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Deposited in DRO:
21 March 2018

Version of attached file:
Published Version

Peer-review status of attached file:
Peer-reviewed

Citation for published item:

Further information on publisher’s website:
https://doi.org/10.1088/1742-6596/888/1/012197

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Leptonic flavour mixing influenced by flavon cross couplings

To cite this article: Ye-Ling Zhou 2017 J. Phys.: Conf. Ser. 888 012197

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Leptonic flavour mixing influenced by flavon cross couplings

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Abstract. We develop an economical approach to connect flavon cross couplings with flavour mixing. A non-zero reactor mixing angle and almost maximal CP violation arise from cross couplings between flavons in the charged lepton and neutrino sectors. Within this approach, we analyze how cross couplings contribute to charged lepton-flavour-violating decays. We find branching ratio sum rules of these processes and observe that the electroweak-scale flavour symmetry is consistent with current experimental data.

1. Introduction
Current neutrino oscillation data support specific lepton flavour mixing patterns [1]. The most famous one is the tri-bimaximal (TBM) mixing, which predicts $\sin \theta_{12} = 1/\sqrt{3}$ and $\sin \theta_{23} = 1/\sqrt{2}$, fit well current oscillation data and has therefore attracted lots of attentions [2]. The exact TBM mixing has been ruled out due to its prediction of a vanishing $\theta_{13}$. To be compatible with current data, corrections to TBM must be introduced, $\sin \theta_{13} = \sqrt{2}r'$, $\sin \theta_{12} = \frac{1}{\sqrt{3}}(1 + s)$, $\sin \theta_{23} = \frac{1}{\sqrt{2}}(1 + a)$ with $r' \sim 0.1$ and $|s|, |a| \lesssim 0.1$ [3].

These specific mixing patterns can be predicted by flavour models, where an underlying non-Abelian discrete symmetry in the flavour space is often assumed. The simplest symmetry to realise TBM is the tetrahedral group $A_4$ [4, 5]. Flavons play a key role in these models. They break the flavour symmetry after they obtain vacuum expectation values (VEVs). To achieve TBM in $A_4$, residual symmetries $Z_3$ in the charged lepton sector and $Z_2$ in the neutrino sector should be preserved by the flavon VEVs. But cross couplings of different flavons break them. In lots of former works, extra dimension [4] or supersymmetry [5] is imposed to forbid these cross couplings. The price is that many degrees of freedom have to be introduced into the model.

While flavons are essential to realise typical flavour structures, they may contribute to any processes involving leptons, and lead to significant deviations from the Standard Model. The most precisely measured processes are charged lepton-flavour-violating (LFV) decays. However, it is hard to make a statement that new physics comes from flavons, even if we found some experimental hints. To prove flavons, we must utilise some special features of flavour models. One interesting viewpoint is residual symmetries and the connection with flavour mixing [6].

In this paper, we introduce our new approach of flavon cross couplings proposed in [7] and list the main results of flavon-induced charged lepton-flavour-violating (LFV) decays given in [8]. Section 2 discusses how flavon cross couplings influence flavon VEVs and flavour mixing. Section 3 reviews the corresponding charged LFV processes. We summarise in Section 4.
2. Flavour mixing modified by cross couplings

We assume the flavour symmetry to be the tetrahedral group $A_4$ [9], which is generated by $S$ and $T$ with the requirement $S^2 = T^3 = (ST)^3 = 1$. It has four irreducible representations $1, 1', 1''$ and $3$. The product of two triplets are reduced as $3 \times 3 = 1 + 1' + 1'' + 3_S + 3_A$, where $S$ and $A$ denote the symmetric and anti-symmetric parts, respectively.

We introduce two flavon triplets $\varphi = (\varphi_1, \varphi_2, \varphi_3)^T$, $\chi = (\chi_1, \chi_2, \chi_3)^T \sim 3$. They are pseudo-real, satisfying $\varphi_1^* = \varphi_1$, $\varphi_2^* = \varphi_3$, and $\chi_1^* = \chi_1$, $\chi_2^* = \chi_3$. The flavon potential is formed of self couplings $V(\varphi)$, $V(\chi)$ and cross couplings $V(\varphi, \chi)$. Phenomenologically, we assume $V(\varphi), V(\chi) \gg V(\varphi, \chi)$, such that the VEVs $\langle \varphi \rangle$ and $\langle \chi \rangle$ are mainly determined by their self couplings. With suitable choices of the coefficients in the self couplings, we can realise the VEVs $\langle \varphi \rangle$ and $\langle \chi \rangle$ are obtained. From the relation in Eq. (4), we can deduce to the value of $m_{\nu}^2$. Due to the flavon shifts of the directions of the VEVs. The relative terms are given by $\langle \varphi \rangle_1 (\varphi \phi)_1 + \text{h.c.}$ and $((\varphi \phi)_3 S(\phi \phi)_3)$. The VEVs are modified to

$$\langle \varphi \rangle \approx (1, \epsilon_\varphi, \epsilon_\varphi^*)^T, \quad \langle \chi \rangle \approx (1 - 2\epsilon_\chi, 1 + \epsilon_\chi, 1 + \epsilon_\chi)^T v_\chi / \sqrt{3},$$

where, $\epsilon_\varphi$ is complex and $\epsilon_\chi$ is real.

The Lagrangian terms for generating lepton masses are given by

$$-\mathcal{L}_l = \frac{y_e}{\Lambda} (\overline{L}_L \varphi)_1 e_R H + \frac{y_\mu}{\Lambda} (\overline{L}_L \varphi)_1 \nu_R H + \frac{y_\tau}{\Lambda} (\overline{L}_L \varphi)_1 \tau_R H + \text{h.c.},$$

$$-\mathcal{L}_\nu = y_d (\overline{L}_L N)_1 H + \frac{y_1}{2} ((\overline{N}^c N)_3 \phi)_1 + \frac{y_2}{2} (\overline{N}^c N)_1 \eta + \text{h.c.},$$

Besides $\varphi$ and $\phi$, we have introduced a third flavon $\eta \sim 1$ to generate suitable neutrino mass spectra. We have also included right-handed neutrinos $N \sim 3$, and implement the seesaw mechanism. The flavour symmetry has been extended to $A_4 \times Z_2^\varphi \times Z_4^\phi$, with $Z_2^\varphi \times Z_4^\phi$ imposed to forbid redundant couplings of flavons and leptons.

After the flavons and Higgs gain VEVs, leptons obtain masses. Applying the seesaw mechanism and diagonalising the lepton mass matrices, we obtain the lepton mass eigenvalues and mixing parameters. Deviations of mixing parameters from those in TBM are expressed as $r' \approx |\epsilon_\varphi| \sin \theta_\varphi|, \quad s \approx -2|\epsilon_\varphi| \cos \theta_\varphi + 2\epsilon_\phi, \quad a \approx |\epsilon_\varphi| \cos \theta_\varphi, \quad \delta \approx \mp 90^\circ - 2|\epsilon_\varphi| \sin \theta_\varphi,$

for $\theta_\varphi >, < 0$, respectively. Sum rules of mixing parameters

$$r'^2 + a^2 = |\epsilon_\varphi|^2, \quad \delta \approx \mp (90^\circ + \sqrt{2}\theta_{13})$$

are obtained. From the relation in Eq. (4), we can deduce to the value of $|\epsilon_\varphi|$ to be around 0.1 – 0.2 and predict an almost maximal CP violation.

3. CLFV induced by cross couplings

In this section, we discuss the flavon-induced charged LFV 3-body decays $l_l^- \rightarrow l_l^+ l_3^- l_4^-$ and radiative decays $l_l^- \rightarrow l_l^- \gamma$. $Z_3$ is a roughly preserved symmetry in the charged lepton sector, with $e, \mu, \tau$ take charges $1, \omega^2, \omega$, respectively and $\varphi_1, \varphi_2$ take charges $1, \omega^2, \omega$, respectively. The LFV processes can be divided into two classes: the $Z_3$-preserving and $Z_3$-breaking.

The only $Z_3$-preserving processes are $\tau^- \rightarrow \mu^+ e^- e^-$, $e^+ \mu^- \mu^-$ and their conjugate processes [6]. Due to the $Z_3$ symmetry, these processes have the same branching ratios, both suppressed by $(m_{\tau}/m_{e})^2 \cos^2 \theta_{13}^2$. Setting the scale of flavour symmetry $v_\varphi$ and the mass of $\varphi_2$, $m_{\varphi_2}$, around the electroweak scale $v = 246 \text{ GeV}$, we obtain the branching ratios $< 10^{-11}$. 

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The rest 3-body decays and all radiative decays are forbidden by $Z_3$, but they can be relaxed by cross couplings. The cross couplings induce three $Z_3$-breaking effects: 1) mixing between charged leptons, 2) mixing between $Z_3$-invariant flavon $\varphi_1$ and $Z_3$-covariant flavon $\varphi_2$, and 3) mass splitting between the two real degrees of freedom of $\varphi_2$. Taking these effects into account, the rest 3-body processes are further suppressed by $|\epsilon_\varphi|^2 = \alpha^2 + \beta^2$. Sum rules
\begin{align}
2(B_{\mu^+\mu^-e^-} - 2B_{\mu^+\mu^-\mu^-})^2 + (5B_{e^+\mu^-\mu^-} + 10B_{\mu^+\mu^-\mu^-} - 6B_{\mu^+\mu^-e^-})B_{e^+\mu^-\mu^-} \approx 0,
B_{e^+\mu^-\mu^-} = 4(\alpha^2 + \beta^2)\text{Br}(\tau^- \rightarrow \mu^+e^-e^-),
\end{align}
are satisfied, with $B_{\mu^+\mu^-e^-}$, $B_{\mu^+\mu^-\mu^-}$, $B_{e^+\mu^-\mu^-}$ branching ratios of $\tau^- \rightarrow \mu^+\mu^-e^-$, $\mu^+\mu^-\mu^-$, $e^+e^-\mu^-$, respectively. The radiative decays are also suppressed by charged lepton masses and $|\epsilon_\varphi|^2$. $\tau^- \rightarrow e^-\gamma$ and $\tau^- \rightarrow \mu^-\gamma$ have the same branching ratios. The most stringent constraint is from the $\mu^- \rightarrow e^-\gamma$ measurement. In Fig. 1, we show the parameter space of $v_\varphi$ and $m_{\varphi_2}$ allowed by the current experiment MEG [10] and testable at the near future experiment MEG II [11]. Fixing the VEV $v_\varphi = 246$ GeV, we obtain the current lower limit: $m_{\varphi_2} > 500$ GeV.

![Figure 1](image_url)

**Figure 1.** The current and near future constraints from the $\mu^- \rightarrow e^-\gamma$ experiments. $|\epsilon_\varphi|$ is fixed at 0.1 for generating the reactor angle $\theta_{13}$. The current constraint of the MEG experiment is set to be $\text{Br}(\mu^- \rightarrow e^-\gamma) < 4.2 \times 10^{-13}$ [10], and the future constraint of MEG II is set to be $\text{Br}(\mu^- \rightarrow e^-\gamma) < 4 \times 10^{-14}$ [11].

4. Conclusion
We exploit a different approach in which we emphasise the importance of flavon cross couplings to flavour mixing. We find that cross couplings between different flavons can break the residual symmetries, shifting the VEVs of flavons and modifying flavour mixing. These couplings provide new origins and result in new sum rule of the non-zero $\theta_{13}$ and CP violation. We study flavon-induced charged LFV processes. Sum rules of branching ratios for different channels are obtained, which is a special feature of non-Abelian discrete flavour symmetries. A relatively low-scale flavour symmetry, not far above the electroweak scale, is consistent with current experimental constraints.

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