Testing star formation laws in a starburst galaxy at redshift 3 resolved with ALMA

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ABSTRACT

Using high-resolution (sub-kiloparsec scale) data obtained by ALMA, we analyse the star formation rate (SFR), gas content, and kinematics in SDP 81, a gravitationally lensed starburst galaxy at redshift 3. We estimate the SFR surface density (ΣSFR) in the brightest clump of this galaxy to be 357+135−85 M⊙ yr−1 kpc−2, over an area of 0.07 ± 0.02 kpc2. Using the intensity-weighted velocity of CO (5–4), we measure the turbulent velocity dispersion in the plane of the sky and find σv,turb = 37 ± 5 km s−1 for the clump, in good agreement with previous estimates along the line of sight. Our measurements of the gas surface density, freefall time, and turbulent Mach number allow us to compare the theoretical SFR from various star formation models with that observed, revealing that the role of turbulence is crucial to explaining the observed SFR in this clump. While the Kennicutt–Schmidt (KS) relation predicts an SFR surface density of ΣSFR,KS = 52 ± 17 M⊙ yr−1 kpc−2, the single-freefall model by Krumholz, Dekel, and McKee (KDM) predicts ΣSFR,KDM = 106 ± 37 M⊙ yr−1 kpc−2. In contrast, the multifreefall (turbulence) model by Salim, Federrath, and Kewley (SFK) gives ΣSFR,SFK = 491±194 M⊙ yr−1 kpc−2. Although the SFK relation overestimates the SFR in this clump (possibly due to the negligence of magnetic fields), it provides the best prediction among the available models. Finally, we compare the star formation and gas properties of this galaxy to local star-forming regions and find that the SFK relation provides the best estimates of SFR in both local and high-redshift galaxies.

Key words: Turbulence – Stars: formation – galaxies: high-redshift – galaxies: starburst – Galaxy: kinematics and dynamics – Submillimetre: galaxies.

1 INTRODUCTION

Numerous star formation relations have been proposed in a quest to universalize the theory of star formation, by linking the star formation rate (SFR) with the mass of gas, its freefall time, virial parameter, magnetic field strength, and turbulence (Silk 1997; Kennicutt 1998a; Elmegreen 2002; Shi et al. 2011; Krumholz, Dekel & McKee 2012 (hereafter, KDM12); Renaud, Kraljic & Bournaud 2012; Elmegreen 2015; Elmegreen & Hunter 2015; Escala 2015; Salim, Federrath & Kewley 2015 (hereafter, SFK15); Nguyen-Luong et al. 2016; Miettinen et al. 2017). While these relations have been shown to be valid for star-forming regions in the Milky Way and local galaxies (Bigiel et al. 2008), the lack of spatial resolution has limited us in testing them on high-redshift sources with z > 1. Thanks to the high spatial resolution of ALMA, several high-redshift galaxies emitting in the submillimetre (sub-mm) regime have been detected and resolved over the last few years (Decarli et al. 2016; Spilker et al. 2016; Brisbin et al. 2017; Danielson et al. 2017; Lu et al. 2017), particularly if they are gravitationally lensed by a foreground source (Smail, Ivison & Blain 1997; Smail et al. 2002; Hezaveh et al. 2013; Bradač et al. 2017; Fudamoto et al. 2017; Johnson et al. 2017; Laporte et al. 2017; Wong et al. 2017). These galaxies are known to be rigorous sites of dusty star formation where molecular gas plays a key role in modifying the structure of clusters where star formation occurs. Tracing molecular gas in these regions can give us valuable insight on the star formation characteristics of these galaxies since...
it is believed that molecular gas has a strong correlation with SFR whereas atomic gas does not (Wong & Blitz 2002; Bigiel et al. 2008; Blanc et al. 2009).

SDP 81 was detected as a lensed galaxy in the $H$-ATLAS survey of bright submillimetre galaxies (SMGs) by Negrello et al. (2010), where the redshift was measured as $z = 3.042 \pm 0.001$ through ground-based CO measurements. It falls in the popular definition of SMGs where the 850 $\mu$m flux density $S_{\nu,850} > 3$ mJy and infrared luminosity $L_{IR} > 10^{12}$ L$_{\odot}$ (Kovács et al. 2006; Coppin et al. 2008; Hayward et al. 2011). SDP 81 has also been established to be a dusty star-forming galaxy in previous works (Negrello et al. 2014; Swinbank et al. 2015; S15; Dye et al. 2015; Hatsukade et al. 2015; Rybak et al. 2015a,b; Tamura et al. 2015; Wong, Suyu & Matsushita 2015; Hezaveh et al. 2016; Inoue et al. 2016). Even though there are significant uncertainties in determining the stellar mass of SDP 81, we note that it lies 1–2 orders of magnitude above the main sequence on the stellar mass–star formation rate ($M_\star$–SFR) plane (Speagle et al. 2014; Chang et al. 2015; Guo et al. 2015; Schreiber et al. 2015). Thus, SDP 81 falls under the category of extreme starburst galaxies and is an ideal candidate to test star formation relations. This is further confirmed by the position of SDP 81 on the star formation rate–gas mass (SFR – $M_{\text{gas}}$) plane (Sargent et al. 2014).

Our goal in this work is to extract the SFR in individual clumps of this galaxy and compare it with that predicted by existing star formation relations. We refer to clumps as giant star-forming regions (Genzel et al. 2006; Elmegreen et al. 2009; Bournaud et al. 2014) substantially more massive and star-forming than typical molecular clouds in the Milky Way (Cowie, Hu & Songaila 1995; Van den Bergh et al. 1996; Shapiro, Genzel & Förster Schreiber 2010), and possibly showing high star formation efficiencies (Freundlich et al. 2013; Zanella et al. 2015; Cibinel et al. 2017). The paper is organized as follows: in Section 2, we summarize the data reduction through lens modelling which was used to create the source plane reconstructed images of SDP 81 (Dye et al. 2015). This section also identifies different clumps extracted in this galaxy by S15. Section 3 entails the dust spectral energy distribution (SED) fitting of a modified blackbody (MBB) and estimation of SFR surface density in the galaxy. We describe the kinematic analysis of CO (5–4) used to estimate the Mach number in Section 4. In Section 5, we present our estimates of the local gas mass and freefall time in the galaxy. Finally, we put all these parameters together to test various star formation relations and compare with the SFR surface density deduced through dust SED fitting, in Section 6. We summarize our findings in Section 7.

We adopt the $\Lambda$CDM cosmology with $h_0 = 72$ km s$^{-1}$ Mpc$^{-1}$, $\Omega_m = 0.27$, $\Omega_{\Lambda} = 1 - \Omega_m$ and the Chabrier IMF (Chabrier 2003). The luminosity distance and scale length corresponding to these parameters is 25.9 Gpc and 7.69 kpc arcsec$^{-1}$, respectively, for $z \approx 3.042$ (Wright 2006).

2 DATA REDUCTION AND ANALYSIS

ALMA observations of SDP 81 (RA = 09°03′11.57″, Dec. = +00°39′06.6″) were taken during the Science Verification cycle in 2014 October. In the calibrated data, the lensed galaxy is seen in the form of an Einstein ring, with two arcs on the eastern and western sides (Dye et al. 2014; ALMA Partnership 2015). Through $uv$ tapering, a resolution of $\sim 150 \times 120$ mas was achieved in the three bands (see tables 1 and 3 of ALMA Partnership 2015) for observed fluxes and noise levels. The CO (5–4) velocity cubes were binned to a velocity resolution of 21 km s$^{-1}$ (ALMA Partnership 2015).

3 MEASUREMENT OF THE STAR FORMATION RATE

We estimate the SFR surface density ($\Sigma_{\text{SFR}}$) in clump A by fitting a modified blackbody (MBB) spectrum to the continuum emission from dust in the three ALMA bands. The MBB spectral law can be written as

$$S_{\nu} = N_{\nu} \left( \frac{2 \hbar \nu^3}{c^2} \right) \frac{1}{e^{h \nu/k_{\text{B}} T} - 1} \nu^{\beta_{\text{MBB}}},$$

where

$$\beta_{\text{MBB}} = \frac{2}{3} \left( \frac{2}{7} \right) \frac{K_{0}(1/\alpha)}{\pi^{5/2}} \left( \frac{h}{2 \pi k_{\text{B}} T} \right)^{1/2} \left( \frac{h}{m_{\text{e}}} \right)^{1/2},$$

with $K_{0}$ being a modified Bessel function of the second kind of order zero.
where \( \nu \) is the rest-frame frequency, \( S_\nu \) is the flux density of the clump, \( N_\nu \) is the normalization parameter (includes dust opacity), \( T \) is the dust temperature, and \( \beta_{em} \) is the emissivity index (\( \beta_{em} = 0 \) corresponds to a blackbody) (Draine & Lee 1984; Da Cunha et al. 2010b). Since clump A lies near the centre of the galaxy where the background contribution may be high, we subtract the underlying (disc) CO emission from clump A by masking the clump and then smoothing the image by convolving it with a large Gaussian kernel. Then, we subtract the smoothed image from the original image to get the background subtracted image. In order to ensure that we do not over or under subtract, we reiterate this procedure multiple times with different kernels.

The flux density can be integrated over the whole infrared (IR) range (8–1000\( \mu m \)) to get the dust luminosity (Humason, Mayall & Sandage 1956; Oke & Sandage 1968; Hogg et al. 2002):

\[
L_{\text{FIR}} = \frac{4\pi D_L^2}{1 + z} \int_{\mu m}^{1000\mu m} S_\nu d\nu,
\]

where \( L_{\text{FIR}} \) is the far-infrared luminosity of the clump, \( D_L \) is the luminosity distance to SDP 81 and \( S_\nu \) is the flux density of the clump. However, the available ALMA observations are insufficient to simultaneously constrain the dust parameters – \( T \), \( \beta_{em} \), and \( L_{\text{FIR}} \). Therefore, we fix \( \beta_{em} = 1.5, 2.0, \) and 2.5, which are the typically used values for starburst galaxies (Hildebrand 1983; Blain, Barnard & Chapman 2003; Casey 2012; Smith et al. 2013). We also include observed fluxes at various other wavelengths in the infrared regime, reported in table 2 of Negrello et al. (2014), which cover a longer wavelength baseline (\( \lambda_{\text{obs}} = 350–2000 \mu m \)). Then, using a two-step fitting process: (1) we fit the galaxy-integrated fluxes to constrain \( T \) and \( \beta_{em} \) and (2) we then adopt the ‘galaxy-wide’ \( T \) and \( \beta_{em} \) to fit clump A and determine its far-infrared luminosity. The conditions of individual clumps might be very different as compared to the whole galaxy. We lack the spatial resolution for a proper decomposition of the various clumps and assume that the clump conditions (of clump A) are identical to the galaxy-wide properties, while being aware that this might not be the case. We include the systematic uncertainty arising from this assumption in our calculation of the far-infrared luminosity.

We derive a best-fitting temperature \( T = 39 \pm 2 \) K for \( \beta_{em} = 2.5 \), where we use Monte Carlo (MC) simulations (by modelling the uncertainties in the observed fluxes according to a Gaussian distribution) to find the best-fitting MBB (Ogilvie 1984; Johnson et al. 2013). The uncertainty on \( T \) arises from the inclusion of the flux at 350 \( \mu m \) (from SPIRE observations, Griffin et al. 2010) which falls partially under the cold temperature dominated regime (see fig. 6 of Dye et al. 2015). Fig. 2 shows the best-fitting MBB we find for clump A from the fits, along with the 1 \( \sigma \) uncertainty range. We can estimate the dust mass of clump A by using equation 6 of Magdis et al. (2011) to find the dust mass. The efficiency factor \( \eta \) is chosen to be 0.5. Although our result is lower than the average median value reported in Hodge et al. (2016) (\( n \approx 1 \pm 0.2 \)), the result is consistent with the SFR densities found in several high-redshift galaxies (see table 1 of Hodge et al. 2016). Through this composite profile, the fluxes we obtain for clump A for the three bands are similar to those obtained through background subtraction discussed above, within \( \pm 12 \) per cent. Since this difference in flux densities is negligible, the resulting \( L_{\text{FIR}} \) and \( \Sigma_{\text{SFR}} \) from this method are similar to those quoted above, within the uncertainties.

3.1 Gas mass and clump size from continuum emission

Apart from the SFR surface density, we can also estimate the gas mass \( M_{\text{gas}} \) and size of the clump \( R \) using the SED fits and continuum maps, respectively. Since we have an excellent coverage of the Rayleigh–Jeans regime, we use our best-fitting MBB to estimate the dust mass of clump A by using equation 6 of Magdis et al. (2012) and appropriate rest-frame dust mass absorption coefficients for the three ALMA bands, from table 6 of Li & Draine (2001). Then, we use a typical gas-to-dust conversion ratio of 150 to get the gas mass in this clump (Dunne et al. 2000; Dye et al. 2015; Brisbin et al. 2017). For the three ALMA bands, the gas masses we thereby obtain are \( M_{\text{gas}} = (2.3–4.9) \times 10^4 \) M\(_\odot\). This is consistent with the value of \( M_{\text{gas}} \) we obtain from CO (5–4) (as we discuss in detail in Section 5).
Table 1. Results for Clump A in SDP 81 (see Fig. 1) with mean and 1 $\sigma$ errors.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{em}$</td>
<td>2.5</td>
</tr>
<tr>
<td>$T$</td>
<td>39 $\pm$ 2 K</td>
</tr>
<tr>
<td>$L_{4.5}$ (observed)$^a$</td>
<td>$(2.3^{+1.0}<em>{-0.5}) \times 10^{31} L</em>{\odot}$</td>
</tr>
<tr>
<td>$\Sigma_{SFR}$ (observed)$^a$</td>
<td>$357^{+135}<em>{-85} M</em>{\odot} \text{yr}^{-1} \text{kpc}^{-2}$</td>
</tr>
<tr>
<td>$A$</td>
<td>$(7.2 \pm 1.5) \times 10^{-2} \text{kpc}^{-2}$</td>
</tr>
<tr>
<td>$R$</td>
<td>$(1.5 \pm 0.2) \times 10^{-1} \text{kpc}$</td>
</tr>
<tr>
<td>$\sigma_{bgs}$</td>
<td>$80 \pm 10 \text{ km s}^{-1}$</td>
</tr>
<tr>
<td>$\sigma_{turb}$</td>
<td>$37 \pm 5 \text{ km s}^{-1}$</td>
</tr>
<tr>
<td>$c_s$</td>
<td>$0.4 \pm 0.2 \text{ km s}^{-1}$</td>
</tr>
<tr>
<td>$\mathcal{M}$</td>
<td>$96 \pm 28$</td>
</tr>
<tr>
<td>$b$</td>
<td>0.4</td>
</tr>
<tr>
<td>$S_{\nu, \text{CO}}(5-4)$</td>
<td>$8.4 \pm 1.8 \text{ mJy km s}^{-1}$</td>
</tr>
<tr>
<td>$L'_{\text{CO}}(1-0)$</td>
<td>$(1.4 \pm 0.1) \times 10^8 \text{ K km s}^{-1} \text{ pc}^{-2}$</td>
</tr>
<tr>
<td>$r_{bg}$</td>
<td>$0.28 \pm 0.05$</td>
</tr>
<tr>
<td>$L'_{\text{CO}}(5-4)$</td>
<td>$(5.0 \pm 1.1) \times 10^8 \text{ K km s}^{-1} \text{ pc}^{-2}$</td>
</tr>
<tr>
<td>$\alpha_{\text{CO}}$</td>
<td>$0.9 \pm 0.2 M_{\odot} \text{ K km s}^{-1} \text{ pc}^{-2}^{-1}$</td>
</tr>
<tr>
<td>$M_{\text{gas}}$</td>
<td>$(6.2 \pm 1.4) \times 10^8 M_{\odot}$</td>
</tr>
<tr>
<td>$\Sigma_{\text{gas}}$</td>
<td>$(8.6 \pm 1.9) \times 10^8 M_{\odot} \text{ pc}^{-2}$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>$(2.9 \pm 0.6) \times 10^{-21} \text{ g cm}^{-3}$</td>
</tr>
<tr>
<td>$\alpha_{\text{vir}}$</td>
<td>$0.6 \pm 0.1$</td>
</tr>
<tr>
<td>$f_{\text{bg}}$</td>
<td>$(1.3 \pm 0.1) \times 10^{6}$</td>
</tr>
<tr>
<td>$\Sigma_{\text{SFR}}(K98)$</td>
<td>$52 \pm 17 M_{\odot} \text{ yr}^{-1} \text{ kpc}^{-2}$</td>
</tr>
<tr>
<td>$\Sigma_{\text{SFR}}(KDM12)$</td>
<td>$106 \pm 37 M_{\odot} \text{ yr}^{-1} \text{ kpc}^{-2}$</td>
</tr>
<tr>
<td>$\Sigma_{\text{SFR}}(SF15)$</td>
<td>$491^{+139}<em>{-28} M</em>{\odot} \text{ yr}^{-1} \text{ kpc}^{-2}$</td>
</tr>
</tbody>
</table>

$^a$Corrected for Chabrier IMF.

$^b$Large-scale velocity dispersion before gradient subtraction.

$^c$Turbulent velocity dispersion after gradient subtraction.

$^d$Integrated CO (5–4) flux after background subtraction.

$^e$16th and 84th percentiles are 37 and 132 M$_{\odot}$ for the PDF of $\Sigma_{\text{gas}}$.

$^f$16th and 84th percentiles are 37 and 132 M$_{\odot}$ for the PDF of $\Sigma_{\text{gas}}$.

$^g$Errors represent the 16th and 84th percentiles.

We use the composite disc profile (n-Sérsic disc + Gaussian clump, as discussed in Section 3) to find an estimate of the size of clump A. Since the clump is defined using the $a_{obs} = 1.0$ mm continuum obtained from ALMA, we use the best-fitting composite disc profile at this wavelength and find the size of this clump by assuming that its diameter is equal to the full width at half-maximum of the composite profile (i.e. $FWHM = 2 R$). Correspondingly, we obtain $R \approx 0.16$ kpc for clump A. This is in good agreement with the size of clump A which we find in Section 5 by summing up the pixels belonging to clump A, as we report in Table 1.

4 MACH NUMBER ESTIMATION

Supersonic turbulence is a key ingredient to star formation because it can compress interstellar gas which leads to the formation of dense cores. On the other hand, it can suppress the collapse of the clouds, thus significantly reducing the SFR (Elmegreen & Scalo 2004; Mac Low & Klessen 2004; McKee & Ostriker 2007; Hennebelle & Falgarone 2012). The root mean square sonic Mach number associated with turbulence in star-forming regions is given by

$$\mathcal{M} \equiv \frac{\sigma_{v, \text{turb}}}{c_s},$$

where $\sigma_{v, \text{turb}}$ is the turbulent velocity dispersion and $c_s$ is the sound speed, $c_s \propto \sqrt{T}$, where $T$ is the gas temperature. It is difficult to estimate the gas temperature with the current data, however, we can assume it to be between 10 and 100 K. This assumption is valid for gas temperatures in dense molecular clouds (Gao & Solomon 2004; Solomon & Vanden Bout 2005; Wu et al. 2005; Batterby et al. 2014; Immer et al. 2016; Krieger et al. 2017). Using the relation for isothermal sound speed from Federrath et al. (2016) for a mean molecular weight of 2.33 (Kauffmann et al. 2008) and $T_{gas} \approx 10$ K, the sound speed is $\sim 0.2$ km s$^{-1}$, whereas it is $\sim 0.6$ km s$^{-1}$ for $T_{gas} \approx 100$ K, so we assume the sound speed to be in the range 0.2–0.6 km s$^{-1}$.

The CO (5–4) velocity map after source plane reconstruction shows a clear, large-scale gradient running diagonally, as we show in Fig. 3 (also, fig. 1 of S15). This systematic gradient can be associated with the rotational or shear motion of the gas. To extract the small-scale turbulent features in this clump, we fit a large-scale gradient to the clump and subtract it; similar to the analysis of turbulent velocity dispersion done on the central molecular zone (CMZ) cloud Brick by Federrath et al. (2016). For this purpose, we use the PLANEFIT routine in IDL which performs a least-squares fit of a plane to a set of $(x, y, z)$ points. In this case, this is a position–position–velocity (PPV) cube with $x$ and $y$ being the position coordinates of pixels forming clump A, and $z$ being the CO (5–4) velocity of each pixel. We use the standard deviation of residuals after gradient subtraction as the turbulent velocity dispersion:

$$\sigma_{v, \text{turb}} = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} \left( \tilde{v}_{\text{bgs}} - v_{\text{fit}} - \mu \right)^2},$$

where $N$ is the number of pixels or resolution elements, $\tilde{v}_{\text{bgs}}$ is the CO (5–4) velocity of the $i^{th}$ pixel before gradient subtraction, $v_{\text{fit}}$ is the velocity of the fitted gradient and $\mu$ is the mean of residuals after gradient subtraction (i.e. $\mu = \langle \tilde{v}_{\text{bgs}} - v_{\text{fit}} \rangle$).

The turbulent velocity before subtracting the gradient is $\sigma_{v, \text{bgs}} = 80 \pm 10$ km s$^{-1}$. From the gradient subtraction algorithm, we obtain the turbulent velocity dispersion in clump A as $\sigma_{v, \text{turb}} = 37 \pm 5$ km s$^{-1}$, where the 1 $\sigma$ error is the standard deviation calculated using (Lehmann & Casella 1998):

$$\sigma_{v} = \frac{\Gamma(\nu-1)}{\Gamma(\nu)/2} \sqrt{\frac{N - 1}{2} - \left( \frac{\Gamma(N/2)}{\Gamma(\nu/2)} \right)^2},$$

where $\Gamma(N)$ is the Gamma function. We also find the uncertainty on $\sigma_{v, \text{turb}}$ through MC simulations and note that the result is consistent with the value we obtain from the analytical equation, within $\pm 8$ per cent. Fig. 3 shows the velocity field across clump A before gradient subtraction, fitted gradient velocities, and velocity field after gradient subtraction. By construction, the residuals after gradient subtraction are evenly spread around 0, as is also clear from the PDF of $\sigma_{v, \text{turb}}$ we plot in Fig. 4. From Fig. 4, we note that the distribution of velocities (in the pixels of clump A) before gradient subtraction is highly non-Gaussian and bimodal, while that of the velocities after gradient subtraction is more consistent with a Gaussian distribution. However, due to low-number statistics, it is hard to infer much information from this distribution; some non-Gaussian contributions may still remain after gradient subtraction, because it only removes the largest scale mode of systematic shear or rotation. None the less, this distribution is in agreement with velocity PDFs obtained for simulations of supersonic turbulence which are also Gaussian in nature (Klessen 2000; Federrath 2013) and the non-Gaussian components can arise from small-scale rotational or shear modes, or due to the intrinsic features of turbulence (see section 3.2.2 of Federrath et al. (2016) and references therein). Additionally, we note that the width of the Gaussian we fit for the PDF of...
Figure 3. Left-hand panel: Centroid CO (5–4) velocity (in km s\(^{-1}\)) in the pixels of clump A before gradient subtraction. The size of each pixel is \(\sim 0.05\) kpc. Middle panel: Fitted large-scale gradient to the clump. Right-hand panel: Velocity of CO (5–4) in the clump after gradient subtraction. The last panel isolates the turbulent velocities in the plane of the sky. The turbulent velocity dispersion obtained for clump A is \(\sigma_{v,\text{turb}} = 37 \pm 5\) km s\(^{-1}\), using which the Mach number calculated is \(M = \sigma_{v,\text{turb}}/c_s = 96 \pm 28\).

Figure 4. PDFs of CO (5–4) velocities of the pixels in clump A before and after gradient subtraction. The distribution of velocities before gradient subtraction \(v_{\text{bg}}\) is bimodal; that of velocities after gradient subtraction \(v_{\text{turb}}\) resembles a Gaussian distribution, however, much information cannot be ascertained due to low number of statistics. The width of the Gaussian we fit (right-hand panel) and the data are similar, thus reinforcing the estimate of turbulent velocity dispersion \(\sigma_{v,\text{turb}} = 37 \pm 5\) km s\(^{-1}\).

The turbulent velocity dispersion we calculate is in agreement with the velocity dispersion of 30 ± 9 km s\(^{-1}\) calculated by S15 for this clump. Given the high redshift of SDP 81 and previous works highlighting intense star formation, it is not unusual to obtain Mach numbers near 100. In fact, it implies that the role of turbulence becomes more important at the epoch near the maximum star formation in the history of the Universe (Springel & Hernquist 2003; Madau & Dickinson 2014; Falgarone et al. 2017).

4.1 Resolution check for gradient fit and subtraction algorithm

The gradient fit and subtraction algorithm works accurately only if the resolution is sufficient. For the CMZ cloud Brick, the resolution scale was in sub-parsecs (Federrath et al. 2016) whereas it is in sub-kiloparsecs for SDP 81. We check if the low number of resolution elements in our data affects our measurements, since the velocity dispersion calculated might vary by more than 20 per cent if sufficient number of pixels are not available to resolve the clump.

To investigate whether we have enough pixels to be operating in the saturated regime (where velocity dispersion does not change by more than 20 per cent when the number of resolution elements are altered), we perform a resolution degradation on clump A by creating artificial ‘superpixels’ (merging nearby pixels to make a bigger pixel) and then applying the gradient fit and subtraction algorithm.

For the first degradation (1/4 resolution), we merge four nearby pixels into one (making a square shape, see Fig. 5). While the centre of a superpixel is the centroid of the four constituent pixels, its CO (5–4) velocity is the flux-weighted average of CO (5–4) velocities in the constituent pixels:

\[ v_{\text{spix}} = \frac{\sum_{i=1}^{4} (S_i v_i)}{\sum_{i=1}^{4} S_i}, \]

where \(v_{\text{spix}}\) is the velocity of the superpixel, \(S_i\) is the flux of the \(i\)th constituent pixel, and \(v_i\) is its velocity. At places where pixels belonging to the clump A cannot make a square by themselves, we use pixels from outside the clump, but mask their flux to be 0. Thus, for such constituent pixels, \(S_i = 0\), so these pixels do not contribute to the sums in equation (6).
we confirm that we have enough pixels to resolve this clump with acceptable accuracy.

5 GAS MASS AND FREEFALL TIME FROM CO (5–4)

The total gas mass is an essential parameter which goes in all the star formation relations we test in a later section (see Section 6). It can be estimated by following the CO (1–0) emission in the star-forming region (Carilli & Blain 2002; Pety et al. 2013; McNamara et al. 2014; Scoville et al. 2017). From fig. 1 in Rybak et al. (2015b), and that in Dye et al. (2015), we notice that there is a significant presence of CO (1–0) emission at the position of clump A. However, the CO (1–0) data was obtained by the Karl G. Jansky Very Large Array (Valtchanov et al. 2011) at a lower resolution than ALMA and cannot be used for kinematic analysis. Thus, we rely on ALMA observations of CO (5–4) transition (observed at a frequency of 142.57 GHz in ALMA band 4), to estimate the gas mass of clump A. It should be noted that CO (5–4) is generally a poor tracer of the total diffuse molecular gas, but is bright and easily observable at high redshift (Daddi et al. 2015; Lu et al. 2015; Yang et al. 2017).

We follow the Solomon, Radford & Downes (1992a); Solomon, Downes & Radford (1992b) relation between line luminosity and integrated flux density of CO (5–4):

$$L'_{\text{CO}(5-4)} = 3.25 \times 10^7 S'_v \Delta v \frac{D_L^2}{v_{\text{obs}}(1+z)}, \quad (7)$$

where $L'_{\text{CO}(5-4)}$ is the line luminosity in K km s$^{-1}$ pc$^2$, $S'_v \Delta v$ is the velocity integrated flux density of CO (5–4) after subtraction of background emission, in Jy km s$^{-1}$, $D_L$ is the luminosity distance in Mpc, and $v_{\text{obs}}$ is the observed frequency of transition in GHz. The line luminosity we obtain is $L'_{\text{CO}(5-4)} = (5.04 \pm 1.10) \times 10^8$ K km s$^{-1}$ pc$^2$. Since the transition we observe with ALMA at $z \approx 3$ is higher than the ground (1–0) transition, we introduce an appropriate line ratio factor (defined as the ratio of line luminosity of CO (5–4) to that of CO (1–0)), $r_{54} = 0.28 \pm 0.05$. This value was derived for clump A by S15, where the authors used velocity and magnification maps from the lens model prepared by Dye et al. (2015). It falls in the typical range of values of $r_{54}$ for SMGs (see Carilli & Walter (2013) and references therein).

To get the gas mass from the line luminosity, we use an appropriate CO to H$_2$ conversion factor $\alpha_{\text{CO}}$. Although there is a high uncertainty in the value of this factor for nearby as well as high-redshift galaxies (Narayanan et al. 2012; Papadopoulos et al. 2012), the suggested values based on observations of SMGs lie in the range $\approx 0.8–1.0$ M$_{\odot}$ per (K km s$^{-1}$ pc$^2$) (Downes & Solomon 1998; Solomon & Vanden Bout 2005; Tacconi et al. 2008; Magdis et al. 2011; Hodge et al. 2012; Bolatto, Wolfire & Leroy 2013; Bothwell et al. 2013; Carilli & Walter 2013), which is less by a factor of $\sim 4$ than the typical value used for Milky Way clouds and nearby galaxies. Dye et al. (2015) used a conversion factor of unity (in the same units) for SDP 81, while Hatsukade et al. (2015) used a value of 0.8. Further, we notice that clump A falls on top of the starburst sequence of the $\Sigma_{\text{H}_{2}}$–$L_{\text{IR}}$ relation populated by local ultra-luminous infrared galaxies and SMGs (Daddi et al. 2010). This further justifies the choice of $\alpha_{\text{CO}} \approx 0.8–1$ M$_{\odot}$ per (K km s$^{-1}$ pc$^2$).

Keeping these studies in mind, we assume $\alpha_{\text{CO}} \approx 0.9 \pm 0.2$ M$_{\odot}$ per (K km s$^{-1}$ pc$^2$), which gives an H$_2$ mass of...
(4.5 ± 1.0) × 10⁶ M⊙. Accounting for the contribution to the gas by H2, we further increase the H2 mass obtained so far by 36 per cent to get the total gas mass for clump A as (6.2 ± 1.4) × 10⁶ M⊙. This value is in good agreement with the gas mass found out using SED fitting in Section 3. The gas surface density we derive is \( \Sigma_{\text{gas}} = (8.6 ± 1.9) \times 10^{2} M_{\odot} \text{kpc}^{-2} \), where we calculate and sum the area of all pixels which constitute clump A. The size of clump A we obtain in this manner is \( R \approx 0.15 ± 0.02 \text{kpc} \), in excellent agreement with the size we find through composite disc profile fitting in Section 3. Assuming clump A to be spherical (see Section 4 for a discussion on the validity of this assumption), we calculate its density to be \( \rho = M_{\text{gas}} / V = (2.9 ± 0.6) \times 10^{-21} \text{g cm}^{-3} \), where \( V \) is the volume of the spherical clump.

To establish whether the cloud is undergoing collapse, we estimate the virial parameter \( \alpha_{\text{vir}} \), which is the ratio of twice the kinetic energy to the gravitational energy (Federrath & Klessen 2012). Using the definition from Bertoldi & McKee (1992), the virial parameter can also be given by

\[
\alpha_{\text{vir}} = \frac{5\sigma_{\text{tot}}^2}{4\pi G \rho R^2},
\]

where the velocity dispersion \( \sigma_{\text{tot}} \) is the total thermal and turbulent velocity dispersion including the shear component (i.e. turbulent velocity dispersion before gradient subtraction, \( \sigma_{v,bgs} \)). However, in this clump, since the turbulent velocity dispersion \( \sigma_{v,bgs} \gg \sigma_{\text{v}} \), it implies that the total velocity dispersion can be approximated as \( \sigma_{\text{v,tot}} \approx \sigma_{v,bgs} \) (Krumholz & McKee 2005; Federrath & Klessen 2012). The virial parameter we thus obtain is \( \alpha_{\text{vir}} = 0.63 ± 0.13 < 1 \), implying the cloud is strongly gravitationally bound and likely undergoing collapse. For such a cloud, the freefall time can be given by (Hennebelle & Chabrier 2011, 2013; Chabrier, Hennebelle & Charlot 2014):

\[
\tau_{\text{ff}} = \sqrt{\frac{3\pi}{32 \alpha_{\text{vir}} G \rho}},
\]

where \( G \) is the gravitational constant. From this equation, we obtain a freefall time of 1.3 ± 0.1 Myr. This value is in agreement with the freefall times calculated for other high \( z \) starbursts (see table 4 of Krumholz, Dekel & McKee 2013).

We summarize all the parameters going into predictions of SFR surface density in various star formation relations in Table 1.

### 6 Comparison of Observed SFR Surface Density with Theoretical Predictions by K98, KDM12, and SFK15

We compare the SFR surface density obtained through dust SED fitting with star formation relations proposed for nearby and high-redshift galaxies in Fig. 7. The PDF of the measured SFR surface density in clump A is shown as the solid line, and we compare it with the predictions of SFR surface density by three popular star formation relations in the same plot. These PDFs were calculated using MC simulations with a sample size of 100 000 and included systematic errors on the SFR surface densities.

1This gas mass is essentially in agreement as that obtained by S15 for clump A. However, due to a typographical error, the gas masses reported in the last column of Table 1 of S15 have to be rearranged. For the 5 clumps, the reported gas masses are in the order D-C-A-B-E.

2The size of 1 pixel is \( \sim 0.05 \text{kpc} \). There are 32 pixels in this clump.

3This distribution does not take into account the uncertainty on the power-law index in equation (10) because we notice that it becomes highly skewed when this uncertainty is randomized. In that case, the 16th and 84th percentile values of \( \Sigma_{\text{SFR}} \) are 1 and 133 M⊙ yr\(^{-1}\) kpc\(^{-2}\), respectively.

![Figure 7. PDF of \( \Sigma_{\text{SFR}} \) estimated from dust SED fitting (solid curve) and those predicted from K98 (dotted curve, equation 10), KDM12 (dot-dashed curve, equation 11) and SFK15 (dashed curve, equation 12) relations. The PDFs were calculated using MC simulations with a sample size of 100 000.](https://academic.oup.com/mnras/article-abstract/477/4/4380/4964761/4386-P.Sharda-et-al.2018)
relations with that observed, the dust temperature (which goes into the MBB function, see equation 1) should be lowered by \( \sim 8 \) K, if emissivity is fixed at the best-fitting emissivity \( \beta_{\text{em}} = 2.5 \). Another way could be to decrease the emissivity to 1.9 such that the original best-fitting temperature (\( T = 39 \) K) can provide a reasonable match of observed SFR surface density with that predicted by KS or KDM12 relations. Although these values of \( (T, \beta_{\text{em}}) \) are not favoured by theSED fit, they are in the typical range of dust temperature and emissivity found out for other high-redshift galaxies (Smith et al. 2013; Ota et al. 2014; Lu et al. 2015, 2017). Furthermore, in order to fit the KS relation to the observed SFR surface density in this clump, we find that the power law should be steeper with an exponent of \( n \approx 1.6 \) in equation 10). This is consistent with scaling in the KS relation estimated for other SMG galaxies and starbursts (Bouché et al. 2007; Kogoperskov & Vasiliev 2017). Moreover, Daddi et al. (2010) also proposed that the KS relation underpredicts the star formation efficiency in starburst galaxies by a factor of 10. Similarly, an equivalence between observed SFR surface density and that predicted by KDM12 relation can be obtained if \( \epsilon_g \) in equation (11) is increased to thrice its best-fitting value. These discrepancies between observed and predicted SFR surface density motivate us to include the role of turbulence in star formation relations, as suggested by SFK15.

The SFK15 relation is the result of the combination of gas surface density and density-dependent freefall time determined by KDM12 (see also Krumholz et al. 2013) and the role of turbulence in star-forming regions (Federrath & Klessen 2012; Federrath 2013). SFK15 correlated the scatter present in the KDM12 relation with turbulent motions in gas clouds and found that \( \Sigma_{\text{SFR}} \) is \( \sim 0.45 \) per cent of the multi-freefall gas consumption rate (MGCR) in star-forming regions:

\[
\Sigma_{\text{SFR}} = \frac{0.45}{100} \frac{\Sigma_{\text{gas}}}{t_{\text{ff}}} \left( 1 + b \frac{\mathcal{M}^3}{\rho_{\text{mag}}} \frac{\beta_{\text{mag}}}{\beta_{\text{mag}} + 1} \right)^{3/8},
\]

where \( b \) is the turbulent driving parameter (\( b = 1/3 \) for solenoidal driving and \( b = 1 \) for compressive driving) (Federrath, Klessen & Schmidt 2008; Federrath et al. 2010; Federrath & Klessen 2012; Federrath et al. 2013). We use a mixed driving mode with \( b = 0.4 \) (Federrath et al. 2010). The turbulence term was derived by Molina et al. (2012). In this term, \( \beta_{\text{mag}} \) is the ratio of thermal to magnetic pressure. It can also be expressed as a ratio of Alfven to sonic Mach numbers: \( \rho_{\text{mag}} = 2 \mathcal{M}^2/\mathcal{M}^2 \). The freefall time used in this equation comes from \( (t_{\text{ff}}, t_{\text{ff}}) \), where \( t_{\text{ff}} \) stands for Toomre and GMC stands for giant molecular clouds, as used by KDM12. KDM12 showed that the Toomre time is shorter than the GMC freefall time for starburst galaxies. We do not have any estimates of magnetic field strength in this galaxy since it requires polarization or Zeeman measurements of the magnetic field, which are unavailable for SDP 81. For simplicity, we neglect magnetic fields and set \( \beta_{\text{mag}} \rightarrow \infty \), leading to \( \rho_{\text{mag}}/(\rho_{\text{mag}} + 1) = 1 \).

The SFK15 relation (equation 12) generates a skewed distribution of predicted SFR surface density, as shown in Fig. 7. The mean of the distribution is \( \Sigma_{\text{SFR}} = 491^{+170}_{-120} \) mJy yr\(^{-1}\) kpc\(^{-2}\). The errors represent the 16th and 84th percentiles. As can be seen from Fig. 7, the distribution of SFR surface density predicted from SFK15 overlaps to a good extent with the distribution of the observed SFR surface density. The overestimation of SFR surface density by SFK15 can be attributed to ignoring the magnetic field strength, which can reduce the SFR by a factor of \( \sim 2 \) (Padoan & Nordlund 2011; Federrath & Klessen 2012; Federrath 2015).

Fig. 8 depicts the observed SFR surface density (\( \Sigma_{\text{SFR}} \)) plotted against the gas surface density (\( \Sigma_{\text{gas}} \)) and single and multi-freefall times, overlaid with the three star formation relations, consistent with the Chabrier IMF. We also plot other star formation relations based on gas surface density (Bigiel et al. 2008 (B08); Heiderman et al. 2010 (H10); Wu et al. 2010 (W10), and Bigiel et al. 2011 (B11)) in the first panel of Fig. 8. While the H10 relation can possibly explain the observed SFR surface density in the clump A in SDP 81, it is not universally applicable. Other star formation relations shown in this panel cannot account for observed SFR surface density in all the molecular clouds. Additionally, we also note that SDP 81 lies on one of the dashed lines in the middle panel of Fig. 8, which represent deviations by a factor of 3 from the best-fitting relation of KDM12 (shown as the solid line). It is evident that there is a large scatter in both the K98 and KDM12 relations, which we calculate from

\[
\chi^2_{\text{red}} = \frac{1}{N_D} \sum \left( \frac{\Sigma_{\text{SFR}} \text{(observed)} - \Sigma_{\text{SFR}} \text{(predicted)}}{E} \right)^2,
\]

where \( E \) is the measured error on \( \Sigma_{\text{SFR}} \) (observed) and \( N_D \) is the number of star-forming regions.\(^4\) We emphasize that we do not fit any relations to compute the scatter but simply perform a \( \chi^2 \) minimization routine. From equation (13), we calculate a reduced \( \chi^2 \) scatter of \( \chi^2_{\text{red}} = 50.1 \) and 7.27 for the KS and KDM12 relations, respectively. The final panel in Fig. 8 illustrates the turbulence-based SFK15 relation. We observe that the characteristics of the clump A in SDP 81 match the SFK15 relation to a good extent. The scatter we obtain for the SFK15 relation is 1.25. It has significantly reduced as compared to the scatter from KS and KDM12 relations because the SFK15 relation includes systematic variations in the Mach number, as were established by Federrath (2013). This highlights the role of turbulence in star-forming regions (Federrath & Klessen 2012; Kraljic et al. 2014). The validity of the multi-freefall star formation relation has been previously supported in an independent work by Braun & Schmidt (2015).

7 CONCLUSIONS

Using high-resolution (sub-kpc) ALMA data of SDP 81 – a high-redshift \((z \sim 3)\) lensed galaxy, we have measured the SFR surface density in its biggest and most isotropic clump, revealed by the lensing analysis of Dye et al. (2015). Through dust SED fitting with an MBB spectrum for this clump (clump A in S15), we find the best-fitting dust temperature \( T = 39 \pm 2 \) K when an emissivity index \( \beta_{\text{em}} = 2.5 \) is used. Taking into account the systematic errors resulting from partially cold temperature dominated flux and assuming that this clump has conditions similar to those in the whole galaxy, we obtain the SFR surface density in this clump as \( \Sigma_{\text{SFR}} = 357^{+115}_{-85} \) mJy yr\(^{-1}\) kpc\(^{-2}\).

Using CO (5–4) flux and velocity data for this galaxy, we obtain a turbulent velocity dispersion of \( \sigma_{\text{turb}} = 37 \pm 5 \) km s\(^{-1}\), corresponding to a turbulent Mach number \( \mathcal{M} = 96 \pm 28 \). This is higher than the typical Mach numbers found for local galaxies, but is in agreement with those estimated for high-redshift starbursts. The turbulent velocity dispersion that goes into estimating this Mach number is obtained from the large-scale gradient subtraction of CO (5–4) velocity in the plane of the sky, which is in good agreement with the velocity dispersion obtained along the line of sight by S15 after correcting for beam smearing. Using an appropriate CO
Figure 8. Left-hand panel: Observations of \( \Sigma_{\text{SFR}} \) (SFR surface density) in local clouds, plotted against gas surface density. The data is extracted from Heiderman et al. (2010, H10), Lada, Lombardi & Alves (2010, L10), Wu et al. (2010, W10), Gutermuth et al. (2011, G11), and Jameson et al. (2016, J16). Data for the Brick molecular cloud in the CMZ is taken from Federrath et al. (2016, F16). The K98 (discs and starbursts) data were adjusted in KDM12, Krumholz et al. (2013), and Federrath (2013) to the Chabrier IMF, similar to that done for the high-redshift data by Daddi et al. (2010). The systematic difference between starbursts and the original K98 relation is due to different \( \alpha_{\text{CO}} \) conversion factors used for starbursts and discs (see section 2 of Daddi et al. 2010 and fig. 1 of Federrath 2013). SFR relations proposed by Kennicutt (1998b, K98) (corrected for Chabrier IMF), Bigiel et al. (2008, B08), Bigiel et al. (2011, B11), Wu et al. (2010, W10), and Heiderman et al. (2010, H10) are also shown. Middle panel: Observations of \( \Sigma_{\text{SFR}} \) plotted against the single-freefall time. Solid line depicts the best-fitting model from KDM12, with \( \epsilon_{\text{ff}} = 0.015 \) (see Krumholz et al. 2013); dashed lines illustrate deviations by a factor of 3 from the best fit. Right-hand panel: Observations of \( \Sigma_{\text{SFR}} \) plotted against the turbulence-based multifreefall model proposed by Salim et al. (2015, SFK15). Solid line represents equation (12). Clump A analysed in this work is marked with an arrow in all the three panels. The scatter obtained through a \( \chi^2 \) minimization routine for K98, KDM12, and SFK15 relations is 50.1, 7.27, and 1.25, respectively.

On testing star-forming relations based on gas mass, freefall times and turbulence (available in literature), we find that the KS relation underpredicts the observed SFR surface density in this clump (\( \Sigma_{\text{SFR,KS}} = 52 \pm 17 \; M_\odot \; \text{yr}^{-1} \; \text{kpc}^{-2} \)) by a factor \( \gtrsim 3.3 \), which can be corrected for if the dust temperature is lowered while keeping the emissivity same or vice versa. It is also clear that the other star formation relations as plotted in first panel of Fig. 8 are not universally applicable. Further, the single-freefall time based KDM12 relation also underestimates the observed SFR surface density in this clump, giving \( \Sigma_{\text{SFR,KDM}} = 106 \pm 37 \; M_\odot \; \text{yr}^{-1} \; \text{kpc}^{-2} \); however, it can explain the observed SFR if deviations up to a factor of 3 from its best-fitting model are considered. We also find that the large scatter present in these star formation relations can be explained by turbulence acting in this clump. The turbulence regulated multi-freefall model by SFK15 predicts the SFR surface density as \( \Sigma_{\text{SFR,SFK}} = 491^{+139}_{-149} \; M_\odot \; \text{yr}^{-1} \; \text{kpc}^{-2} \). The overestimation of SFR surface density by SFK15 can be attributed to ignoring magnetic fields while calculating the SFR through equation (12). Our findings emphasize the role of turbulence giving rise to the multifreefall model of the SFR and its consistency with the observed SFR in molecular clouds in local as well as high-redshift galaxies.

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5www.bits-pilani.ac.in/university/ipcd/home
6https://automeris.io/WebPlotDigitizer/index.html
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