Abstract: This paper aims to bring clarity to the term topology as it has been deployed in human geography. We summarize the insights that geographers have garnered from thinking topologically about space and power. We find that many deployments of topology both overstretch topology's conceptual merit and limit its insights for spatial thinking. We show how topology, with its structuralist and modernist baggage, requires some theoretical reworking to be put to work by poststructuralist geographers. Our purpose is not to consolidate a specific topological approach for geographers, but to call for an ongoing consideration of what topology offers poststructuralist spatial theories.

Keywords: Mobius strip, post-structuralism, spatial theory, topography, topology

Introduction

Topology, it seems, is everywhere. A recent special issue on “Topologies of Culture” in Theory, Culture, and Society (29:4-5) includes articles from sociology, psychology, urban studies, philosophy, cultural studies, computer science, mathematics, and science and technology studies, while a Space and Culture (16:2) special issue on “Topological Sense-making” draws authors from performance studies, architecture, and media studies. Geographers, for their part, have characterized phenomena as diverse as borders, networks, computing, security, memory, power, cities, bodies and mobility as topological. What is the attraction of this term? Topological thinking holds, for many, the promise of a post-Euclidean spatial theory, a way of thinking about relationality, space, and movement beyond metrics, mapping, and calculation. Topology “can shed
new insight into familiar social science objects of research by mapping out how such objects change and how they relate, in this process, to other changing objects in multiple, relational spaces” and “provides the mental hand-holds for working with situations where relationships are changed, distanciated, collapsed or distorted, reshaping the ‘diagram’ one might draw of the situation” (Shields, 2012: 48). Topology is heralded both as a theoretical approach, and “a new order of spatio-temporal continuity for forms of economic, political, and cultural life” that represents “an ephochal transformation in the intersection between the form and content of cultural expression” (Lury et al., 2012: 4,5). Topology, we are told, is a new language, better suited for our current conjuncture: “The idiom of the topological offers a language for articulating the instabilities and fluctuations of state territory” (Harvey, 2012: 77), and overcomes the “limited ability of conventional geometric concepts to account for recent spatial shifts in the architecture of power” (Allen, 2011a: 284). Accompanied by a vocabulary of flows, deformations, twists, folds, torsions, severations, and cuts, topology captures, for example, how social relationships are remapped by information technology or how governments excise waiting zones from national territory. The diversity of this work is dizzying, while the significance of topology for geography today remains unclear.

In this article we ask what it is about topological thinking that attracts geographers, how geographers have thought with and through it, and what traction the concept may (or may not) have for theories of space and spatialization. Topology is not new to the discipline. Topology has been critical to the development of GIS software, for example, but what GI scientists do with topology has been of little interest to human geographers. In human geography, we find that there are many topologies with different analytic, ontological, and metaphorical implications. Topology is used as a metaphor, a heuristic device, an analytical approach, a figure, and an ontological relationship. Appropriating concepts from mathematics via diverse social theorists has produced, we think, a great deal of ambiguity about what geographers mean when they say that something is,
becomes, or acts topologically. What precisely is being topological (an object, network, or process) has significant implications for geographers’ theorizations of space, and for this reason we hope to bring some clarity to the term.

Topology's recent (re)emergence in human geography tracks from a range of social theorists and may be seen as part of a broader “mathematical turn” in critical social theory (Lash, 2012). Gilles Deleuze, Michel Foucault, Michel Serres, and Jacques Lacan, for example, have translated topology from a diverse field of mathematics, reformulated the concept and deployed it to new ends. Others, like Martin Hiedegger and Jeff Malpas, begin from the Greek notion of topoi and place to theorize the ontotopology of being. This latter approach, in particular, offers a non-mathematical philosophical basis for conceptualizing socio-spatial ontologies, without reference to geometry, measurement, or calculation. Elaborating the implications of Heidegger’s ontotopology for geography’s theories of space is indeed a rich line of inquiry, but lies beyond the aims of this paper (and is taken up by others; Joronen, 2012, n.d.). We do think that much work remains to be done elaborating the philosophical grounding of topological approaches to space, particularly given philosophers’ recent rethinking of concepts of space and place (e.g. Schatzki, 2002; Malpas, 1999, 2012a; Casey, 1993). We do not attempt, however, to review the emergence of topological thinking across social theory. Our aim is to critically review geographers’ topological approaches to space, and so we limit ourselves to several prominent strands that trace their roots to mathematical topology.

Geographers have long borrowed, appropriated, and remade concepts from mathematics and many other fields. In their 1981 paper, for example, Agnew and Duncan argued that geographers have a habit of borrowing concepts without fully investigating the context and controversies of their original formulation. Massey (1999) argues that these appeals to other disciplines (particularly physics and so-called ‘hard sciences’) often seek to invoke a higher scientific authority, to lend geographic theory a patina of incontrovertible truth. Geographers have
also been on the other side of this trade, exporting cartographic metaphors to cultural studies in the late 1980s and early 1990s (Smith and Katz, 1993). Just as other disciplines’ appropriations of spatial metaphors often work to depoliticize space (Massey, 1992), borrowed concepts smuggle with them ontological and epistemological assumptions that bear significant implications for geographers’ understandings of space, the social, relationality, change, and power (Agnew and Duncan, 1981; Elden, 2011). Further, geographers are not alone in their susceptibility to the allure of science, as the current fashion for neuroscience in cultural studies and the humanities attests (e.g., Massumi, 2002; Connolly, 2002; Smail, 2008). In the wake of such enthusiasm, critics have argued that these appropriations are often partial and in service of extrinsic theoretical or ideological agendas (Leys, 2011; Sokol and Bricmont, 1998). And so, taking on a mathematically inspired topology does not come without risks.

Yet we do think that topology has something to offer geographers. If there is something that unites geographers’ uses of topology, it is a move to conceptualize the dialectic between continual change and enduring relations. In the rest of the paper, we provide a brief introduction to mathematical topology, its introduction to the social sciences, and its recent uptake in critical social theory. Our aim is not to provide an exhaustive genealogy but to show that topological thinking has migrated from mathematics to human and social sciences. From there we review how human geographers are using topology to think about space in different ways. We find that geographers are attracted to topology because it provides a way of conceptualizing non-Euclidean space, but these approaches rely upon a problematic dichotomization of topographical versus topological space, wherein topography becomes an analog for fixity and topology for flow. In the third section, we unpack the theoretical implications of opposing topographical and topological space in this way. Specifically, we argue that invoking topology does not automatically allow us to avoid modern presumptions of subject and object as is often claimed. Rather, much theoretical work must be done to develop a poststructuralist topology. In the fourth section, we explore how we might do this by
presenting an argument for how topology allows human geographers to conceptualize immanent structure that remains open to new relations.

**From mathematical to post-mathematical topologies**

Topology is a broad branch of mathematics that has its roots in the 19th century studies of Carl Gauss (1777-1855), G.F.B. Reimann (1826-1866), and Felix Klein (1849-1925), thinkers whose work destabilized the sedimented assumptions of Euclidean geometry (Merzbach and Boyer, 2011). Reimann in particular generalized non-Euclidean geometry by showing that Euclidean space was not an absolute truth or scientific foundation. Instead of focusing on the Euclidean metric, Reimann urged the study of manifolds of any number of dimensions and with different curvatures; and indeed, Reimann’s insights in this regard made the theory of relativity possible (Merzbach and Boyer, 2011; Jammer, 1969). Topology, however, did not become its own branch of study until the end of the 19th century and beginning of the 20th, when Henri Poincare (1854-1912) systematically developed and elaborated concepts derived from Reimann and others in his study of analysis situs, or combinatorial topology, “the study of intrinsic qualitative aspects of spatial configurations that remain invariant under continuous one-to-one transformations” (Merzbach and Boyer, 2011: 553). For example, a circle, a square and a triangle can all be deformed into one another without cutting or adding to the figure. Likewise, a coffee cup and a donut are topologically equivalent, both of them being 3-dimensional surfaces with one hole (the handle/the donut hole).

Topologists thus treat figures as manifolds – spaces whose coordinates are not extrinsic, as in a line embedded within Cartesian grid, but rather intrinsic to the surface itself – and focus on what aspects of a figure remain constant (such as the figure’s dimensionality, or number of edges) when the surface is bent, stretched, rotated, but not cut or augmented. While many consider Poincare the founder of topology, the field is diverse in its orientations, and others point to other origins, such as the set theory of Cantor (1845-1918). Indeed, twentieth century mathematicians
such as L.E.J. Brouer (1881-1966) and Hermann Weyl (1885-1955) brought set-point topology together with Poincare’s analysis situs. Among other insights, point-set topology focuses on continuity, connectedness, and compactness and derives from set theory the principle that the relations among the elements, rather than the nature of the elements themselves, are the important aspects of a topological space.

Given that challenges to the hegemony of Euclidean geometry have been percolating in mathematics for almost two centuries, it is not surprising that topology found its way into social and human sciences in the first half of the twentieth century. Anssi Paasi (2011a) points to sociologist Lundberg’s use of topology in his text, The Foundation of Sociology (1939). Paasi quotes an extended passage that demonstrates that Lundberg turned to topology, and especially Reimann’s idea of the manifold, to suggest that space could be understood in terms of the qualitative social relations that compose it (characterized in terms of connection) rather than (merely) in terms of distance. As Paasi argues, Lundberg was a “neopositivist” who “strived to make sociology a science in the spirit of law-seeking physical sciences” (2011a: 300). During the high point of quantitative geography and after, geographers too have turned to topology in their search for a mathematical language that would capture the spatial, social, and cultural structures of urban life (Atkin et al., 1971; Yeats, 2001).

Likewise during this period, topology (which was having a heyday in US mathematics departments) was picked up by the Gestalt school psychologist, Kurt Lewin, who argued for an understanding of the relationship between the person and his environment as a topological “life space” in his Principles in Topological Psychology (1936). Lewin’s treatise was an attempt to establish topology as the basis for a truly scientific psychology -- though contemporaneous reviews of the book show that the psychological community was duly skeptical of the project, with one critic pointing out that “the mathematical symbols add no transcendent conceptual quality to the simpler verbal characterization,” and “the unambiguity of the mathematics does not guarantee the
unambiguity of the underlying psychological relationships” (Garrett, 1939, p. 517). Importantly, for both Lundberg and Lewin, topology was a way to make more scientific claims in the study of social relations.

Topology's more recent resurgence in critical human geography and other social sciences has a different flavor than the topology of the early to mid-twentieth century scholars who enlisted mathematics in the hope of bringing greater scientific rigor to their fields. Topology has become a poststructuralist buzzword, associated more with the flux of the Deleuzian virtual than with any élan of positivist precision. It is part of the broader rethinking of space as relationally constituted in critical human geography, as well (Amin, 2002; Lefebvre, 1991; Massey, 2005; Paasi, 2011b; Thrift, 2006). Insofar as Lacan and Deleuze use topology to place the subject within the space of the object rather than in a position of transcendence, theirs is more accurately termed a “post-mathematical topology” (Rosen, 2006:13; Secor, 2013). Our review considers topology in geography in these terms – not as an application of mathematical topology to social scientific inquiry, but as part of the development of a post-mathematical topology that is less concerned with fidelity to mathematical principles than with articulating a poststructuralist idea of space.

**Geography's topologies**

Much of the work on topology in geography has been associated with Actor-Network Theory (ANT) and draws from Serres’ and Deleuze’s topological imaginations (Latour, 1987; 2005; Callon, 1986). Topology for Serres accounts for the “multiple proliferations of spaces” that are populated by embodied subjects such as: “the Euclidean house, the street and its network, the open and closed garden, the church or the enclosed spaces of the sacred, the school and its spatial varieties containing fixed points, and the complex ensemble of flow-charts, those of language, of the factory, of the family, of the political party, and so forth” (Serres, 1982: 44-5; see also Serres, 1972). In fact, topology is important throughout Serres’ work, not only as a way of conceptualizing
multiple space-times (Connor, 2004). Serres uses topology as a method of analyzing texts. Just as mathematical topology does not impose an external metric system on a spatial form, but looks for rules of transformation within the topological figure itself, Serres argues that the rules for deciphering knowledge systems come from within those ordering systems themselves (Serres, 1982). Each contains its own spatial ontology, its own rhythms, its own theories of matter, systems of circulation, and rules of exchange. The problem, for Serres, is "to find the single space or the set of operators by which these spatial varieties in impractical, inconceivable vicinity will be joined together" (ibid., 51). Not only does this approach level the authoritative hierarchies of truth-making between, for example, science and literature, but it offers a shift in analytic perspective. In a move picked up by ANT scholars and a number of geographers, Serres proposes that we look for the endemic rules of connection and disconnection within a network or field of circulation, for the invariant structures that come to be repeated, whether within a piece of literature (e.g. Don Juan) or between field of philosophy and science (see Brown 2002 for a review; Serres and Latour, 1995). It is in this sense that Serres thinks of his work as structural, arguing that topology is the "true structuralism" (Serres and Latour, 1995: 35-6).

For both Serres and Deleuze, topology is invoked to unseat the primacy of Euclidean space and to assert a multiplicity of spaces. As Manuel DeLanda influentially laid out in his Intensive Science and Virtual Philosophy (2002), Deleuze's concept of multiplicity is related to the manifold, a space of some number of dimensions whose coordinates are intrinsic to it (rather than indexed to an extrinsic grid). For Deleuze, the manifold or multiplicity is understood as a nonmetric, continuous space within which a given topological figure at a given instant of time is but a single point. Such an understanding of the manifold goes beyond the basic topological definition of a manifold as a specific surface (e.g. a circle is a one-dimensional manifold, a torus or a Klein bottle is a two dimensional manifold) to draw on theories of configural space (or state space) in which the manifold is the space of all possible states. The Deleuzian manifold is thus a virtual continuum of
topological transformations, of becoming, that gives rise to differentiated multiplicities of Euclidean and non-Euclidean spaces. Deleuze’s purpose here is to rethink the ontological relationships between space, representation, and the material world, and to destroy the appeal to transcendent universal space on which cartographic representation relies. For Deleuze (1990, 1993; Deleuze and Guattari, 1987), conceptualizing space topologically is crucial to his spatial ontology.

Though their differences are rife, ANT and others drawing on Serres and Deleuze build upon the insight that spaces are multiple, processual, relational, and without a transcendent metric. Annemarie Mol and John Law deploy topology to argue for a multiplicity of social spaces with variable logics and dimensionalities (Mol and Law, 1994; Law and Mol, 2001). Just as Reimann’s argument for the study of curved metric spaces laid the groundwork for the general theory of relativity in physics, Mol and Law show that topology can enable the apprehension of “different kinds of space in which different ‘operations’ take place” (Mol and Law, 1994: 643). They suggest three different operative topologies for understanding the social space of anemia – that of the region (bounded clusters), the network (distance as a function of relations between elements), and the fluid (flowing, mixing, and leaking) (Mol and Law 1994). In a later paper, Law and Mol (2001) explain Latour’s idea of the “immutable mobile” in terms of an object’s dual inscription in Euclidean (regional) and relational (network) space: an object may move in Euclidean space but be immobile in network space, so long as the relations that define it are sustained. As they did in their previous essay on anemia, Law and Mol once again move beyond ANT’s focus on the regional and the network to suggest that other non-Euclidean topologies may be useful to get beyond “the network metaphor which links an appreciation of relationality to a specific image of connectivity” (Law and Mol, 2001: 613). Thus once again they introduce other topological figures with different rules for continuity and change, such as fluid spaces, in which any continuity is dependent upon gradual change, and a fire, in which continuity is an effect of constant, abrupt change and the “flickering relationship between absence and presence” (Law and Mol, 2001: 615). Other scholars have built
upon this approach, both by applying fire and fluid topologies in their own analyses and suggesting yet other kinds of spaces, such as smoke or gel topologies (Moreira, 2004; Sheller, 2004).

Law and Mol and others have thus used the topological insight of the manifold – of there being a multiplicity of spaces within which specific rules for continuity and transformation obtain – to generate creative new ideas about how social space works. In a similar fashion, Whatmore and Thorne (1998) deploy topology in their analysis of the overlapping, coexisting commercial and wildlife networks. Just as Law and Mol argue that different kinds of connectivity and relation produce different spaces, Whatmore and Thorne argue that wildlife networks are “fluid topologies” that result from relational, heterogeneous networks, each with their own spatial and temporal modes of ordering. Conceptualizing wildlife in this way refigures the relations of interiority and exteriority that have defined human and animal, thus offering an explicit critique of classificatory knowledge practices that dominate contemporary wildlife ecology. Arguably, these alternative spatialities permit more nuanced analyses of the dialectics of absence and presence and the complexity of connections than either commonsensical (mostly Euclidean) or network topologies allow (Callon and Law, 2004). These scholars’ work is clearly post-mathematical; topology here is an inspiration, but the specifics of mathematical topology are left far behind. Despite the fertility of these new spatial imaginaries, however, the sheer scope of these alternative topologies (fluid, fire, gel or smoke) raises the possibility that their full potential for the development of spatial thinking may be lost in their endless proliferation. To move spatial theory somewhere, our theories need to be able to build on one another, rather than continually producing new terms (Paasi, 2011b).

The suggestion of multiple topologies aside, ANT-influenced scholars often work with a simpler binary between intensive (qualitative, topological) and extensive (metric) relations. For these scholars, Deleuze’s virtual continuum is topological, while discrete points, regions, or territories are seen as temporary stabilizations of generative topological processes (Murdoch, 2006). Thus, rather than defining particular topological spaces in terms of specific logics and modes
of connectivity, those working in this mode tend to equate topology with the virtual field of movement, flux, and emergence (Deleuze and Guattari, 1987). Privileging this “topological hyperspace” (Massumi, 2002: 185) as the virtual multiplicity out of which extensive geographies are differentiated, geographers drawing on ANT have variously argued that size, scale, cities, and even space itself are “network effects,” arrested snapshots of intensive topological becoming (Thrift, 1993; Latham, 2002; Murdoch, 2006). John Allen, for example, argues that intensive power relations generate a certain power-geometry; he writes, “Distanciated ties and real-time connections are not understood as lines on a map which cut across territories, but rather as intensive relationships which create the distances between powerful and not so powerful actors,” (Allen, 2011a). In the same vein, Hinchcliffe et al. (2012) critique the geometrical and Euclidean conceptions of disease, which demonstrate a will to enclose ‘clean’ territories from diseased ones, in favor of a relational, topological “diagram” of “an entangled interplay of environment, hosts, pathogens and humans” (8). For them, topology is deployed to disrupt commonsensical assumptions about distance and proximity, past and present.

Matthew Hannah’s work on geographical imaginations of terrorism, though in a different register than Allen’s (and indexed to Foucault rather than to Latour and Serres), resonates with Allen’s ideas about how subjects’ affective capacities may be spatially extended. Hannah (2006) contrasts subjects whose everyday activities he defines as “proportional-point topologies,” in which the subject’s ability to affect her material surroundings is constrained and predictable, to terrorists, whose sphere of effective action is much wider than ordinary individuals’. Terrorists perform what he names an “expanding-point topology,” because they enact “damaging expansion of the scale of impacts of a person or object far beyond its normally modest sphere of expected effects” (Hannah, 2006: 628). For Hannah, these topologies define the spatio-temporal ontology through which biopolitical subjects are made legible and normalized, and it is the abnormality of terrorism’s
expanding-point topology that justifies extreme responses like torture, rendition, and indefinite detention.

Topology's field of deployment in geography is not cut from a single fabric. Other geographers have engaged with topology drawing on Giorgio Agamben's work on sovereignty and the space of exception. Agamben's topological imagination focuses on "the sovereign exception," which he argues is "the fundamental localization (Ortung), which does not limit itself to distinguishing what is inside from what is outside but instead traces a threshold (state of exception) between the two, on the basis of which outside and inside, the normal situation and chaos, enter into those complex topological relations that make the validity of the juridical order possible" (Agamben, 1998: 19). The shape of "those complex topological relations" is given by the camp, the "hidden matrix of the political space in which we are still living" (Agamben, 1998: 175). For this reason, Agamben writes, "we must admit that we find ourselves virtually in the presence of a camp every time such a structure is created, independent of the kinds of crime that are committed there and whatever its denomination and specific topography...[I]t is this structure of the camp that we must learn to recognize in all its metamorphoses" (Agamben, 1998: 174-175).

The movement between sovereignty's topological figure and the material localizations of spaces of exception in Agamben's work has inspired two slightly different emphases: 1) sovereignty's topology as a process that structures the materialization of specific manifestations of spaces of exception; 2) the theorization of the camp as a threshold and zone of indistinction between inclusion and exclusion. The former approach parallels the Deleuzian work noted above in that (material) spatial formations emerge from (virtual) topological processes. However, while the Deleuzian approach frames the topological as a "space of spaces," or a manifold in which any potential topological figure is but a point, those following Agamben focus on one specific topological figure (the Mobius inclusion/exclusion of sovereignty) from which a particular kind of space (the camp) is manifest. For example, for Mountz (2011), sovereignty's topology produces "distinct
material topographies of exclusion,” against which she mobilizes a feminist counter-topography of cross-border solidarity. In this way, sovereign topology works as a process producing specific material geographies.

In their analysis of Auschwitz’ spatial orderings, however, Giaccaria and Minca (2011) emphasize the camp as a threshold. Rather than producing the fixed coordinates of the camp, topology for Giaccaria and Minca necessarily erupts from the Nazi’s failure to secure the camp as a seamless, rational order. For them, Nazi spatial planning attempted to materialize calculative rationality, imbued with ideologies valorizing pre-modern Germany, in the camp. Yet the completion of the camp’s project required certain zones of ambiguity: ‘Mexico’, a sub-camp in which detainees were never entered in rolls; and the comingling of civic and camp administration. “In and on that threshold, in fact, the calculative and topographical aspirations of the Nazi project meet a crucial limit and a real challenge, and open up to the irruption of the topological” (Giaccaria and Minca, 2011: 4). In this way, “the topological spatialities of the camp-as-a-threshold” thereby become “a constitutive element of the overall biopolitical Nazi project of ‘protective custody’ and extermination.” Gregory (2006) finds a similar inspiration in this aspect of Agamben’s work. Tracing Israel’s reordering of Palestinian time-space across technical, cultural, and political juridical registers, he links cartographic representation of Palestinian land to discourses of civilization and barbarism to the internal banishment of Palestinian citizens. In a series of moves, Palestinian national space has been splintered, he argues, first through “a series of severations in Euclidean space” (Gregory, 2006: 123) and then through a “violent fragmentation and recombination of time and space” (132). For Gregory, these deformations of Palestinian daily life, these torsions of time and space, constitute the space of exception’s zone of indistinction (124).

For Gregory or Giaccaria and Minca, topological and topographical spatialities operate according to different logics and these spatial logics are brought into tension in the camp-as-threshold. For Mountz (2011), in contrast, detaining asylum-seekers in extra-territorial detention
centers is the spatial expression of sovereignty’s topology. Likewise, Belcher et al. (2008) contrast the topology of the exception, in which the law and its suspension become indistinguishable, to the “topographical juridical-territorial order” that it produces. On the surface, these may seem to be semantic differences, but as we will discuss in more detail below, topology’s relationship to topography has significant implications for topology’s specific potential for theories of space.

While they emphasize different aspects of Agamben’s topological conception of sovereignty, Mountz, Giaccaria and Minca share an interest in how state governments manipulate cartographic boundaries in order to strip detainees of rights. Didier Bigo garners similar insights, but does not rely on Agamben’s topological sensibility. Bigo (2001) argues that securitization weaves together the government of internal and external threats, metaphorically represented as a Mobius strip on which inside and outside run seamlessly together. For Bigo, the Mobius strip provides a figure through which to understand “the transversal dispositif” that connects “previously unconnected conceptual worlds—internal security, external security, war and conflict, and crime and delinquency” (Bigo 2008: 30). Bigo’s Mobius strip metaphor has been widely cited to describe how contemporary border security practices blur boundaries between inside and outside. This blurring of the border is often accompanied by analyses of a mobile border, a deterritorialized border that is no longer marginal but “transported into the middle of political space” (Balibar, 2004: 109). For Mezzadra and Nielson, these interiorized borders “could be said to work as topological functions, which at once connect and divide, cross and cut political space, include and exclude” (Mezzadra and Nielson, 2012: 63). Thus for Mezzadra and Nielson, borders are not topological in an Agambenian sense; the camp is not the defining figure shaping border practices for them. Instead, topological borders operate through “differential inclusion,” the filtering and selecting out of preferred and risky subjects and the distribution of citizenship benefits according to hierarchies of labor market skill-levels (see also Mezzadra, 2011). It is through specific bordering practices, such as excision, externalization, and points-based immigration systems that states experiment with how “the timing
and tempo of migration can be more precisely regulated” through “degrees of internality and externality” (Mezzadra and Nielson, 2012: 68). For many border scholars, then, topological figures like the Mobius strip have offered useful metaphors for conceptualizing paradoxical relationships between law, security, and territory. In contrast, Mezzadra and Nielson argue that the spectrum of bordering practices themselves operate topologically, so that topology is not merely a heuristic device, but the current ontological situation.

Given the diversity of scholarship sketched here, what is the status of topology in geography? Is the discourse on topology ontological (suggesting that being’s spatiality is topological), epistemological (suggesting that topological language works as an analytic tool, a heuristic or even a metaphor), or empirically descriptive (aptly representing emergent phenomenon such as time-space compression, the blurring of boundaries, and new forms of connectivity)? In other words, is topology useful because it helps us to explain what space is or how space works (in all times and places), or is it useful because it corresponds to the dominant trends of our current historical conjuncture? Reading across the texts we have reviewed here and others as well, it appears that there is a diversity of (mostly implicit) positions on this question. For Deleuzian topologists, the manifold includes a multiplicity of Euclidean and non-Euclidean spaces. For Law and Mol (1994, 2001), topological figures describe different arrangements of connectivity and change that create social spaces. For Allen (2011a, 2011b) and Hannah (2006), topology allows us to conceptualize how certain actors have effects at or in spite of distance. For Agambenian and border scholars, topological figures of the camp and the Mobius strip capture reconfiguring relationships between interiority and exteriority. And for still others, topology is one form of spatial logic, to be analyzed for its material and discursive effects. The matter is made all the more confusing by those that argue that the spread of mathematical topology in computing has occurred alongside cultural theory's becoming-topological (Lury et al., 2012; Shields, 2012). What is at stake for geographers, we think, is the clarity and precision of our theories of space.
The limits of topology versus topography

A genealogy of the commonplace distinction between topology and topography would have to examine the historical development of what Henri Lefebvre (1991) identified as the impasse between mathematical theories of space and the “social” space that geographers, urban planners, and architects have taken as their object. The divergence between these fields, in which “real” (lived, experienced) space is understood to be a thing apart from the “true” (mental) space of the mathematicians and philosophers is, according to Lefebvre, an error that perpetuates a distorted view of space. This impasse continues to resurface in the guise of a distinction between the topological (true, conceptual) and the topographical (real, lived) that often grounds definitions of topology in geography. As it was for Lefebvre, the problem remains to articulate the processes through which space is at once both real and conceptual, both geographical and mathematical. It is with this goal in mind that we critically interrogate the work that topography does in contemporary topological approaches. As such, our analysis falls short of tracing the history and meaning of topography, itself a term richly laden with connotations of land, landscape, place and region. Instead, we focus narrowly on how topography is deployed as the ‘other’ to topology in the (post)mathematical spatial turn.

When geographers and other social scientists introduce the concept of topology in their texts, they often do so by way of a contrast to topographical space (e.g., Belcher et al. 2008; Amin, 2002; Giaccarria and Minca, 2012; Hinchcliffe et al., 2012; Mountz, 2011; Blum and Secor, 2011; Harvey, 2012; Whatmore and Thorne, 1998). For example, Jonathan Murdoch’s introduction to Post-structuralist Geography (2006) includes a text box laying out the dictionary definitions of topography and topology. Murdoch writes that topographical spaces “are seen as ‘contained spaces’, in which space is seen in terms of its surface (maps, points, lines, contours and so forth)” (Murdoch, 2006: 12). He goes on to say that topographical space “is also sometimes called
‘Euclidean’ space because of this concern for contained surfaces” (12). Topology, on the other hand, he asserts, “refers not to surfaces but to ‘relations’ and to the interactions between relations,” (12). Setting aside the violence that these particular definitions do to mathematical topology (which in fact is all about understanding surfaces as spaces), Murdoch’s summary reflects a common tendency to embark on topological geographies by situating topology in relation to its apparent other. Massumi writes that the difference between topology and geometry lies “between the process of arriving at a form through continuous deformation and the determinate form arrived at when the process stops” (2002: 184). Belcher et al., following Massumi, argue that “topologies, unlike topographies, do not map discreet locations or particular objects” but “create the conditions for particular materialized sites” (2008: 499). Hinchcliffe et al., also define their topology against topography, arguing that “this is not a world of flat surfaces, with well-defined proximities... but rather a topological landscape of embeddings and disembeddings.” (Hinchcliffe et al. 2012, p. 8).

The temptation to dichotomize (topological) becoming and (topographical) being is fueled by easily extractable statements that seem to confirm this dichotomy in texts by Deleuze and Guattari, Agamben, and Serres. Take, for example, the following: “Perhaps we must say that all progress is made by and in striated space, but all becoming occurs in smooth space” (Deleuze and Guattari, 1987: 486). Similarly, Agamben repeatedly refers to the camp as a “principle of localization” and ”hidden structure” of politics that operates virtually, implying that any actually existing space of exception is derivative of the topological form. Serres explains his topological approach to time: “Classical time is related to geometry...it’s simply the difference between topology (the handkerchief is folded, crumpled, shredded) and geometry (the same fabric is ironed out flat)” (Serres and Latour, 1995: 60). Yet Deleuze and Guattari are also careful to explain the co-dependence of topological and topographical, which they use to “model” smooth and striated space respectively. For them, “no sooner do we note a simple opposition between the two kinds of space then...we must remind ourselves that the two spaces in fact exist only in mixture” (Deleuze and
Guattari, 1987: 474). For Deleuze and Guattari, the crucial questions concern how smooth and striated space pass from one to the other, the rules of connection and disconnection that characterize them, and how they articulate with one another (ibid., 475).

But what are the consequences of constructing and deploying this dualism (topology/topography, or topological space/Euclidean space) to talk about space? At worst, the dichotomization of these two kinds of space reintroduces into spatial theory a host of dualisms that poststructuralist theory has sought to deconstruct. While some topological work explicitly engages the relationship between ‘topology’ and ‘topography,’ other work valorizes the topological, a move that is part of a wider trend to celebrate mobility, flows, and change over sedentarism, localization, and stasis. Indeed topology often seems to stand in for a general ‘relational’ view of space, resulting in vague theoretical points where spaces and places become “a heady swirl of spatial trajectories and flows, in which boundaries, if they remain at all, take on a highly uncertain status” (Malpas, 2012: 4). Topography becomes a secondary effect of topology, or perhaps an over-coding. Massumi, for example, explains that, "To get a static, measurable, accurately positioned visual form, you have to stop the movement...leaving a Euclidean form as a static witness to its arrested dynamism" (2002: 183). However heuristic such an understanding may be, it can be argued that privileging the field of multiplicity and movement over identity and stasis basically re-inscribes a Platonic becoming/being or body/mind dualism into spatial theory (Rosen, 2006).

So what happens if we do not focus on counterpoising topology to topography, and instead interrogate how topological operations crosscut these concepts? For in fact, topology can be mobilized precisely to work against the becoming/being dualism that the topology/topography distinction seems to replicate. This potential is emergent in the work of a number of scholars deploying the concept. Mezzadra and Nielson write, for example, that "no matter how much topology draws our attention to unexpected forms of connection and continuity, it must also account for processes of partition, filtering and hierarchization" (2012: 59). Echoing Malpas’
caution against valorizing emergence over boundedness, they seek to rethink how borders order national space but in ways that no longer rely upon or reproduce cartographic, calculated territory. For them, the point is not that topological spatialities supercede or replace topographical mappings, but that “borders, far from serving simply to block or obstruct global flows, have become essential devices for their articulation” (Mezzadra and Nielson, 2012: 64). The calculative territorial logics others associate with topography is, in their analysis, one topological figure among many. Gregory, too, understands Euclidean space as space that can be “splintered,” severed and twisted “into ever more violent constellations” (2006: 123-4). This is much like Joronen's (2012) analysis of different spatial ontologies, i.e. how different modes of being-in-the-world shape the ways in which being is revealed. For example, global capitalism requires that things are put in relation to similar things everywhere (trees come to be understood in terms of a global lumber market, for example), and scientific practices enact a different mode of gathering-together than the gathering of a jug (Malpas, 2008: 240). For these authors (and see also Lash, 2012), it makes little sense to oppose topology and topography. Rather, the point is to understand Euclidean space as one possible topology among others.

A basic insight of topology is that there are a multiplicity of possible spaces, or manifolds, of which the Euclidean plane is but one, existing halfway between a convex and concave curved space: “Inasmuch as the plane is a surface with constant zero curvature, Euclidean geometry can be regarded as an intermediary between the two types of non-Euclidean geometry” (Merzbach and Boyer, 2011: 498). As soon as we suggest that topology is to be defined in opposition to topography, and define topography in terms of Euclidean space, we have privileged Euclidean space as the one geometry to be contrasted to all others. But our point is not that it is 'bad mathematics' to rely on a contrast to topography to define topology. We recognize the impulse behind the dichotomization: the topography/topology distinction is often the quickest way to explain to the uninitiated what topology is; indeed we have used it ourselves in previous work. Yet, no matter how useful the
Towards a Poststructuralist Topology

Without doubt, the desire for a language and theory of space that would be thoroughly poststructuralist has worked to raise the profile of topology in geography. Callon and Law, for example, introduce a special issue of EPD saying that the authors collected therein (e.g. Thrift, 2004; Moriera, 2004; Sheller, 2004; Urry, 2004) are concerned with: “What happens if presence and absence -- or proximity and distance -- are not opposed to one another; and, second, what happens if there is no spatial totality or shared context” (Callon and Law, 2004: 3). For these scholars and many others, thinking about space topologically dynamites any metaphysical foundation that would fix space or secure its dialectical completion. We have seen that, for many, topology is used to unmoor the presumed fixity of the structural grid and to open up space to a multiplicity of modes of connection, continuity, and discontinuity. Further, we recall that Deleuze and Guattari (1987) mobilize the idea of the manifold – in which coordinates are no longer Cartesian and external to the surface, but rather mapped directly onto it self-referentially -- to unseat the transcendent structure of symbolic signification. Thus topology has been positioned as an entry point not only to a poststructuralist idea of space, but to a spatial idea of poststructuralism.

Topology does not merely direct us to the (well worn) idea that space emerges from the relations between things; it directs us to understand the spatial operation of continuity and change, repetition and difference. In other words, topology directs us to consider relationality itself and to question how relations are formed and then endure despite conditions of continual change. As Lury et al. (2012) note, topologically speaking, “invariance and intrinsic change (understood as deformation) are not incompatible; rather, they are rigorously inter-related” (p. 8). When, for example, a donut is deformed into a mug, the topological structure is ‘the same’ but this wouldn’t be
worth mentioning except for that it is also different (more suitable to receive coffee). The topological space is ‘repeated’ by the homomorphic transformation, but with a difference. A manifold of all mappings that share a topological structure resonates with Derrida’s field of freeplay, “a field of infinite substitutions in the closure of a finite ensemble” (Derrida, 2007 [1970]: 260). Yet, for Derrida, this freeplay of substitutions is permitted not by the fixed presence of an organizing center, but rather by the lack of such a center. We have said, along with the topologists, that the mug and the donut are the same, that they and a multitude of substitutes share a structure. The question then must be asked, is topological ‘structure’ an absence at the center of the field of transformation, or is it an organizing presence governing the freeplay of topology? Has topology smuggled structure into spatial theory?

Our point is not that topology is simply the latest substitute for a structural foundation, but that one cannot count on topology to bring with an inherently poststructuralist conceptualization of space without attending to it with theoretical precision. For example, in the first chapter of Post-structuralist Geography, Murdoch refers to Law and Urry (2004) to argue that “structuralist theory sees space as a surface configured by the play of underlying structures” (Murdoch, 2006: 12) and then, on the same page, with a seeming lack of reflexivity, he proposes a distinction between topology and topography in which “any spatial coherence that is achieved (on the surface) serves to disguise the relational complexities that lie ‘underneath’ spatial forms” (12). In this formulation, topology becomes very much like the “base” to the topographical “superstructure” -- except, to be fair, topological space is not defined by a set of given relationships (such as a capitalist mode of production) but rather by the multiplicity of potential relationships that comprise that space.

Nor should this equation of topology with structure be surprising. As Deleuze writes in Difference and Repetition, “the reality of the virtual is structure” (Deleuze, 1994: 209). Lacan likewise refers to the “strict equivalence between topology and structure” (Lacan, 1998: 9) and repeatedly insisted that the topological structure of the subject is not an analogy or a metaphor
(Lacan, 2007 [1970]; 1973). Yet for Deleuze and Lacan, the structure that they propose is immanent, which is to say that it is not separate from its effects. Lacan for example argues for an understanding of structure “not as a theoretical model” but as that which operates within the scene, [met en scene] to direct the subject (Lacan, 2006 [1966]: 544).

Another way to understand this immanent idea of structure is to focus on the mutual constitution of the virtual and the actual. Deleuze introduces the quasi-cause in order to explain how the virtual and the actual give rise to one another. The quasi-cause is the operator that splits every present moment into the future and the past, giving rise in the process both to ‘what actually happens’ (the actual) and its excess -- the field of potentialities, or the virtual (Deleuze, 1990). Thus for Deleuze, it is not the case that the virtual is the determining realm of “relational complexity” that is “disguised” by topographical coherence (as Murdoch's parsing of topology and topography would have it). Instead, the quasi-cause is like the twist in the Mobius strip, an immanent supplement that inaugurates the differentiation of the virtual field that leads to actualization and back again. Thus for both Lacan and Deleuze, the topological may be a structure, but it is not a transcendent field or a timeless essence -- it is, rather, what we could call a poststructuralist spatial structure.

It is helpful, perhaps, to consider an example of how topology has been used to conceptualize immanent structure. For this, we turn to the Mobius strip, a single-sided topological surface (a manifold) that nonetheless maintains a distinction between its two sides. Although topologically speaking a Mobius strip does not even have the thin depth of a piece of paper (it is two-dimensional), we can model it by taking a strip of paper, twisting it, and then connecting the ends. The result is sometimes called an inverted figure eight, and has been drawn on by many theorists to explain a relationship in which difference and identity become indistinguishable. That the Mobius works in this way is easily demonstrated; a person can grasp the paper strip with fingers at the same point but on ‘opposite’ sides, thus affirming that the figure has two distinct sides. And yet, if one runs one's finger over the twisted strip, it will cover 'both' sides and end up
where it started, thus proving that the figure has but one side. In 2-dimensions, this travelling ‘finger’ would meet its mirror image at the point where it had started (Weeks, 2002). Lacan refers to the Mobius as “something which is at the same time one or two” (Lacan, 2007 [1970]: 192) and argues that it is the basic symbol of the subject. Other topological surfaces, such as the Klein bottle or the cross-cap, embed this Mobius structure, and likewise have been used to diagram the continuity of the discontinuous. The Klein bottle, for example, is another two-dimensional figure (constructed from a 2-dimensional torus) in which self-intersection (an impossibility in three dimensional space, but topologically imaginable) allows the interior space to open out and become the exterior of the bottle; like the Mobius, one can imagine fluid poured into the bottle flowing over its exterior, and vice-versa. In Rosen’s words, “The Klein bottle’s ontological blending of the contained object and its containing space confounds the idealized division of these terms into separate dimensions...[A] dialectic of continuity and discontinuity is enacted with the Klein bottle” (Rosen, 2006: 34-35).

It is because of this dialectical operation that the Mobius and related figures continue to be called upon to rethink relationships such as sovereignty and the ban, the mind and the body, inclusion and exclusion, presence and absence. For example, Elizabeth Grosz (1994) introduces Volatile Bodies by indicating that she will use the Mobius strip as a model that gets beyond dualisms (mind/body, interior/exterior) or monisms (the idea of singular mind/body substance, which fails to capture dissonance and disarticulation). Variously mobilized to explain the operations of subjectivity or sovereignty, the Mobius is thus a spatial structure, a specific ‘shape of space,’ that operates by different rules of connectivity and transformation than, say, a plane or a sphere.

But what is poststructuralist about the Mobius relation? Zizek takes on the Mobius as a post-Hegelian dialectic in The Parallax View (2006). He argues that the Mobius is a model of a dialectic in which there is no totality and no synthesis, but rather a continual transformation that constitutes both difference and identity (Secor, 2008). The two sides of the single-sided Mobius, for Zizek,
represent the problem of antinomy in the Kantian sense: a difference that cannot be mediated or negated, because there is no common ground between them. Yet, there is also no polarity between these two sides; the difference between these two sides is, Zizek argues, the minimal difference that separates the One from itself, the subject from the void, the signifier from its place of inscription. Thus the difference between the two sides of the single-sided figure is not a difference between two positively existing things, but difference as such (Zizek, 2006). The Mobius topology is thus one that enacts a poststructuralist dialectic in which it is difference itself that initiates figure. Unlike structuralist dialectics, in which two distinct objects act on each other, a Mobius relation describes a movement between two inseparable states of being. The Mobius strip locates difference, through the twist, within a single object. This figure should not, however, be taken as a model for all relations of difference everywhere or as a structuring mechanism for a transcendental subject, but as a way to describe a particular kind of connection, relation, and movement. The Mobius thus demonstrates how the insights of topology -- from the multiplicity of manifolds to the interplay of transformation and continuity -- can be harnessed for the development of poststructuralist spatial theory.

Conclusion: Trailing Assumptions and Topological Desire

Yet to return to the skepticism with which we began our investigation, we must ask whether topology has re-emerged today less because of its theoretical potential and more as part of a renewed attempt to lend a scientific veneer to humanist and social scientific work. This possibility is more apparent in some deployments than others; few of the papers we have reviewed have attempted to present the reader with mathematical formula or theorems, and even an example of one that does (Sha, 2012) emphasizes the poetics, not the axiomatics, of topology. But given that there are periodic passions for the appropriation of scientific authority in the social sciences and humanities, we must ask ourselves: What if topology is not only the object of our
barely repressed desire for scientific authority, but moreover has come dragging along with it a host of decidedly positivist and even Cartesian assumptions?

In his brilliant and strange book, *Topologies of the Flesh*, Rosen (2006) argues that the assumptions of mathematical topology are actually in conflict with its poststructuralist reworking. He argues that, while thinkers like Deleuze celebrate topological space as having been freed from the transcendent dimension, in fact the distinction between intrinsic and extrinsic properties in mathematical topology does perpetuate divisions between container and contained and subject and object. He uses the example of the Klein bottle, a two-dimensional figure that cannot be viewed as whole except for in four-dimensional space (because three dimensional space does not allow its self-intersection). Likewise, the 2-dimensional Mobius strip could not be constructed in the two dimensions of ‘Flatland’ because the twist could not be performed without ripping the strip, thereby destroying the figure’s topological integrity (remember, no breaking!). Thus he points out that insofar as the Mobius and the Klein bottle are 2-dimensional manifolds that must be embedded within other manifolds of different dimensions (of 3 and 4 dimensions respectively), they are objects contained within spaces apprehended by externally located (3rd or 4th dimensional) subjects. So much, then, for topology’s negation of the external dimension; instead, here the viewer and the object cannot coexist on a single plane.

Where does this leave us? Rosen does not abandon topology even though he understands that some of its basic assumptions are in tension with his own philosophical project. Instead, he suggests that we do it differently; rather than embedding the Klein bottle in an imaginary 4-dimensional space, he suggests that we “stick with the hole” that it requires in 3-dimensions and to understand this hole as in fact the structure of the subject. Without going any further into Rosen’s own topological project, we would like to suggest learning from how he handles the baggage of topology in order to build the kind of understanding he desires (which happens to be a phenomenological topology). He examines the way in which mathematical topology fails to provide
the object-space-subject relationship that he desires, and he suggests a different understanding that, while it is enriched by the topologically defined figure of the Klein bottle, nevertheless does not rely on mathematical topological assumptions.

The fact that topology comes trailing historical and intellectual associations does not mean that we have to drop it like soiled tender. More productively, following Xin Wei Sha (who is mathematically trained), we may aim to use topological terms “mildly but responsibly loosened from the contexts in which they traditionally have been defined” (Sha, 2012: 222). We cannot expect that topological terms are neutral; as Derrida put it, “Concepts are not elements or atoms and since they are taken from a syntax and a system, every particular borrowing drags along with it the whole of metaphysics” (Derrida, 2007 [1970]: p. 251). Yet, for Derrida, “there is no sense in doing without the concepts of metaphysics in order to attack metaphysics. We have no language -- no syntax and no lexicon -- which is alien to this history; we cannot utter a single deconstructive proposition which has not already slipped into the form, the logic, and the implicit postulations of precisely what it seeks to contest” (p. 250). This inability to shake the concepts that we interrogate (whether the sign, or, in our case, space) from their moorings is the paradox of critique, a condition that one cannot avoid but about which, Derrida suggests, one can be more or less naive. Perhaps, then, what spatial theory needs is simply a less naive topological critique.

Our goal here has not been to consolidate a particular approach to topology, nor is it our mission to call for stricter allegiance to mathematical topology. There is little to be gained from such claims when the disciplines have such different orientations, questions, and theoretical traditions. We want, however, to push geographers to think more carefully about what it means--for theories of spatialization, structure, dialectics, and causality--to take up a topological approach to space. For example, it is not enough to begin or end an article by saying that something (or some process) is topological. This amounts to saying little more than that a thing can be described by topology, while saying nothing about the rules of connection, disconnection and transformation
through which it works. Nor is topology a satisfactory synonym for relationality, since topology—both its mathematical and post-mathematical guises—is about how and through what process those relations are repetitively reproduced, and yet continually changed. For topology to be more than a fashionable buzzword, we need to think carefully about the ontological and epistemological assumptions undergirding topology’s allure.

Topology’s “post-mathematical” potential resides, we think, in how it is reworked through postructuralist theories of space. To move spatial theory in geography forward, our engagements with topological thinking need to build on one another, rather than to produce an endless proliferation of singular deployments (see also Paasi, 2011b). Here we have pointed out a few ways that topology can open up spatial theory to a sustained consideration of difference and repetition, poststructuralist dialectics, and multiple shapings of space. Such a topological approach could, for example, enrich our understanding of how spaces of racial or class inequality are reproduced despite changing social, economic, or political frameworks, or how political logics travel but produce drastically different outcomes when mobilized in different contexts. There are many more possibilities, of course, that remain to be conceptualized. Others need not agree with our analysis here, but without sustained conversation about what post-mathematical topology can do for our theories of space, we risk missing rich opportunities to expand our spatial vocabulary. Our call is for a topology that enables an ongoing critique of spatial truths, a topology that is not a substitute but a spur for the development of poststructuralist spatial theory.

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