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Probing the type-II seesaw mechanism through the production of Higgs bosons at a lepton collider

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We investigate the production and decays of doubly-charged Higgs bosons for the Type-II seesaw mechanism at an $e^+e^-$ collider with two center of mass energies, $\sqrt{s} = 380$ GeV and 3 TeV, and analyze the fully hadronic final states in detail. Lower mass ranges can be probed during the 380 GeV run of the collider, while high mass ranges, which are beyond the 13 TeV Large Hadron Collider discovery reach, can be probed with $\sqrt{s} = 3$ TeV. For such a heavy Higgs boson, the final decay products are collimated, resulting in fat-jets. We perform a substructure analysis to reduce the background and find that a doubly-charged Higgs boson in the mass range 800–1120 GeV can be discovered during the 3 TeV run, with integrated luminosity $\mathcal{L} \sim 95$ fb$^{-1}$ of data. For 380 GeV center of mass energy, we find that for the doubly-charged Higgs boson in the range 160–172 GeV, a 5$\sigma$ significance can be achieved with only integrated luminosity $\mathcal{L} \sim 24$ fb$^{-1}$. Therefore, a light Higgs boson can be discovered immediately during the run of a future $e^+e^-$ collider.

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I. INTRODUCTION

With the discovery of the Higgs boson at the Large Hadron Collider (LHC), we start to develop an understanding of how the standard model (SM) fermion and gauge boson masses are generated in terms of the Brout-Englert-Higgs (BEH) mechanism. However, one of the main puzzles that still remains unclear is the origin of light neutrino masses and mixings. The same BEH mechanism can, in principle be employed to generate Dirac mass of SM neutrinos, or even via a cascade decay[16–18]. Such operator is not forbidden as the lepton number is only a classical symmetry of the SM, violated by quantum effects.

There are three proposed categories, commonly known as, Type-I, -II, and -III seesaw mechanisms in which the SM is extended by a $SU(2)_L$ singlet fermion $[3–9]$, $SU(2)_L$ triplet scalar boson $[10–13]$, and $SU(2)_L$ triplet fermion $[14]$, respectively. In particular, the second possibility, i.e., where a triplet scalar field with the hypercharge $Y = +2$ is added to the SM, is the simplest model with an extended Higgs sector. The neutral component of the triplet acquires a vacuum expectation value (vev) $v_\Delta$, and generates neutrino masses through the Yukawa interactions. Perhaps, the most appealing feature of this model is its minimality. The same Yukawa interaction between the lepton doublet and the triplet scalar field generates Majorana masses for the neutrinos, and also dictates the phenomenology of the charged Higgs bosons.

A number of detailed studies have already been performed for the Hadron colliders like, Tevatron $[15]$ and LHC $[15–29]$ to search for the triplet Higgs scenario. One attractive feature of this model is the presence of the doubly-charged Higgs boson, and its distinguishing decay modes. Depending on the triplet vev, the doubly-charged Higgs boson can decay into same-sign dilepton, same-sign gauge bosons, or even via a cascade decay $[16–18]$.
The details of the Higgs spectrum have been discussed in [30–32]. For the branching ratios and collider signatures, see [16–20]. The CMS and ATLAS collaborations have searched for the same-sign dilepton final states for all flavors, and constrained the mass of the doubly-charged Higgs as $M_{H^\pm\pm} > 820$, 870 GeV at 95% C.L. [33,34]. However, this is only relevant for a very tiny vev $v_\Delta < 10^{-4}$ GeV, where the doubly-charged Higgs boson decays into the same-sign dilepton with 100% branching ratio. For larger triplet vev such as $v_\Delta \gtrsim 0.01$ GeV, this branching ratio is negligibly small. Therefore, a direct bound on the mass of the $H^{\pm\pm}$ from the same-sign dilepton final state cannot be obtained. An alternative search where the $H^{\pm\pm}$ is produced in association with two jets (vector boson fusion channel) gives relaxed constraints [35,36]. For $v_\Delta \gtrsim 10^{-4}$ GeV, the doubly-charged Higgs boson predominantly decays into same-sign diboson. The collider signatures and the discovery prospect of this scenario have been discussed in [37–39], and [40,41] (see [42] for the discussion on the composite Higgs model and [43] for discussion on flavor violating $\tau$ decays). Previous searches for $H^{\pm\pm}$ in the pair-production channel and their subsequent decays into same-sign leptons at LEP-II has put a constraint $M_{H^{\pm\pm}} > 97.3$ GeV at 95% C.L. [44].

While a number of searches at the LHC are ongoing to experimentally verify the presence of the doubly-charged Higgs boson, in this work we perform a detailed collider analysis to explore the discovery prospects at a future lepton collider. For a large mass of the doubly-charged Higgs boson, the pair-production cross-section at the LHC becomes small. Furthermore, the presence of numerous backgrounds weakens its discovery prospects. Therefore, a lepton collider with a much cleaner environment will be more suitable to search the high mass regime of the doubly-charged Higgs boson. In addition, we also exhaust the low mass regime, yet unconstrained by the LHC, and by LEP-II measurements.

We consider the pair-production of the doubly-charged Higgs boson at a lepton collider and its subsequent decays into same-sign gauge bosons. We focus on the hadronic decays of the produced gauge bosons and analyze the multijet final states in detail. As a prototype example, we consider the future $e^+e^-$ collider Compact Linear Collider (CLIC) [45–48], that will operate with the center of mass energies $\sqrt{s} = 380$ GeV, 1.4 and 3 TeV. We first analyze the discovery reach of the doubly-charged Higgs boson at 380 GeV center of mass energy. Subsequently, we carry out a detailed simulation for the very heavy doubly-charged Higgs boson with a mass around and beyond one TeV. For such a heavy Higgs, its final decay products are collimated, leading to fat-jets. We perform a jet-substructure analysis and tag the gauge bosons. We find that a heavy Higgs boson with a mass up to 1120 GeV can be most optimally discovered with $5\sigma$ significance at the 3 TeV run of CLIC with 95 fb$^{-1}$ of data. For lower masses, the range 160–172 GeV can be covered with only $L \sim 24$ fb$^{-1}$ of luminosity. For the earlier discussions on Higgs triplet model at a linear collider, see [49–52]. For the other SM and BSM searches at CLIC and other linear colliders, see [47,53–67] for Higgs physics and effective field theory, [68–72] for different BSM scenarios, and [73–80] for seesaw and radiative neutrino mass model searches. For the discussion on probing dark-sector at $e^+e^-$ collider, see [81,82].

Our paper is organized as follows: we briefly review the basics of the Type-II seesaw model in Sec. II. In Sec. III, we discuss existing experimental constraints. In the subsequent subsections, Secs. IV.A and IV.B, we analyze in detail the production cross-sections and the discovery potential of the multi-jet final states at the $e^+e^-$ collider. Finally, we present our conclusions in Sec. V.

II. MODEL DESCRIPTION

In addition to the SM Higgs field $\Phi$, the type-II seesaw model [10–13] contains an additional $SU(2)_L$ triplet Higgs field

$$\Delta = \left(\begin{array}{c}
\Delta^+ \\
\Delta^0 \\
\Delta^-
\end{array}\right) \sim (1, 3, 2). \quad (2.1)$$

We denote the neutral components of the SM doublet and triplet Higgs fields as $\Phi^0 = \frac{1}{\sqrt{2}}(\phi^0 + i\delta^0)$ and $\Delta^0 = \frac{1}{\sqrt{2}}(\delta^0 + i\phi^0)$, respectively. The real scalars $\phi^0$ and $\delta^0$ acquire vevs denoted as $v_\Phi$ and $v_\Delta$ with $v^2 = v_\Phi^2 + v_\Delta^2 = (246 \text{ GeV})^2$. The light neutrino mass is proportional to the triplet vev $v_\Delta$. The new scalar field $\Delta$, being a triplet under $SU(2)$, interacts with the SM gauge bosons. The relevant kinetic term has the form

$$L_{\text{kin}}(\Delta) = \text{Tr}[(D_\mu \Delta)^\dagger (D^\mu \Delta)], \quad (2.2)$$

with the covariant derivative $D_\mu \Delta = \partial_\mu \Delta + i\frac{g}{2} [W^\mu_R, \Delta] + igB_\mu \Delta$. The Yukawa interactions of $\Delta$ with the lepton fields are

$$L_Y(\Phi, \Delta) = Y_\Delta \overline{L}i\tau_2 \Delta L + \text{H.c.} \quad (2.3)$$

In the above, $Y_\Delta$ is a $3 \times 3$ matrix and $c$ denotes charge conjugation. The triplet field $\Delta$ carries lepton number +2 and hence the Yukawa term conserves lepton number. The scalar potential of the Higgs fields $\Phi$ and $\Delta$ is

$$V(\Phi, \Delta) = m_\Phi^2 \Phi^\dagger \Phi + \frac{\lambda}{4} \text{Tr}(\Delta^\dagger \Delta) + (\mu \Phi^\dagger \Phi \Delta^\dagger \Phi + \text{H.c.}) + \lambda_1 (\Phi^\dagger \Phi)^2 + \lambda_2 (\Delta^\dagger \Delta)^2 + \lambda_3 \text{Tr}[(\Delta^\dagger \Delta)^2] + \lambda_4 \Phi^\dagger \Delta \Delta^\dagger \Phi. \quad (2.4)$$
where \( m_{\Phi} \) and \( \tilde{M}_\Delta \) are real parameters with mass dimension 1, \( \mu \) is the lepton-number violating parameter with positive mass dimension and \( \lambda, \lambda_1-4 \) are dimensionless quartic Higgs couplings.

There are seven physical Higgs states in mass basis, that arise after diagonalization of the scalar mass matrix written in the gauge basis. They are: the charged Higgs bosons \( H^{\pm \pm}, H^{\pm} \), the neutral Higgs bosons \( h^0, H^0 \) and \( A^0 \). The two charged scalar fields \( \Phi^\pm \) of \( \Phi \) and \( \Delta^\pm \) of \( \Delta \) mix to give singly-charged states \( H^\pm \) and the charged Goldstone \( \chi^\pm \) bosons. Similarly, the mixing between the two \( CP \)-odd fields \( (\chi^0 \text{ and } \eta^0) \) gives rise to \( A^0 \), and the neutral Goldstone boson \( \rho^0 \). Finally, we obtain the SM Higgs boson \((h)\) and a heavy Higgs boson \((H)\) via the mixing of the two neutral \( CP \)-even states \( \Phi^0 \) and \( \delta^0 \).

The physical masses of the doubly and singly charged Higgs bosons \( H^{\pm \pm} \) and \( H^\pm \) can be written as

\[
m_{H^{\pm \pm}}^2 = M_\Delta^2 - v_\Phi^2 \lambda_3 - \frac{\lambda_4}{2} v_\Phi^2,
\]

\[
m_{H^\pm}^2 = \left( M_\Delta^2 - \frac{\lambda_4}{4} v_\Phi^2 \right) \left( 1 + \frac{2 v_\Phi^2}{v_\Phi^2} \right).
\]

The \( CP \)-even and \( CP \)-odd neutral Higgs bosons \( h \) and \( H \) have the physical masses

\[
m_h^2 = T_{11}^2 \cos^2 \alpha + T_{22}^2 \sin^2 \alpha - T_{12}^2 \sin 2\alpha,
\]

\[
m_H^2 = T_{11}^2 \sin^2 \alpha + T_{22}^2 \cos^2 \alpha + T_{12}^2 \sin 2\alpha.
\]

In the above \( T_{11}, T_{22} \) and \( T_{12} \) have the following expressions:

\[
T_{11}^2 = \frac{v_\Phi^2 \lambda}{2},
\]

\[
T_{22}^2 = M_\Delta^2 + 2 v_\Phi^2 (\lambda_2 + \lambda_3),
\]

\[
T_{12}^2 = - \frac{2 v_\Phi^2}{v_\Phi^2} M_\Delta^2 + v_\Phi v_\Delta (\lambda_1 + \lambda_4).
\]

The \( CP \)-odd Higgs field \( A^0 \) has the mass term

\[
m_A^2 = M_\Delta^2 \left( 1 + \frac{4 v_\Phi^2}{v_\Phi^2} \right), \quad \text{with} \quad M_\Delta^2 = \frac{v_\Phi^2 \mu}{\sqrt{2} v_\Delta}.
\]

The difference between \( H^{\pm \pm} \) and \( H^\pm \) masses is dictated by the coupling \( \lambda_4 \) of the scalar potential. For a positive \( \lambda_4 \), the \( H^{\pm \pm} \) is lighter than \( H^\pm \). The mass difference \( \Delta M^2 \) is

\[
\Delta M^2 = M_{H^{\pm \pm}}^2 - M_{H^\pm}^2 \sim \frac{\lambda_4}{2} v_\Phi^2 + O(v_\Delta^2).
\]

Throughout our analysis, we consider the mass hierarchy \( M_{H^{\pm \pm}} < M_{H^\pm} \). We have chosen \( v_\Delta = 0.1 \). For this value of \( v_\Delta \), the \( H^\pm \) decays to \( W^+ W^+ \) with 100% branching ratio.

The other Lagrangian parameters are chosen as—\( \lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 1.0, \lambda = 0.52 \). To change the \( H^{\pm \pm} \) mass we vary \( \mu \). Here we vary \( \mu \) from 0.105 to 0.15 for 380 GeV center of mass energy and from 1.56 to 4.65 for 3 TeV center of mass energy.

Due to the nontrivial representations of \( \Delta \), the Higgs triplet has interactions with a number of SM fermions and gauge bosons. This opens up a number of possible decay modes that can be explored at the LHC, and at future linear colliders. In the next section, we summarize the different direct experimental constraints on the doubly-charged Higgs boson mass and triplet vev.

### III. DECAY MODES AND EXPERIMENTAL CONSTRAINTS

The most characteristic feature of the type II seesaw model is the presence of the doubly-charged Higgs boson \( H^{\pm \pm} \), that can decay into the leptonic or bosonic states and gives unique signatures at high energy colliders. The different decay modes and the branching ratios of the \( H^{\pm \pm} \) depend on the triplet vev \( v_\Delta \). For smaller triplet vev, the \( H^{\pm \pm} \) predominantly decays into the same-sign leptonic states \( H^{\pm \pm} \to l^+ l^+ \), whereas for larger \( v_\Delta \), the gauge boson mode \( H^{\pm \pm} \to W^\pm W^\pm \) becomes dominant [16,17]. The relevant decay widths are calculated to

\[
\Gamma(H^{\pm \pm} \to l_i l_j) = \frac{M_{H^{\pm \pm}}}{(1 + \delta_{ij}) 8 \pi} \left| \frac{M_{\nu}}{v_\Delta} \right|^2,
\]

\[
M_{\nu} = Y_\Delta v_\Delta.
\]

\[
\Gamma(H^{\pm \pm} \to W^\pm W^\pm) = \frac{g^2 v_\Delta^2}{8 \pi M_{H^{\pm \pm}}} \sqrt{1 - \frac{4}{r_W^2}} \left[(2 + (r_W/2 - 1)^2)\right].
\]

In the above \( M_{\nu} \) denotes the neutrino mass matrix, \( i, j \) are the generation indices, \( \Gamma_{l_i l_j} \) and \( \Gamma_{W^\pm W^\pm} \) are the partial decay widths for the \( H^{\pm \pm} \to l_i l_j \), and \( H^{\pm \pm} \to W^\pm W^\pm \) channels, respectively. The parameter \( r_W \) denotes the ratio of \( H^{\pm \pm} \) and the \( W \) gauge boson masses, \( r_W = \frac{M_{H^{\pm \pm}}}{M_W} \). The branching fraction of the leptonic and bosonic mode becomes equal around the triplet vev \( v_\Delta \sim 10^{-4} \) GeV [16,17].

A number of searches have been proposed at the LHC to discover \( H^{\pm \pm} \) using multilepton signatures. The searched modes in [16–18,40] are pair and associated production with the \( H^{\pm \pm} \) decaying into leptonic or gauge boson states. Below we discuss the existing constraints on \( H^{\pm \pm} \) from LEP and LHC searches.

(i) Constraint from LEP-II: The search for doubly-charged Higgs boson \( H^{\pm \pm} \) decaying into charged...
leptons have been performed at LEP-II. This constrains the mass parameter $M_{H^{\pm\pm}} > 97.3$ GeV \cite{44} at 95% C.L.

(ii) Constraints from pair and associated production: Stringent constraint on the $M_{H^{\pm\pm}}$ by analyzing $H^{\pm\pm} \rightarrow l^\pm l^\pm$ have been placed at the 13 TeV LHC. The CMS collaboration looked for different leptonic flavors including $ee, e\mu, e\tau, \mu\mu, \mu\tau$ and $\tau\tau$. In addition, the CMS searches also include the associated production $pp \rightarrow H^{\pm\pm}H^\mp$ and the subsequent decays, $H^\pm \rightarrow l^\pm \nu$. This combined channel of pair-production and associated production gives the stringent constraint $M_{H^{\pm\pm}} > 820$ GeV \cite{33} at 95% C.L. for $e, \mu$ flavor. The constraint from ATLAS searches comes from pair-production. The bound is $M_{H^{\pm\pm}} > 870$ GeV at 95% C.L \cite{33}. Note that these limits are valid only for a small triplet vev $v_\Delta < 10^{-4}$ GeV.

(iii) Constraint from VBF: For larger values of the triplet vev $v_\Delta \geq 10^{-1}$ GeV, the leptonic branching ratio becomes smaller. Instead the decay mode $H^{\pm\pm} \rightarrow W^\pm W^\pm$ is dominant. Therefore the searches in vector boson fusion (VBF) become more important. A search for $pp \rightarrow jjH^{\pm\pm} \rightarrow jjW^\pm W^\pm$ at the 8 TeV LHC in the VBF channel sets a constraint on the triplet vev $v_\Delta \sim 25$ GeV for $M_{H^{\pm\pm}} \sim 300$ GeV \cite{35}. This constraint has been updated \cite{36} using 13 TeV data at the LHC.

Note that, for extremely small $v_\Delta$, the mass of the doubly-charged Higgs boson is very tightly constrained from pair-production searches. For a larger triplet vev, this constraint significantly relaxes. The VBF cross-section scales quadratically with the triplet vev and hence, increases for a very large vev. However, the range of $v_\Delta \sim 10^{-4}$ to $10^{-1}$ GeV cannot be probed at the 13 TeV LHC in VBF channel, as the cross-section becomes extremely small in this range. Recently, in \cite{41}, the authors have looked for pair-production of $H^{\pm\pm}$ in large $v_\Delta$ region and analyzed the signature where the final state contains dilepton, multijet, and missing energy. The lighter mass $M_{H^{\pm\pm}} \sim 190$ GeV can be probed at the 14 TeV LHC with 3000 fb$^{-1}$ of data. In \cite{39}, the authors have used LHC 8 TeV run-I result of same-sign dilepton to derive a bound $M_{H^{\pm\pm}} \geq 84$ GeV, relevant for large $v_\Delta$. For large mass of the doubly-charged Higgs, the LHC cross section however becomes significantly smaller, as shown in Fig. 2. On the other hand, the fall in the cross-section at a $e^+ e^-$ collider is relatively smaller. This motivates us to explore the signatures of doubly-charged Higgs at a lepton collider, where the cross-section still remains larger for heavy charged Higgs masses. In the following sections, we explore the scope of a future linear collider to probe large $v_\Delta$ region with (a) a very low mass range of $H^{\pm\pm}$, that is still experimentally allowed, and (b) a very heavy highly boosted $H^{\pm\pm}$.

IV. LARGE TRIPLET VEV AND COLLIDER SIGNATURES

In this section, we analyze the collider signatures of a doubly-charged Higgs boson at an $e^+ e^-$ collider and explore the sensitivity reach to probe low and high mass regimes. Throughout our analysis, we consider a large triplet vev $v_\Delta \geq 10^{-4}$ GeV, where the present experimental constraints are weak. As the prototype example, we consider CLIC \cite{45-48} that will operate with three different center of mass energies $\sqrt{s} = 380, 1400$ GeV and 3 TeV. We present our simulation for 380 GeV and 3 TeV center of mass energies. The doubly-charged Higgs boson, $H^{\pm\pm}$, can be produced at $e^+ e^-$ collider via photon and Z-boson mediated diagrams, as shown in Fig. 1. We show in Fig. 2 the respective production cross sections. As both of the diagrams are s-channel processes, the cross section reduces with increasing center-of-mass energy. For a relatively small center-of-mass energy $\sqrt{s} \sim 380$ GeV, the maximum cross section reaches up to $\sigma \sim 506$ fb for $M_{H^{\pm\pm}} = 102$ GeV. A rapid decline in the cross section occurs near $M_{H^{\pm\pm}} \sim 190$ GeV, close to kinematic threshold. For the choice of large $v_\Delta$, the produced particles $H^{\pm\pm}$ will decay into $W^\pm W^\pm$ gauge bosons with almost 100% branching ratio. In the following, we will first discuss the low-mass regime, that can be probed in the $\sqrt{s} = 380$ GeV run. Following that we discuss the high-mass regime, that can be explored at 3 TeV center of mass energy and gives rise to specific signatures of boosted Higgs boson. In both cases we focus on multijet final states.

![Diagram](https://via.placeholder.com/150)

**FIG. 1.** The Feynman diagram for $H^{++}H^{--}$ pair-production and its subsequent decays into gauge bosons.
we demand a high jet multiplicity, i.e., the number of jets $N_{\text{jets}} \geq 7$. For the signal, the production processes are
(i) $e^+e^- \rightarrow H^{\pm}H^\mp \rightarrow 4W \geq 7j$ for $M_{H^{\pm}} \gtrapprox 2M_W$
(ii) $e^+e^- \rightarrow H^{\pm}H^\mp \rightarrow W^\pm jj W^\mp jj \geq 7j$ for $M_{H^{\pm}} < 2M_W$

In the former scenario the $H^{\pm}$ decays predominantly into on-shell $W^\pm W^\pm$, while in the latter case $H^{\pm}$ decays into one on-shell and one off-shell gauge bosons with subsequent decays into jets.

To simulate the events, we use first FeynRules [83] and generate the model file via Universal FeynRules Output (UFO) [84,85]. We compute the hard processes with the package MADGRAPH5_AMC@NLO [86], and pass the output (in LHE format) through PYTHIA 6 [87] for showering and hadronization. The detector simulation has been taken into account by DELPHES-3.3.0 [88], where we use the ILD card. Here we use anti-$k_t$ jet clustering algorithm [89] to form jets. Similar final states will be generated from a number of SM processes. We consider the following sets of backgrounds and perform a detailed simulation:
(i) $e^+e^- \rightarrow \ell\ell \rightarrow 6j$
(ii) $e^+e^- \rightarrow W^+W^- + 3j$, $W^\pm \rightarrow 2j$, and $e^+e^- \rightarrow ZZ + 3j$, $Z \rightarrow 2j$
(iii) $e^+e^- \rightarrow 7j$
(iv) $e^+e^- \rightarrow W^+ + 5j$, $W^\pm \rightarrow jj$, and $e^+e^- \rightarrow Z + 5j$, $Z \rightarrow jj$

Among the backgrounds, $e^+e^- \rightarrow 7j$ includes diagrams of coupling order $\alpha_w^2 \alpha_s^3$ with quarks and gluons as intermediate particles. As listed above, we treat the $\ell\ell$ and gauge boson mediated backgrounds separately. For the partonic event generation, we implement the following

### Table I

The cross sections for the signal and background for the fully hadronic final states, arising from $e^+e^- \rightarrow H^{\pm}H^\mp$. $\sigma_p$ refers to the partonic cross section. $\sigma_d$ is the cross section after taking into account detector effects. The last column represents the cross section with $b$-veto. The center-of-mass energy is $\sqrt{s} = 380$ GeV and kinematic cuts are specified in the text.

<table>
<thead>
<tr>
<th>Mass (GeV)</th>
<th>$\sigma_p$ (fb)</th>
<th>$\sigma_d(N_{\text{jets}} \geq 7j)$ (fb)</th>
<th>$\sigma_d(N_{\text{jets}} \geq 7j + b$-veto) (fb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>121</td>
<td>0.80</td>
<td>0.30</td>
<td>0.20</td>
</tr>
<tr>
<td>137</td>
<td>2.08</td>
<td>0.94</td>
<td>0.66</td>
</tr>
<tr>
<td>159</td>
<td>5.45</td>
<td>2.58</td>
<td>1.82</td>
</tr>
<tr>
<td>172</td>
<td>5.04</td>
<td>2.48</td>
<td>1.74</td>
</tr>
<tr>
<td>184</td>
<td>1.11</td>
<td>0.53</td>
<td>0.38</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Processes</th>
<th>$\sigma_p$ (fb) $\times 10^{-2}$</th>
<th>$\sigma_d(N_{\text{jets}} \geq 7j)$ (fb) $\times 10^{-2}$</th>
<th>$\sigma_d(N_{\text{jets}} \geq 7j + b$-veto) (fb) $\times 10^{-2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+e^- \rightarrow \ell\ell \rightarrow 6j$</td>
<td>10341.0</td>
<td>338.0</td>
<td>36.0</td>
</tr>
<tr>
<td>$W^+W^- 3j$, $W^\pm \rightarrow 2j$</td>
<td>8.89</td>
<td>1.18</td>
<td>0.88</td>
</tr>
<tr>
<td>ZZ $+ 3j$, $Z \rightarrow 2j$</td>
<td>0.98</td>
<td>0.13</td>
<td>0.10</td>
</tr>
<tr>
<td>$7j$</td>
<td>30.32</td>
<td>1.13</td>
<td>0.88</td>
</tr>
<tr>
<td>$W^+ + 5j$, $W^\pm \rightarrow jj$</td>
<td>30.18</td>
<td>4.64</td>
<td>3.54</td>
</tr>
<tr>
<td>$Z + 5j$, $Z \rightarrow jj$</td>
<td>18.32</td>
<td>2.15</td>
<td>1.61</td>
</tr>
</tbody>
</table>
sets of cuts at MadGraph level both for the signal and backgrounds: the transverse momentum of light jets \( p_T(j_j) > 20 \text{ GeV} \) for all the final state partons, the pseudorapidity \( |\eta| < 5.0 \), and the separation between the light jets \( \Delta R(j_i,j_j) > 0.4 \).

We consider few illustrative mass points between \( M_{H^\pm} \sim 121 \text{ GeV} \) and the kinematic threshold \( M_{H^\pm} \sim 184 \text{ GeV} \), and display the signal cross sections in Table I. The cross sections \( \sigma_p \) refers to the partonic cross section, while \( \sigma_j \) is after taking into account reconstruction and detector effects. In addition to the cuts at the partonic level, we further implement few more selection cuts: the transverse momentum of jets \( p_T(j_j) > 20 \text{ GeV} \) for all the jets, pseudo-rapidity \( |\eta| < 4.5 \) for jets, and the number of jets \( N_j \geq 7j \). The largest background arises from \( t \rightarrow 6j \), where the cross section is about 103 fb at the partonic level. This is much larger than the largest signal cross section 5.45 fb, corresponding to \( M_{H^\pm} \sim 159 \text{ GeV} \). For other mass points, the ratio is even bigger. However, demanding high jet multiplicity \( N_j \geq 7j \) reduces this background to \( \sigma_j \sim 3 \text{ fb} \). For the masses of the doubly-charged Higgs boson \( M_{H^\pm} = 159 \) and 172 GeV, the signal and background cross sections become almost equal after demanding higher jet multiplicity. A few comments are in order:

(i) Between the higher and lower mass ranges, i.e., \( M_{H^\pm} > 2M_W \) and \( M_{H^\pm} < 2M_W \), the former scenario corresponds to larger pair-production cross sections. The fall in cross section in the higher mass range occurs when \( M_{H^\pm} \sim 184 \text{ GeV} \), where it approaches the kinematic threshold. For lower mass ranges, \( M_{H^\pm} \sim 121 \text{ GeV} \), the reduction of cross-section after the detector effect occurs due to stronger kinematic cuts. The produced jets from a \( H^\pm \) with mass \( M_{H^\pm} \sim 121 \text{ GeV} \) are often quite soft. With the constraint on jet transverse momentum \( p_T > 20 \text{ GeV} \) the reconstruction efficiency becomes smaller.

(ii) The signal comprises of hadronic final states with higher jet multiplicity. For the signal, \( H^\pm \) decays into two \( W^\pm \) with subsequent decay into quarks, resulting in a final state with \( N_j = 8 \). At a \( e^+e^- \) collider, there are only a few SM processes that can generate a similar final state. A full reconstruction of the signal results in a fairly low reconstruction efficiency. Thus, we allow for one jet to be too soft or out of the kinematic cuts range.

In Table II, we derive the statistical significance \( n_s = \sigma_d(S)/\sqrt{\sigma_d(S) + \sigma_d(B)} \) for our benchmark points corresponding to Table I. Here \( \sigma_d(S) \) and \( \sigma_d(B) \) represent the final cross-sections for the signal and background after all the selection cuts. Additionally, we also show the required luminosity to achieve a 5\( \sigma \) significance. Other than the extreme low and high mass ranges \( M_{H^\pm} = 121 \) and 184 GeV, all other mass points have a large discovery prospect with 125 fb\(^{-1}\) of data. In particular, we show that the doubly-charged Higgs boson with intermediate masses of 159 GeV (172 GeV) can be discovered with 5\( \sigma \) significance with only \( \mathcal{L} \sim 22(24) \text{ fb}^{-1} \), respectively. From Table III, we can see that it further improves to \( \mathcal{L} \sim 16(17) \text{ fb}^{-1} \) after applying a b-veto (50–60% efficiency and 1% miss-tag efficiency), which helps in reducing the dominant top-quark pair background.

### B. Boosted heavy \( H^\pm \) at \( \sqrt{s} = 3 \text{ TeV} \)

We now consider heavy \( H^\pm \) with a mass \( M_{H^\pm} \sim 1 \text{ TeV} \) and its decay into like-sign \( W^\pm W^\pm \) gauge bosons. The produced \( W^\pm \) decays into hadronic as well as leptonic states. As before, we focus on the purely hadronic final states, which has the largest branching ratio. For such heavy \( H^\pm \), each of the produced \( W^\pm \) boson will have large transverse momentum. For a 1.1 TeV \( H^{\pm} \), their transverse momentum peaks around \( p_T \sim 1 \text{ TeV} \), and most of the \( W^\pm \) are produced in the central region. We show the transverse momentum, and the pseudo-rapidity distribution of \( H^{\pm} \) in Fig. 3, for the illustrative benchmark points \( M_{H^{\pm}} = 800 \text{ GeV}, 1120 \text{ GeV} \) and 1.4 TeV.

The final decay products of such heavy Higgs bosons are highly collimated, and can be reconstructed as fat-jets, see Fig. 4. Therefore, our model signature for such high mass \( H^{\pm} \) is

(i) \( e^+e^- \rightarrow H^{\pm\pm} \rightarrow W^+W^-W^\pm W^\mp \rightarrow 4\text{fat-jet} \).
To generate signal and backgrounds we use the same tool-chain as in Sec. IVA except the use of DELPHES. Here we analyze the output of PYTHIA8 [90] (in HEPMC [91] format) and reclustering fat-jets using Cambridge-Aachen algorithm [92] in FASTJET-3.0.0 [93] with radius parameter $R = 1.0$. In Fig. 5, we show the transverse momentum of the leading fat-jet $j_1$ and the 4th leading fat-jet $j_4$. A number of backgrounds can lead to the final states with multiple fat-jets. These are: 4j (includes both the QED and QCD contributions), $W^+W^-2j$, and $W^+/W^-3j$, $W^+W^-Zjj$ and $it$, with subsequent decays of $W$ boson and the top quark into jets. The partonic cross sections of the signal and backgrounds are shown in Table IV. The cross-sections for $W^+W^-Zjj$ and $it$ are small compared to other backgrounds. Therefore, we do not include these backgrounds in our final analysis. Below we discuss in detail the preselection and selection cuts for the signal and backgrounds:

(i) Most of the signal events are in the central region with pseudorapidity distributed around $\eta \sim 0$, as can be seen in Fig. 3. Additionally, the signal jets have a very high $H_T$ (scalar sum of transverse momentum of all final state particles), as shown in Fig. 6. We consider no cuts on the signal at the parton level. While generating the backgrounds, we consider the following partonic cuts for 4j—the transverse momentum of the jets $p_T > 60$ GeV, and the jet-jet separation $\Delta R(j; j) > 0.6$; for $W^+W^-2j(W^\pm > 2j)$ and $W^+3j(W^\pm > 2j)$—$p_T > 60$ GeV for the leading 4-jets, the transverse momentum $p_T > 20$ GeV for the remaining jets, and the jet-jet separation $\Delta R(j, j) > 0.4$. The $H_T$ and pseudo-rapidity cut is the same for all the backgrounds, $H_T > 1000$ GeV and $|\eta| < 2.5$. For $it$ samples we have put $\Delta R(b, j) > 0.4$ separation cut and transverse momentum cut on leading two light jet as $p_T > 60.0$ GeV. Additionally, we also

FIG. 3. The normalized distribution of the transverse momentum and the pseudorapidity for the produced $H^{\pm\pm}$.

FIG. 4. The Feynman diagram for $H^{++}H^{--}$ pair-production and its subsequent decays to 4 fat-jets.

FIG. 5. The $p_T$ distribution of the leading and 4th leading fat-jets. For signal, we consider $M_{H^{\pm\pm}} = 1120$ GeV.
TABLE IV. The cut-flow for the signal and backgrounds. The cross sections are in fb. $\sigma_p$ refers to the partonic cross section. In the backgrounds the decays of the $W^{\pm}$ boson and top quark to jets are included. Here MD refers to mass-drop. See text for details.

<table>
<thead>
<tr>
<th>Masses (GeV)</th>
<th>$\sigma_p$ (ab)</th>
<th>$4j_{fat}$ (&gt;120 GeV)</th>
<th>4 MD</th>
<th>1 tagged</th>
<th>2 tagged</th>
<th>3-tagged</th>
<th>4-tagged</th>
</tr>
</thead>
<tbody>
<tr>
<td>800</td>
<td>1250</td>
<td>812.9</td>
<td>758.0</td>
<td>757.9</td>
<td>748.9</td>
<td>671.8</td>
<td>389.0</td>
</tr>
<tr>
<td>1000</td>
<td>850.6</td>
<td>527.0</td>
<td>492.5</td>
<td>492.3</td>
<td>486.1</td>
<td>436.6</td>
<td>258.9</td>
</tr>
<tr>
<td>1120</td>
<td>670.0</td>
<td>380.0</td>
<td>358.4</td>
<td>358.3</td>
<td>354.2</td>
<td>321.9</td>
<td>193.1</td>
</tr>
<tr>
<td>1350</td>
<td>167.1</td>
<td>80.4</td>
<td>75.54</td>
<td>75.52</td>
<td>74.88</td>
<td>68.2</td>
<td>42.0</td>
</tr>
<tr>
<td>1400</td>
<td>94.36</td>
<td>45.54</td>
<td>42.85</td>
<td>42.84</td>
<td>42.42</td>
<td>38.6</td>
<td>24.0</td>
</tr>
</tbody>
</table>

Backgrounds

<table>
<thead>
<tr>
<th>Processes</th>
<th>$\sigma_p$ (ab)</th>
<th>$4j$ (&gt;120 GeV)</th>
<th>4 MD</th>
<th>1 tagged</th>
<th>2 tagged</th>
<th>3-tagged</th>
<th>4-tagged</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4j$</td>
<td>6900.0</td>
<td>1310.0</td>
<td>895.0</td>
<td>360.0</td>
<td>68.0</td>
<td>5.5</td>
<td>0.0</td>
</tr>
<tr>
<td>$W^+W^-3j$ &amp; $W^-3j$</td>
<td>1900.0</td>
<td>320.0</td>
<td>220.0</td>
<td>166.0</td>
<td>44.0</td>
<td>4.8</td>
<td>$1.52 \times 10^{-1}$</td>
</tr>
<tr>
<td>$W^+W^-2j$</td>
<td>190.0</td>
<td>25.6</td>
<td>17.7</td>
<td>15.6</td>
<td>8.3</td>
<td>1.23</td>
<td>$5.7 \times 10^{-2}$</td>
</tr>
<tr>
<td>$W^+W^-Zjj$</td>
<td>4.23</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t\bar{t}$</td>
<td>42</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Other coming from another $H^{--}$. This leads to smaller number of 4 fatjet events. From 2nd column and 3rd column in Table IV, this is evident that after demanding four fatjet with $p_T > 120$ GeV the drop in the cross section is larger for higher masses as compare to that of the lower masses.

(iii) The model signature contains four fat-jet with high momentum. We show in Fig. 5, the transverse momentum of leading and 4th leading fat-jet for $M_{H^{\pm \pm}} = 1120$ GeV. Additionally, we also show the distributions of the backgrounds. It is evident that most of the jets have larger transverse momentum for signal, with $p_T \gg 100$ GeV. Therefore, we design our selection cuts as (a) the number of fat-jets $N_{fat} = 4$, (b) $p_T > 120$ GeV for all the fat-jets.

(iv) We further carry out substructure analysis for the fat-jets. To reconstruct the W bosons we use the

FIG. 6. The $H_T$ distribution of the jets at the partonic level. We consider three illustrative benchmark points $M_{H^{\pm \pm}} = 800, 1120, and 1400$ GeV.
mass-drop tagger \cite{94} of which compares the energy-sharings of subjets to indicate if the fat-jet was initiated by a $W$ boson or a parton. For the signal and background, we show the invariant mass of the two subjets inside the fat-jet in Fig. 8. For the signal, the subjets inside a fat-jet are generated from the $W$. Therefore, the distribution peaks around the $W$ mass. For the different backgrounds, $4j$ gives flat distribution, while $W^+W^-2j$ and $W^+3j$ shows smaller peak around $M_W$. As shown in Table IV, the backgrounds are significantly reduced after applying the selection cut $|M_{J_1J_2} - M_W| \leq 20$ GeV. Here, $M_{J_1J_2}$ is the invariant mass of the subjets $J_1$ and $J_2$ inside a fat-jet. A detailed cut-flow chart is given in Table IV. If at least one fat-jet passes the invariant mass selection cut, we have 1-tagged event; if at least two fat-jet pass the cut, we have 2-tagged event and so on.

From Table IV, the effect of the substructure analysis is clearly evident. The largest background arises from the $e^+e^- \rightarrow 4j$ events. At the partonic level we find a cross section of $\sigma_p(4j) \sim 6.9$ fb $\gg \sigma_p$(signal). The higher transverse momentum cut on jet $p_T$ reduces the signal nominally, and the background by more than $\mathcal{O}(5)$ for $4j$, $WW2j$ and $W + 3j$. Demanding that 4 fat-jets have a nontrivial substructure (referred to as mass-drop MD in Table IV)
TABLE V. The statistical significance $n_s$ for $\mathcal{L} = 500$ fb$^{-1}$ and the required luminosity to achieve $5\sigma$ significance. The c.m.e.
ergy is $\sqrt{s} = 3$ TeV. In the 2nd column, to derive significance, we consider 2 tagged events for 800–1120 GeV mass range and 3 tagged events for the higher mass range. Here 2-tag implies two or more than two fat-jet masses are within the window of 60–100 GeV, and the fat-jets are tagged as $W$ jets. Similar criteria applies for 3-tagged jets.

<table>
<thead>
<tr>
<th>Masses (GeV)</th>
<th>$n_s$ (2,3-tagged)</th>
<th>$\mathcal{L}$ (fb$^{-1}$) (with 2,3-tagged)</th>
</tr>
</thead>
<tbody>
<tr>
<td>800</td>
<td>17.96 (2-tag)</td>
<td>38.75</td>
</tr>
<tr>
<td>1000</td>
<td>13.95 (2-tag)</td>
<td>64.23</td>
</tr>
<tr>
<td>1120</td>
<td>11.49 (2-tag)</td>
<td>94.68</td>
</tr>
<tr>
<td>1350</td>
<td>5.40 (3-tag)</td>
<td>428.66</td>
</tr>
<tr>
<td>1400</td>
<td>3.85 (3-tag)</td>
<td>843.31</td>
</tr>
</tbody>
</table>

reduces the background even more. Finally, with the invariant mass cut for the subjets, all backgrounds become almost negligible. For the $H^{\pm\pm}$ masses between 800 GeV to 1.1 TeV one can achieve a $S/B \sim \mathcal{O}(10)$. We show the required luminosity to achieve a discovery in Table V. The 800–1120 GeV doubly-charged Higgs boson can be discovered with 39–95 fb$^{-1}$ of data with at least 2 fat-jet tagged as $W$-bosons. However, for higher masses, such as 1.4 TeV a minimum 3 tagged jets will be required.

V. DISCUSSION AND CONCLUSIONS

The Type-II seesaw model consists of an extension of the scalar sector by a Higgs triplet field $\Delta$ with hypercharge $Y = \pm 2$. The neutral component of the triplet acquires a vev and generates the light neutrino mass. One of the most attractive features of this model is the presence of the doubly-charged Higgs boson $H^{\pm\pm}$. Depending on the triplet vev, $H^{\pm\pm}$ can decay into a number of final states, including same-sign leptons, same sign gauge bosons, and via cascade decay to three body final states. For the lower triplet vev where $H^{\pm\pm} \rightarrow \ell^\pm j^\pm$ decays are predominant, the doubly-charged Higgs boson mass is tightly constrained by LHC pair and associated production searches, $M_{H^{\pm\pm}} > 820, 870$ GeV. However, the higher triplet vev region is poorly constrained by the VBF searches. Moreover, the LHC search is limited in the very high mass region $M_{H^{\pm\pm}} \sim 1$ TeV, where the cross section is tiny.

In this work, we consider an $e^+e^-$ collider operating with two center-of-mass energies $\sqrt{s} = 380$ GeV and 3 TeV, and probe the large $v_\Delta$ region $v_\Delta \geq 10^{-2}$ GeV. We consider two mass regimes, (a) light $H^{\pm\pm}$ with mass $M_{H^{\pm\pm}} \lesssim 180$ GeV, and (b) a very heavy $H^{\pm\pm}$ with mass $M_{H^{\pm\pm}} \sim 800$–1400 GeV. We consider fully hadronic decays of the produced $W$’s and perform a detailed analysis for the multi-jet final states.

For the 380 GeV center of mass energy, we look into multijet final states with $N_j \geq 7j$. We find that a doubly-charged Higgs boson with mass $M_{H^{\pm\pm}} \sim 160$–172 GeV can be discovered in the immediate run of the $e^+e^-$ collider, with only integrated luminosity $\mathcal{L} \sim 24$ fb$^{-1}$. This improves considerably once we apply a $b$-veto, reducing the $t\bar{t}$ background to $\sigma \sim \mathcal{O}(0.1)$ fb.

The higher mass range $M_{H^{\pm\pm}} \geq 1$ TeV can be probed in the $\sqrt{s} = 3$ TeV run of the $e^+e^-$ collider. Note that, for such high masses of $H^{\pm\pm}$ the pair-production cross section at 13 TeV LHC is significantly smaller. Therefore, an $e^+e^-$ collider with large center of mass energy is more suitable to probe the high mass range. For such heavy mass, the produced $W$’s are boosted and their subsequent decay products will be collimated, resulting in fat-jets. A number of SM processes, including $4j, W^{\pm}j, W^2/2j$ can mimic the signal. To reduce backgrounds, we carry out a jet-substructure analysis with $W$-tagging. We find that for the 800–1120 GeV mass range, a minimum of two tagged jets can effectively reduce the total backgrounds to a level of $\sigma \sim \mathcal{O}(0.1)$ fb, whereas the signal cross section is $\sigma \sim \mathcal{O}(0.3–0.7)$ fb. For higher masses, three tagged jets are needed. A doubly-charged Higgs boson with mass between 800–1120 GeV can be discovered with $\mathcal{L} \lesssim 95$ fb$^{-1}$ of data. For even higher masses, such as $M_{H^{\pm\pm}} \sim 1400$ GeV, a discovery will require much higher integrated luminosities.

Thus, a future high-energy $e^+e^-$ collider can provide an outstanding opportunity to probe weakly-coupled heavy particles, which are beyond the reach of the LHC.

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