A New Industry Concentration Index

August 24, 2015

Abstract

We propose and analyse a new concentration index alternative to the Herfindahl-Hirschman Index ($HHI$). This new index emphasizes the concept of competitive balance. It is designed to preserve the convexity property of the $HHI$ when a merger involves one of the $m$ largest firms, but to decrease and thus to indicate an increase in competition when a merger is purely among the $(n-m)$ smallest firms.

**Keywords:** Horizontal mergers, dominant firms, small firms, competitive balance, concentration index.

**JEL Classification** Number: L10
1 Introduction

Merger analysis often involves a comparison between the pre- and post-merger degrees of concentration in a market. This degree of concentration matters since a high concentration measure is supposed to proxy for lack of competitiveness in that market. The standard index that is used to measure the level of concentration in an industry is the Herfindahl-Hirschman Index ($HHI$). The $HHI$ possesses the so-called convexity property in that it increases whenever there is a ‘mean preserving spread’ of firms’ market shares in an industry. Consequently, it yields a higher concentration level in response to any merger between firms.

Suppose that there are three firms with percentage market shares of 70, 25 and 5 in Industry 1. Ceteris paribus, the $HHI$ would deem that this industry is more concentrated and thus less competitive than another one, Industry 2, which has market shares of 70, 15 and 15. This can be far from obvious, since in Industry 2, a dominant firm facing two relatively small and potentially insignificant rivals may simply ‘follow’ the lead of the dominant firm whereas, in contrast, in Industry 1, the dominant firm facing a competitor with a 25% market share may be able to provide greater competitive restraint to the dominant firm than two equally-sized but smaller rivals of Industry 2 can.

Similarly, using the $HHI$ a merger of two 15% market-share firms when there is a single rival would raise the $HHI$ and may be deemed anti-competitive. This too can be far from obvious, since a 30% rival may prove to be far more vigorous and competitive against the dominant firm than two 15% firms.\(^1\)

\(^1\)With such a merger, the $HHI$ would rise from 5,350 pre-merger (using the standard convention of normalizing the $HHI$ to be out of 10,000) to 5,800 post-merger. Consequently, as can be seen from ‘safe harbour’ examples below, this merger would certainly fall outside any safe harbours established in merger guidelines issued by competition regula-
Given this issue with the HHI (and the other concentration indices that will be briefly discussed in the next section), in most jurisdictions a merger that would lead to significant cost savings and so raise welfare would have a path to legal clearance (for example ‘authorisation’ in Australia, the ‘rule-of-reason’ in the United States). Hence, in reality an increase in the HHI would not stop a merger but rather would force the merging parties to prove their cost savings, although it may be possible that a merger of small firms can raise competition directly, even if there are no cost savings.

Noting these issues with the HHI (and other indices below), we propose and analyse an alternative index which emphasizes the concept of ‘competitive balance’. This new index is designed to have the convexity property of the HHI when a merger involves one of the $m$ largest firms, but to decrease and thus to indicate an increase in competition when a merger is purely among the $(n - m)$ smallest firms.

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3Gugler et al. (2003) provide strong evidence that among mergers that increase profits, those involving larger firms achieve these profits by increasing their market power, while mergers involving smaller firms achieve higher profits by increasing efficiency.
2 A Brief Review of Concentration Indices

Let the market shares of \( n \) firms be listed as \( v_1 \geq v_2 \geq ... \geq v_n > 0 \) where \( \sum v_i = 1 \). As mentioned above, the standard, most-prominent industry concentration index is the (\( HHI \)): 

\[
HHI(v_1, ..., v_n) = (a_1v_1 + ... + a_nv_n),
\]

where \( a_i = v_i \) so that the weights, \( a_1, ..., a_n \), sum to one.

Another notable concentration index, the four-firm concentration ratio (\( C4 \)), does not depend on the market shares of firms which are not the largest four firms: 

\[
C4(v_1, v_2, v_3, v_4) = (v_1 + v_2 + v_3 + v_4).
\]

Neither does it assign different weights to different market shares of the firms.

There are a few other notable concentration indices. One, proposed by Hall and Tideman (1967), stresses the need to include the number of the firms in the calculation when measuring the concentration level of an industry (the number of firms measures the ease of entry into that particular industry). The Hall-Tideman concentration index (\( HTI \)) is 

\[
\frac{1}{(2\sum_{i=1}^{n} v_i)^{-1}}.
\]

The other one, an index of entropy, 

\[
E = -\sum_{i=1}^{n} v_i \log v_i
\]

is discussed by Hart (1967, p. 78). Unlike the other indices considered thus far, it does not have a range of 0 to 1. Rather, it takes the value 0 when the market structure is a monopoly and takes a value far exceeding 1 when the market structure is perfect competition.

Finally, Dansby and Willig (1979) introduced alternative performance indices that measure the potential social gains from appropriate government interventions (such as anti-trust, regulatory, and deregulatory actions). Their performance indices establish a welfare theoretic basis for indices such as \( C4, HHI, \) and others. Essentially, Dansby-Willig versions of these indices

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incorporate a weight that is the inverse of the price elasticity of the industry demand. Alternatively, Blackorby, Donaldson and Weymark (1982) (who use a Cobb-Douglas functional form) provided an index that assigns weights to not only firms’ market shares but also to total output.

Note that any merger would increase the measure of industry concentration according to all of the indices above, except for $C_4$ in which any merger beyond the largest four firms would not have a neither negative nor positive effect on the measure of concentration unless the newly merged firm itself becomes one of the largest four firms.

3 CB* - The Competitive Balance Index

The ‘competitive balance’ index we propose has different implications than the indices discussed above when horizontal mergers do not include the largest firm(s). Denote this index when there are $m$ dominant firms in an industry as $CB^*(m)$, where $1 \leq m \leq n$. When $m = 1$, Firm $i$’s market share relative to that of the sole dominant firm is $\frac{v_i}{v_1}$. It follows that the total market shares, relative to the largest firm’s market share is $\frac{v_1}{v_1} + \frac{v_2}{v_1} + \ldots + \frac{v_n}{v_1}$. We first consider this index when market shares of firms are measured in terms of only the largest firm’s market share, $CB^*(1)$.

$$CB^*(1) = \frac{1}{\left(\frac{v_1}{v_1}\right)^2 + \left(\frac{v_2}{v_1}\right)^2 + \ldots + \left(\frac{v_n}{v_1}\right)^2}$$

$$= \frac{1}{\frac{1}{(v_1)^2} + \frac{1}{(v_2)^2} + \ldots + \frac{1}{(v_n)^2}}$$
\[ = \frac{(v_1)^2}{(v_1)^2 + (v_2)^2 + \ldots + (v_n)^2}. \]

Observe that \( CB^*(1) = \frac{(v_1)^2}{HHI} \).

Table 1 provides a few examples to illustrate the stark differences between the \( HHI \) and \( CB^*(1) \).
Table 1: A Comparison of the $HHI$ and $CB^*(1)$ under Different Market Share Profiles

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Although there may be industries in which increasing the market share of the second largest firm could cause a reduction in the industry price, in many industries a reduction in price could not be achieved until a higher critical number of large firms is reached. For example, Lamm (1981, p. 75) reports empirical findings from the food retailing industry that in many urban markets “growth in the 3 largest firms’ shares have a significant positive effects on prices... In contrast, an increase in the market share of the fourth largest firm causes a reduction in food prices.” This clearly indicates that the number of dominant firms in a market may be greater than one which is critically important to the analysis of a potential merger. Thus, we now explore our index with $m > 1$ dominant firms, $CB^*(m)$. 
\[ CB^*(m) = \frac{1}{v_1^2 + v_2^2 + \ldots + v_m^2} \]

\[ = \frac{1}{[v_1^2 + (v_2)^2 + \ldots + (v_n)^2] \cdot \frac{1}{v_1^2 + \ldots + v_m^2}} \]

\[ = \frac{v_1^2 + \ldots + v_m^2}{(v_1)^2 + (v_2)^2 + \ldots + (v_n)^2}. \]

Observe that when \( m > 1 \), \( CB^*(m) = \frac{v_1^2 + \ldots + v_m^2}{HII} \).

The following proposition describes how \( CB^*(m) \) behaves in mergers that do and do not involve the largest firm.

**Proposition 1** (1) If merger does not involve the \( m \) largest firms and does not make the new firm one of the \( m \) largest firms, then \( CB^*(m) \) decreases.

(2) If a merger involves one or two of the \( m \) largest firms, then \( CB^*(m) \) increases.

**Proof:** (1) Since \([v_1^2 + (v_2)^2 + \ldots + (v_n)^2]\) increases in any merger, \( CB^*(m) = \frac{v_1^2 + \ldots + v_m^2}{(v_1)^2 + (v_2)^2 + \ldots + (v_n)^2} \) must decrease in any merger that does not involve any of \( v_1, v_2, \ldots, v_m \). (2) Consider \( CB^*(m) = \frac{v_1^2 + \ldots + v_m^2}{(v_1)^2 + (v_2)^2 + \ldots + (v_n)^2} \). Suppose Firm \( i \) and Firm \( j \) merge such that \( i \leq m \) and \( j > m \). Thus, after the merger \( CB^{*'}(m) = \frac{v_1^2 + \ldots + v_{i-1}^2 + v_{i+1}^2 + \ldots + v_m^2 + (v_i + v_j)^2}{(v_1)^2 + (v_2)^2 + \ldots + (v_n)^2 + (v_i + v_j)^2 + (v_{j+1})^2 + \ldots + (v_n)^2} \). Let \( A = (v_1)^2 + \ldots + (v_m)^2 \) and \( B = (v_1)^2 + \ldots + (v_m)^2 + \ldots + (v_n)^2 \); hence, \( B > A \). Thus, \( CB^*(m) \) becomes \( \frac{A}{B} \) and \( CB^{*'}(m) \) becomes \( \frac{A + (v_i + v_j)^2 - v_i^2}{B + (v_i + v_j)^2 - v_i^2 - v_j^2} \). Then \( CB^{*'}(m) \)

\[ \Rightarrow \] \( CB^*(m) \) reduces to \( Bv_j^2 + 2Bv_iv_j \) \( \Rightarrow \) \( 2Av_i v_j \). Since \( B > A \), we obtain \( CB^{*'}(m) > CB^*(m) \).
For the second case where Firm $i$ and Firm $j$ merge such that $i, j \leq m$, slightly modify the above argument. ■

The next proposition describes how much $CB^*(m)$ increases when Firm $j$ merges with Firm $i$ instead of with Firm $i'$ where $v_i > v_{i'} > v_m > v_j$.

**Proposition 2** Consider a merger $M$ between Firm $i$ and Firm $j$, and a merger $M'$ between Firm $i'$ and Firm $j$, where $v_i > v_{i'} > v_m > v_j$. Then $CB^*(m, M) > CB^*(m, M')$.

**Proof:** $CB^*(m, M) = \frac{v_i^2 + \ldots + v_{i-1}^2 + v_{i+1}^2 + \ldots + v_m^2 + (v_i + v_j)^2}{v_i^2 + \ldots + v_{i-1}^2 + v_{i+1}^2 + \ldots + v_m^2 + (v_i + v_j)^2 + \ldots + (v_n)^2}$ and $CB^*(m, M') = \frac{v_i^2 + \ldots + v_{i-1}^2 + v_{i+1}^2 + \ldots + v_m^2 + (v_{i'} + v_j)^2}{v_i^2 + \ldots + v_{i-1}^2 + v_{i+1}^2 + \ldots + v_m^2 + (v_{i'} + v_j)^2 + \ldots + (v_n)^2}$.

Let $A = (v_1)^2 + \ldots + (v_m)^2$ and $B = (v_1)^2 + \ldots + (v_m)^2 + \ldots + (v_n)^2$. Thus, $CB^*(m, M)$ becomes $\frac{A + (v_i + v_j)^2 - v_i^2}{B + (v_i + v_j)^2 - v_i^2}$ and $CB^*(m, M')$ becomes $\frac{A + (v_{i'} + v_j)^2 - v_i^2}{B + (v_{i'} + v_j)^2 - v_i^2}$. Then $CB^*(m, M) \geq CB^*(m, M')$ reduces to $\frac{A + v_{i'}^2 + 2v_{i'}v_j}{B + 2v_{i'}v_j} \geq \frac{A + v_{i'}^2 + 2v_{i'}v_j}{B + 2v_{i'}v_j}$. Since $B > A + v_{i'}^2$ and $2v_{i'}v_j > 2v_{i'}v_j$, we obtain $CB^*(m, M) > CB^*(m, M')$. ■

The implication of the preceding proposition is that according to $CB^*(m)$, a merger between a small firm and a relatively large dominant firm will increase the concentration level in that industry more than will a merger between the same small firm and a relatively small dominant firm. Thus, Propositions 1 and 2 verify that $CB^*(m)$ satisfies the convexity property of the $HHI$ when a merger involves one of the $m$ largest firms, but decreases and thus indicates an increase in competition when a merger is purely among the $(n - m)$ smallest firms.
Table 2 considers several different market settings in an attempt to gauge how the $HHI$ and $CB^*$ respond to proposed mergers. In each row, merging firms’ pre-merger market shares are denoted with a box around them. For instance, Rows 1 – 5 entail a situation in which there is one dominant firm and six identical smaller firms. All of the smaller firms merge in Row 1, five of the smaller firms merge in Row 2, and so on. The last two columns furnish the predicted changes in the two indices. Observe that $HHI$ increases while $I^*$ decreases for each merger.

Table 2: Changes in $HHI$ and $CB^*$ Resulting from Mergers

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4 Discussion: An Example

The general idea behind our index is that a merger between two small firms that creates a more competitive mix of firms should be allowed, even if that mix yields a net reduction in the number of firms by one and an increase
in the $HHI$. The main purpose of this discussion section is to provide an example of mergers that would increase the level of concentration according to the $HHI$, although these mergers are capable of increasing competitive balance and economic welfare. Not surprisingly, they decrease the level of concentration according to $CB$.

Heubeck et al. (2006) discuss the inadequacy of the $HHI$ and other concentration indices that simply add pre-merger market shares of merging firms to approximate post-merger shares. The basic problem is that this method ignores second-order “industry wide strategic effects” that arise from post-merger competition where firms strategically alter quantity or pricing decisions.

The following example begins with the standard dominant/fringe firm model. We next allow a subset of the fringe firms to merge at a level that makes them competitive against the previously unique dominant firm. Pre-merger status quo is such that the dominant firm sets its price based on residual demand, leaving the fringe firms to take that price and choose output accordingly. In post-merger setup, however, the newly merged firm is on equal footing with the previously dominant firm and engages in Bertrand competition where the remaining fringe firms take the price that results from that competition as given.

The profit motive for the merging firms is akin to Caveat 3 (page 1245) of Levin (1990): they merge in order to eliminate redundancies in fixed costs. We adopt this motive for two reasons. First, the elimination of fixed-cost redundancies is sufficient to guarantee the profitability of the merger. Second, we wish to avoid variable production efficiencies as their presence would possibly lead to merger approval even if the $HHI$ suggests otherwise.
(recall the discussion in the Introduction).

Our specific example is as follows.\textsuperscript{4} Demand in the market is $Q = 90 - P$, where $Q$ is the total quantity produced by all firms and $P$ is the market price. The dominant firm has a cost of $C = 50 + \frac{1}{2} q_d^2$ where $q_d$ is quantity it produces, and each of the four smaller fringe firms has a cost of $C = 45 + q_f^2$ where $q_f$ is quantity each such firm produces. Inverse Residual Demand for the dominant firm’s product is therefore $P = 30 - \frac{1}{3} q_d$. Then in this market the dominant firm sets a price of $24$ and produces $18$ units, generating a profit of $220$. Suppose the fringe firms follow the dominant firm by accepting that price; then each produces $12$ units and earn profits of $99$.

Now let two of the fringe firms merge in an effort to eliminate fixed cost redundancies and become strategically more competitive against the dominant firm. Assume that the remaining two fringe firms remain on the fringe, taking the equilibrium price arising between the previously dominant firm and the newly merged firms as given. That price equilibrium is arrived at via Bertrand competition for the residual demand left by the two remaining fringe firms.\textsuperscript{5}

The marginal cost curves of the firms that merged yield the cost function of this newly-created dominant firm, $C = 50 + \frac{1}{2} q_d^2$, where the reduction

\textsuperscript{4}Although this example is stylized and simple for illustrative purposes, the qualitative results will hold for many other parameter values as well.

\textsuperscript{5}The one caveat to the standard Bertrand model here is that rather than assuming that the firm with the lower price serves the entire residual market, we assume that the firm with the lower price has the option of serving the entire residual market but may choose to only serve a portion of the market if serving the entire market becomes prohibitively costly. If the lower-price firm serves less than the entire residual demand, then the higher-price Bertrand competitor picks up the remaining residual demand. This caveat is necessary because marginal cost is increasing in this example rather than being constant as is commonly assumed in standard Bertrand models.
in fixed costs has put the merged firm on equal footing with the previously unique dominant firm. Given that there are only two remaining fringe competitors, the Inverse Residual Demand curve facing the two Bertrand competitors is now \( P = 45 - \frac{1}{2} q_d \).

The Bertrand equilibrium price in the game between the dominant firms can easily verified to be $22.50. At that price the previously-dominant firm as well as the newly-merged dominant firm each produces 22.5 units and each earns $203.13 in profits. Each of the two remaining fringe firms now produces 12.25 units and each earns $92.81 in profits. For all prices above $22.50, each firm finds it profitable to lower its price if the other firm matches or goes below that price.\(^6\) Once the price reaches $22.50, no firm finds it profitable to lower the price further as doing so will only give them the opportunity to sell additional units at a price below the marginal cost of providing them.

The reduction in fixed costs that accompanies the merger leads to joint profits for the merged firm that are greater than the summed individual profits they would have earned by remaining on the fringe. Because the market price is lower than the pre-merger price and marginal costs have not changed, the merger leads to an increase in welfare. Pre-merger, the market shares were \((\frac{1}{3}; \frac{1}{6}; \frac{1}{6}; \frac{1}{6})\). Post-merger, they become \((\frac{1}{3}; \frac{1}{3}; \frac{1}{6}; \frac{1}{6})\), leading to pre- and post-merger HHI measures of 0.22222 and 0.27777 respectively. Alternatively, the pre- and post-merger \(CB^*(1)\) measures are 0.5 and 0.4.

\(^6\)Once again, the argument deviates slightly from the standard Bertrand argument that the firms find this profitable because they pick up the entire market. For instance, hypothesize that both firms charge $22.51. The result then is that they split residual demand, each supplying 44.49 units and making $203.12 in profits. By lowering their price to $22.50, either firm can now sell 22.5 units at a profit, and slightly increase profits. It is worth noting that the firm does not pick up the entire residual demand of 45 units at the price of $22.50 because supplying any units beyond 22.5 units incurs a marginal cost of \(Q\) that is greater than the price.
Thus, this example illustrates how a merger that is welfare enhancing can decrease $CB^*$, but increase the $HHI$.

Extending the example further, we can use $CB^*(2)$ by then allowing the two remaining fringe firms to merge in order to compete with the two dominant firms, putting all three firms on equal footing. It can be easily confirmed that once the final two fringe firms merge, the equilibrium Bertrand outcome is for each firm to charge $22.50, resulting in output by each firm of 22.5 units. There is no welfare loss since the pre- and post-merger prices are the same. Using the pre- and post- merger market shares of $(\frac{1}{3}, \frac{1}{6}, \frac{1}{6})$ and $(\frac{4}{9}, \frac{1}{3})$ yield the respective pre- and post- $HHI$ measures of 0.27777 and 0.33333. Likewise, the pre-and post-merger $CB^*(2)$ measures are 0.8 and 0.66666, once again illustrating how a merger that is welfare enhancing can decrease $CB^*$, but increase the $HHI$.

Finally, in our index the analyst must make a judgement about how many firms to include in $m$, but once $m$ is chosen, a merger involving two small firms decreases the index, and a merger involving at least one of the large firms raises the index. One relevant question then is ”what determines $m$?” In some cases, the industry analysts may have already determined it empirically, as reported in Lamm (1981) that was mentioned above (as observed in the food retailing industry, “growth in the 3 largest firms’ shares have a significant positive effects on prices... In contrast, an increase in the market share of the fourth largest firm causes a reduction in food prices”). In some other cases, like in the example we have just provided above, a natural gap between the market shares of firms may provide strong clues about $m$. E.g., if the market shares profile is $(\frac{1}{3}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6})$, then it would be straightforward to deduce that
\( m \) is 1, whereas if that profile is \((\frac{1}{3}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7})\), then one can deduce that \( m \) is 2.

5 References


