\text{min}} \) and \( P_{G}^{\text{max}} \) are minimum and maximum power generations of CPP.
Regarding power output of VPP, uncertainties of DG and IL are described as intervals. For the dispatchable DG units, due to the intermittency of renewable resources, the predictions of DG units tend to be inaccurate, which can be reflected by an upper and lower bound of output for energy and reserve markets, respectively. Thus, in addition to the deterministic inner constrain \( P_{DG}^{min,U} \leq (P_{DG}^{j} + R_{DG}) \leq P_{DG}^{max,L} \), an outer constraint \( P_{DG}^{min,L} \leq (P_{DG}^{j} + R_{DG}) \leq P_{DG}^{max,U} \) is adopted to represent maximal regulation capacity. Hence, the constraint for DG in energy and reserve market is:

\[
[P_{DG}^{min,L} - P_{DG}^{max,U}] \leq (P_{DG}^{j} + R_{DG}) \leq [P_{DG}^{max,L} - P_{DG}^{min,U}]
\]

where \([P_{DG}^{min,L} - P_{DG}^{max,U}]\) and \([P_{DG}^{max,L} - P_{DG}^{min,U}]\) denote lower and upper bounds of power output of \( j \)th DG in the energy market. The inner and outer constraints reflect the conservative and optimistic uncertain levels and risks afforded by decision makers.

3) IL Constraint:

Similarly, the uncertainties of IL due to the variations of load curtailments are reflected as an interval:

\[
0 \leq (P_{IL}^{j} + R_{IL}^{j}) \leq [P_{IL}^{max,L} - P_{IL}^{min,U}]
\]

where \([P_{IL}^{min,L} - P_{IL}^{max,U}]\) is the upper bound of IL.

4) Ramp Rate Constraints:

\[
-R_{i}^{down} \leq P_{DG}^{i} \leq P_{DG}^{i \downarrow} \leq R_{i}^{up}
\]

where \(R_{i}^{down} \) and \(R_{i}^{up}\) denote the ramp-down and ramp-up rates of \( i \)th CPP. The ramp rate of the DG is faster than CPP due to rapid regulation capacities. Thus, the ramp rate constraint of VPP is not taken into consideration.

III. STOCHASTIC MODEL TRANSFORMATION

This section illustrates the conception of probability degree, so that the stochastic EED can be transferred into deterministic MOP. The MOIA algorithm to solve the MOP is also presented.

A. Probability Degree

The probability degree [19] is employed to solve the interval-based MOP. The probability degree can represent the risk levels which decision makers are willing to take based on interval-based MOP. The probability degree can be expressed as:

\[
P(a \leq B) = \begin{cases} 
1, & a \leq b \\
\frac{b-a}{\overline{b}-a}, & b \leq a \leq \overline{b} \\
0, & a \leq \overline{b} 
\end{cases}
\]

where \(P(a \leq B)\) represents the probability degree of \( a \leq B \). The variable within B is assumed to obey the uniform distribution.

![Figure 1. Relationship between a real number and an interval.](Image)

Furthermore, depending on the risk tolerance of decision makers, the probability degree \( \lambda \in [0,1] \) can be defined as a threshold on the condition of \( a \leq B \). Therefore, \( P(a \leq B) \geq \lambda \) can be transferred into:

\[
a \leq \overline{b} + \overline{b}(1 - \lambda)
\]

According to Eq. (16), when \( \lambda = 0 \), the interval constraint \( a \leq [\overline{b}, \overline{b}] \) becomes to be \( a \leq \overline{b} \), which means that the decision maker is optimistic to focus on upper bound of the interval. By contrast, when \( \lambda = 1 \), the interval constraint becomes to be \( a \leq \overline{b} \). Hence, the decision maker is pessimistic to reduce uncertainties. Thus, a higher probability degree represents a lower risk level would be afforded by decision maker.

B. Transformation of Stochastic Model

Building on the aforementioned probability degree, the stochastic interval-based constraints in Eq. (10), Eq. (12), and Eq. (13) can be transferred into deterministic constraints:

\[
\sum_{r=1}^{NoCPP} P_{DG}^{r} + \sum_{j=1}^{NoDG} (P_{DG}^{j} + R_{DG}^{j}) + \sum_{j=1}^{NoIL} (P_{IL}^{j} + R_{IL}^{j}) = D_{xt}^{LAB} \lambda_{DL} + D_{xt}^{LAB}(1 - \lambda_{DL}) \tag{17}
\]

\[
P_{DG}^{j} + R_{DG}^{j} \leq P_{DG}^{min,U} \lambda_{DG} + P_{DG}^{min,L}(1 - \lambda_{DG}) \tag{18}
\]

\[
P_{DG}^{j} + R_{DG}^{j} \leq P_{DG}^{max,U} \lambda_{DG} + P_{DG}^{max,L}(1 - \lambda_{DG}) \tag{19}
\]

\[
P_{IL}^{j} + R_{IL}^{j} \leq P_{IL}^{max,U} \lambda_{IL} + P_{IL}^{min,U}(1 - \lambda_{IL}) \tag{20}
\]

where \(\lambda_{DL}, \lambda_{DG}, \) and \(\lambda_{IL}\) are the assigned probability degree of load, DG, and IL constraints, respectively.

C. Methodology

The MOP is solved by MOIA (See TABLE I) for the purpose of obtaining pareto front (PF) [7]. The PF is the image of all nondominated solutions as shown in Fig. 2. If a point is able to provide better performance to at least one objective without sacrificing other objectives, it becomes the pareto optimal (PO) [7].

IV. CASE STUDIES

In order to demonstrate the proposed model, case studies have been conducted using the IEEE 30-bus system which consists of 6 generators [20]. The generators from G1 to G5 are CPPs, and the G6 is replaced by a VPP. The static
The simulation uses system original data to compare the results between deterministic approach and stochastic approach. Moreover, the real-time simulation uses the scaled-down UK daily generation and consumption data in proportion to present the results of daily power dispatch in CPP and VPP as well as corresponding carbon emissions. The coefficients are selected based on practical experience and [15].

### TABLE I. MOIA ALGORITHM

<table>
<thead>
<tr>
<th>Input:</th>
<th>Objective functions: Eq (8), (9); initial solution size $n$; maximum iteration time.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Generate a group of antibodies as initial population to represent the power dispatch over constraints Eq. (11), (14), (17), (18), (19), and (20):</td>
</tr>
<tr>
<td>2.</td>
<td>Remove dominated antibodies and remain nondominated antibodies.</td>
</tr>
<tr>
<td>3.</td>
<td>Perform mutation operation over the remaining nondominated antibodies to produce a set of antibodies.</td>
</tr>
<tr>
<td>Repeat</td>
<td>4. Remove dominated antibodies.</td>
</tr>
<tr>
<td></td>
<td>5. Evaluating the remaining antibodies through satisfying the constraints and removing infeasible antibodies.</td>
</tr>
<tr>
<td></td>
<td>6. if The population size is larger than the nominal size then</td>
</tr>
<tr>
<td></td>
<td>7. Update to normalize the antibodies</td>
</tr>
<tr>
<td>End if</td>
<td>Until The maximum iteration time is reached.</td>
</tr>
<tr>
<td>Output:</td>
<td>A solution which is able to maximize the minimum improvement in all dimensions.</td>
</tr>
</tbody>
</table>

### A. Static Simulation

The uncertainties of load predictions, DG, and IL are reflected in the probability degrees for quantifying uncertain intervals under six conditions. TABLE II shows the comparison between deterministic and six conditions of stochastic results in EED. The corresponding PFs of MOP are shown in Fig. 2. It can be seen that the highest cost and emission reach £537.8525 and 206.5426 ton/h respectively in case 1, whereas the relatively lower cost and the lowest emission drop to £511.8148 and 190.4036 ton/h respectively in case 5. This is because a lower probability degree of load uncertainty ($\lambda_L$) indicates a higher load level, whereas higher probability degrees of DG ($\lambda_{DG}$) and IL ($\lambda_{IL}$) in the VPP indicate a lower output, which causes the highest cost and emission. Additionally, the deterministic results are closer to case 2 with medium load uncertainty and lower VPP output.

### TABLE II. TOTAL COST AND EMISSION OF SYSTEM

<table>
<thead>
<tr>
<th>Approach</th>
<th>Case</th>
<th>$\lambda_L$</th>
<th>$\lambda_{DG}$</th>
<th>$\lambda_{IL}$</th>
<th>Cost (£/h)</th>
<th>Emission (ton/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stochastic</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>537.8525</td>
<td>206.5426</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.5</td>
<td>1</td>
<td>1</td>
<td>527.5326</td>
<td>198.4871</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>508.8145</td>
<td>193.7966</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1</td>
<td>0.5</td>
<td>0.5</td>
<td>508.2081</td>
<td>192.8227</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>511.8148</td>
<td>190.4036</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.8</td>
<td>0.7</td>
<td>0.6</td>
<td>536.0419</td>
<td>202.1907</td>
</tr>
<tr>
<td>Deterministic</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>528.2506</td>
<td>197.2691</td>
</tr>
</tbody>
</table>

The uncertainties of load predictions, DG, and IL are reflected in the probability degrees for quantifying uncertain intervals under six conditions. TABLE II shows the comparison between deterministic and six conditions of stochastic results in EED. The corresponding PFs of MOP are shown in Fig. 2. It can be seen that the highest cost and emission reach £537.8525 and 206.5426 ton/h respectively in case 1, whereas the relatively lower cost and the lowest emission drop to £511.8148 and 190.4036 ton/h respectively in case 5. This is because a lower probability degree of load uncertainty ($\lambda_L$) indicates a higher load level, whereas higher probability degrees of DG ($\lambda_{DG}$) and IL ($\lambda_{IL}$) in the VPP indicate a lower output, which causes the highest cost and emission. Additionally, the deterministic results are closer to case 2 with medium load uncertainty and lower VPP output.

### B. Real-time Simulation

The aforementioned condition in case 6 is selected as an example for real-time simulation with the scaled down UK generation and demand data [21]. The daily MOP results of EED for CPPs and VPP in both energy and reserve markets are shown in Fig. 3 and Fig. 4, respectively. The power curve of CPP is closer to the lower-bound of load interval, because the selected probability degree of load uncertainty is relatively high ($\lambda_L=0.8$). The total daily generated power of VPP is presented in Fig. 5. It is clear that the dispatching VPP output falls into the uncertain interval during the periods from 12h to 14h and from 16h to 18h, which means that the EED is confronted with risks due to those uncertainties. Furthermore, Fig. 6 shows the daily EED. There is the same trend of variation between daily costs and emissions, but the costs during the peak-time (8h to 18h) present a more dramatic increase than carbon emissions.

![Figure 2. The comparison of EED between determinstic and stochastic approaches.](image1)

![Figure 3. EED power curves of CPPs.](image2)

### V. CONCLUSION

This paper proposes a stochastic EED model in power systems considering the VPP in both energy and reserve markets. The generation dispatch problem is considered as a MOP solved by the MOIA. The uncertainties of load prediction, DG and IL are taken into consideration as uncertain intervals, so that the stochastic optimization problem can be converted into deterministic optimization. The
static simulation demonstrates the various optimization results considering different levels of probability degrees. Moreover, the results indicate that the EED is confronted with risks caused by uncertainties.

Figure 4. EED power curves of VPP.

Figure 5. Total generated power curves of VPP.

Figure 6. Daily EDD results of cost and emission.

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