Abstract—Recently, we designed and commissioned a ‘crossboard’ sample holder which can apply biaxial strains in the plane of a (RE)BCO coated conductor. It allows us to measure the critical current density \( J_C \) for arbitrary combinations of \( x \)- and \( y \)-strain. Understanding the in-field, in-plane, biaxial strain dependence of a tape’s \( J_C(B, \varepsilon_{xx}, \varepsilon_{yy}) \) is crucial for applications such as CORC® or Roebel cables, as the cables are subjected to multiaxial strains during manufacturing and operation. Here we present experimental data for \( J_C(B, \varepsilon_{xx}, \varepsilon_{yy}) \) on a SuperPower SCS4050 APC tape in magnetic fields up to 0.7 T, at 77 K. We also outline a theoretical model for the biaxial strain dependence of \( J_C \) and use it to parameterise our data and show that the fraction of A-domains and B-domains are roughly equal (\( \theta = 0.49 \pm 0.03 \)) and that the strain sensitivity of the critical temperature is 1.8 ± 0.1 K%/\( \varepsilon \) and -1.3 ± 0.1 K%/\( \varepsilon \) along their \( a \)- and \( b \)-axes respectively, for all the domains in this (RE)BCO tape. For the first time, we show both parabolic and linear strain dependences of \( J_C \) in a single tape by changing the angle between the applied strain direction and the twin boundaries in the (RE)BCO layer.

Index Terms—Critical current, strain measurement, 2G HTS conductors, cuprates.

I. INTRODUCTION

THERE are many applications of (RE)BCO coated conductors (commonly known as (RE)BCO tapes) in which the tapes are subjected to multiaxial loads during manufacturing and operation. For example, the (RE)BCO tapes in ‘conductor on round core’ (CORC®) cables, which are constructed from tapes wound helically around a round core [1], are subjected to axial, transverse, and shear strains [2]. Roebel cables are also subjected to these strains, due to the material discontinuities across the widths of the cables [3]. Finite element models of CORC® or Roebel cables under strain can be used to predict their electrical performance completely if the effect of strain on the critical current density \( J_C \) is comprehensively understood for an arbitrary strain direction (i.e. in three dimensions) [3, 4]. Therefore, there is a need to develop experimental equipment and mathematical models to measure and explain the multiaxial strain dependence of \( J_C \), as this will allow cable manufacturers to optimise their cable designs to minimise the suppression of \( J_C \) with strain. The effect on \( J_C \) by uniaxial strain along the length of a (RE)BCO tape is well known [5-12]. It has led to the tapes in CORC® cables being wound at 45° to minimise the suppression of \( J_C \) with that type of strain [1, 5]. However, there have been comparatively fewer studies on the effect of the other types of strain on \( J_C \) [3, 13-16] and even fewer on the effect of having strains along multiple axes of a tape simultaneously [13, 16]. Single crystal data suggest that the in-plane strain dependence of the critical temperature of (RE)BCO is large (along the \( a \)-axis and \( b \)-axis) but very weak along the \( c \)-axis of (RE)BCO [17]. This means that biaxial measurements and understanding may provide a very significant improvement in our ability to predict the performance of these tapes in advanced applications with complex strain distributions. To this end, we have recently designed and commissioned a biaxial sample holder known as a ‘crossboard’ which can apply in-plane, biaxial strains to a tape [16]. The crossboard allows us to measure the in-plane biaxial strain dependence of \( J_C \) for arbitrary \( x \)- and \( y \)-strains (where the \( x \)-axis is parallel with the length of the tape and the \( y \)-axis is orthogonal to the length of the tape). The crossboard is manufactured from Beryllco® 25 and has \( x \)-strain and \( y \)-strain limits of \(-0.50\% \leq \varepsilon_{xx} \leq 0.50\% \) and \(-0.20\% \leq \varepsilon_{yy} \leq 0.15\% \) respectively.

In this paper, in Section II we present \( J_C(B, \varepsilon_{xx}, \varepsilon_{yy}) \) measurements as a function of \( x \)-strain and \( y \)-strain using the cross-board at 77 K in magnetic fields \( B \) up to 0.7 T. We then present the Biaxial Strain model for \( J_C \) (BSJ) and parameterise our results in Section III. The model considers (RE)BCO layer as a 1D chain of (RE)BCO A- and B-domains which have their \( a \)- and \( b \)-axes aligned with the length direction of the tape respectively. The in-field, in-plane, biaxial strain dependence of a whole tape’s critical current density, \( J_C(B, \varepsilon_{xx}, \varepsilon_{yy}) \), is governed by the \( J_C(B, \varepsilon_{xx}, \varepsilon_{yy}) \)'s of the A- and B-domains. The model and experimental results are then discussed in Section IV and then the paper concludes in Section V.

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The dashed lines are fits to the strain data. The open data points and dashed linear fits correspond to the strains that the tape were fixed to at 300 K and solid linear fits and closed data points correspond to the strains at 77 K.

Fig. 1. The strains that were applied to the (RE)BCO tape. The open data points and dashed linear fits correspond to the strains that the tape were fixed to at 300 K and solid linear fits and closed data points correspond to the strains at 77 K.

### TABLE I
FREE PARAMETER VALUES FROM THE LINEAR FITS IN Fig. 1

| Strain Dataset | Temperature (K) | $\frac{\partial \varepsilon_{yy}}{\partial \varepsilon_{xx}}$ | $\varepsilon_{yy}(\varepsilon_{xx} = 0)$ (%)
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Dataset I</td>
<td>300</td>
<td>1.03±0.02</td>
<td>-0.002 ± 0.001</td>
</tr>
<tr>
<td></td>
<td>77</td>
<td>0.94±0.01</td>
<td>0.034 ± 0.066</td>
</tr>
<tr>
<td>Dataset II</td>
<td>300</td>
<td>-0.33±0.007</td>
<td>-0.100 ± 0.002</td>
</tr>
<tr>
<td></td>
<td>77</td>
<td>-0.33±0.002</td>
<td>-0.066 ± 0.004</td>
</tr>
<tr>
<td>Dataset III</td>
<td>300</td>
<td>-0.33±0.007</td>
<td>0.000 ± 0.001</td>
</tr>
<tr>
<td></td>
<td>77</td>
<td>-0.31±0.01</td>
<td>0.015 ± 0.003</td>
</tr>
</tbody>
</table>

II. EXPERIMENTS

A. Experimental Procedure

$J_C(B, \varepsilon_{xx}, \varepsilon_{yy})$ d.c. current transport measurements have been performed on a single SuperPower SCS4050 APC tape sample in fields up to 0.7 T with the current density $J$ orthogonal to the field at 77 K, with the field aligned parallel to the plane of the tape ($\theta = 0^\circ$) and with it perpendicular to the plane of the tape ($\theta = 90^\circ$). The tape’s Hastelloy substrate is ~50 μm thick and its (RE)BCO layer is ~1 μm thick and contains artificial pinning centers (APC’s). The shape and dimensions of the crossboard on which the sample is mounted have been described previously [16]. Put simply, the crossboard is a sample holder for the tape that is shaped like a cross. The pairs of opposite arms of the cross can be used to apply either tensile or compressive strains in orthogonal directions. The (RE)BCO tape sample had a width of 4 mm, a length of 24 mm and it was soldered with its substrate side facing downwards onto the center of a crossboard. A pair of voltage taps with a separation of 9 mm were soldered about the center of the tape. We have used the convention that the length of the tape was aligned with the $x$-axis of the crossboard. The strains $\varepsilon_{xx}$ and $\varepsilon_{yy}$ were measured using a HBM 1-XY91-1.5/120 2D T-rosette strain-gauge [18], which was glued onto the upper surface of the tape. The strain state of the tape was noted at room temperature before it was cooled to 77 K.

B. $J_C$ Results

Fig. 1 shows the strains that were measured in this paper. At room temperature a strain was applied in each of both directions and the strains noted. The sample was then cooled to 77 K and the strain was noted again. $J_C$ was then measured at 77 K every 0.1 T up to 0.7 T with $J$ orthogonal to the field and with the field applied either parallel or orthogonal to the surface of the tape. An electric field criterion of 100 μV/m was used to calculate $J_C$. The sample was then warmed up to 300 K so the strain state of the tape could be changed. Three sets of strain data were obtained at 77 K and at room temperature as shown in Fig. 1. Table I gives the free parameter values of the linear fits to the strain values for each set of data. The error on the strains in Figs. 1-4 is $\Delta \varepsilon_{xx} \approx \Delta \varepsilon_{yy} \approx 0.02\%$ and is dominated by the uncertainty of the resistance of the strain gauge at zero strain. The free parameters and their errors were calculated by performing least-squares straight-line fits on each of the strain data sets. The associated sets of $J_C$ data are shown in Figs. 2-4. In Fig. 2 strains were applied so that the $x$-strain and $y$-strain were the same (which we refer to as Dataset I). In Fig. 3, the crossboard was first strained along the $y$-axis, then different strains were applied along the $x$-axis (referred to as Dataset II). In Fig. 4, strains were applied along the $x$-axis only (which we refer to as Dataset III). $J_C$ was measured as a function of field once at each of the strains in Dataset I first, followed by once for each strain in Dataset II and then once for each strain in Dataset III. Figs. 2-4 show data for fields every 0.2 T from 0.1 T up to 0.7 T, although data was obtained for 0.2 T, 0.4 T and 0.6 T as well. To ensure that $J_C$ was reversible over the experimental strain range, $J_C(B, \varepsilon_{xx} = 0, \varepsilon_{yy} = 0)$ was measured between Datasets. There was no permanent degradation of $J_C(B, \varepsilon_{xx} = 0, \varepsilon_{yy} = 0)$ over the course of the experiment. Dataset I’s $J_C$ data in Fig. 2 is linear with respect to $x$-strain around $\varepsilon_{xx} = 0\%$. In contrast, Dataset II’s and Dataset III’s $J_C$ data...
have parabolic behaviour with respect to applied x-strain around $\varepsilon_{xx} = 0\%$.

III. THEORY AND PARAMETERISATION

Here we outline a straightforward biaxial strain model for these (RE)BCO tapes that extends the one-dimensional models in the literature [5, 6, 10]. We assume the superconducting layer can be represented by a 1D chain of A- and B-domains that have their crystallographic $a$- and $b$-axes aligned with the length direction of the tape respectively. The standard convention is that the Cu-O chains lie along the $b$-direction. In single crystals uniaxial tension along the $a$-direction increases $T_C$. The critical current density of the tape, $J_C$, can be calculated by considering the electric field contributions from each of the A- and B-domains. This leads to:

$$J_C = \left[f J_{C_{A}}^{-N} + (1 - f) J_{C_{B}}^{-N}\right]^{\frac{1}{N}},$$

where $f$ and $(1 - f)$ are the fractions of A- and B-domains, $J_{C_{A}}$ and $J_{C_{B}}$ are the critical current densities of the A- and B-domains and $N$ is the tape’s quality index in each $t$th domain which is defined using the well-known $E - J$ power law

$$\frac{E_{i}}{E_{C}} = \left(\frac{J}{J_{C_{i}}}ight)^{N},$$

where $E_C$ is the electric field criterion usually taken to be 10 or 100 $\mu$V/m, $E_i$ is the electric field and $i$ is A or B. $J_{C_{i}}$ of domain type $i$ can be calculated using the engineering scaling law [6, 19-23]

$$J_{C_{i}} = A[B_{C_{i,l}}]^{n-3} b^{p-1}(1 - b)^{q}T_{C_{i}}^{2}(1 - t^2)^{2}.$$  

(3)

The parameters $A$, $p$, $q$, and $n$ are fitting constants, $B_{C_{i,l}}$ is the upper critical magnetic field of field domain type $i$, $b$ is the reduced field $B/B_{C_{i,l}}$, $T_{C_{i}}$ is the critical temperature of domain type $i$ and $t$ is the reduced temperature $T/T_{C_{i}}$. In this paper $n$ was set at 2.5 [24, 25], $B_{C_{e}}(0,0)$ at 98.7 T for $\theta = 90^\circ$ and 185 T for $\theta = 0^\circ$ [6, 26] and $T_{C}(0,0)$ was set at 90.1 K [26, 27]. The critical temperature in domain type $i$ is parameterised using the upper critical field where [6, 20, 21]:

$$\frac{B_{C_{i,l}}(T, \varepsilon_{xx, yy})}{B_{C_{i,l}}(0,0,0)} = \frac{\left(T_{C_{i}}(\varepsilon_{xx}, \varepsilon_{yy})\right)^{w}}{\left(T_{C_{i}}(0)\right)^{w}}(1 - t)^{s},$$

where $B_{C_{i,l}}(0,0,0)$ is the upper critical magnetic field of an A- or B-domain at zero temperature and zero strain and $w$ and $s$ are fitting constants. Here we have set $w$ at 2.2 [6, 20] and $s$ at 1.26 [6] to constrain the fits to data. Measurements of the uniaxial strain dependence of $T_C$ of single crystals of (RE)BCO have shown that $T_C$ varies linearly with planar strain [17, 27, 28]. Hence, we define the biaxial strain dependencies of $T_{C_{A}}$ and $T_{C_{B}}$ using:

$$T_{C_{A}}(\varepsilon_{xx}, \varepsilon_{yy}) = T_{C}(0,0)(1 + g_{A}(\varepsilon_{xx} - \varepsilon_{xx0}) + g_{B}(\varepsilon_{yy} - \varepsilon_{yy0})),$$

$$T_{C_{B}}(\varepsilon_{xx}, \varepsilon_{yy}) = T_{C}(0,0)(1 + g_{A}(\varepsilon_{xx} - \varepsilon_{xx0}) + g_{B}(\varepsilon_{yy} - \varepsilon_{yy0})).$$

(5)

(6)

In this paper, $T_{C}(0,0)$ is a constant equal to the critical temperature in both domains when $\varepsilon_{xx} = \varepsilon_{xx0}$ and $\varepsilon_{yy} = \varepsilon_{yy0}$. The parameter $g_{A} = (\partial T_{C_{A}}/\partial \varepsilon_{xx})_{\varepsilon_{yy0}} = (\partial T_{C_{A}}/\partial \varepsilon_{yy})_{\varepsilon_{xx0}}$ and $g_{B} = (\partial T_{C_{B}}/\partial \varepsilon_{xx})_{\varepsilon_{yy0}} = (\partial T_{C_{B}}/\partial \varepsilon_{yy})_{\varepsilon_{xx0}}$. $g_{A}$ and $g_{B}$ are the sensitivities of a (RE)BCO single crystal’s $T_C$ when there are only strains along the $a$- or $b$-axis respectively. The constants $\varepsilon_{xx0}$ and $\varepsilon_{yy0}$ are included to account for prestrains that can be induced in a (RE)BCO layer during a tape’s manufacturing process and cool down, due to the mismatch in coefficients of thermal expansion between the different layers in the tape [6, 10, 29, 30]. In this paper we have chosen to set
The data in Figs. 2-4 demonstrate a change from linear behaviour in Fig. 2 to parabolic strain behaviour in Fig. 3 and Fig. 4 for the same tape. A linear behaviour occurs in Fig. 2 because the $I_C$'s and $T_C$'s of the A- and B-domains both increase with increasing tensile strain. If $|g_A| > |g_B|$, then the tape's $I_C$ increases with tensile strain and if $|g_A| < |g_B|$, the tape's $I_C$ decreases with increasing tensile strain. Parabolic behaviour occurs in Figs. 3-4 because the $I_C$'s of the A- and B-domains increase and decrease respectively with increasing tensile strain. Hence, the $I_C$ of the tape has a maximum value at which the sum of $E$-fields produced by the fractions of A- and B-domains are minimized for any transport current. The twin boundaries in the (RE)BCO layers of SuperPower's tapes are aligned with the [110] direction, however, for other manufacturers such as Fujikura, the twin boundaries are aligned with the [100] direction. The uniaxial strain dependencies of SuperPower's and Fujikura's tapes are parabolic and linear respectively [10, 31] and van der Laan et al. have shown that the in-plane strain dependence of $I_C$ is anisotropic with respect to the direction of applied strain [5]. For the first time, we have been able to show both linear and parabolic strain behaviour on a single tape sample by applying strain in different directions with respect to the twin boundary orientation. Figs. 2-4 show that our theoretical description can produce the main features of the data but more work is needed to provide confidence in the theory described here.

The scatter on the $I_C$ data in this paper is about ~5% in Figs. 2-4. We attribute the scatter to an increase in contact resistance with (RE)BCO layers of the tape due to thermal cycling. The contact resistance increased during the course of this experiment but did not significantly affect $I_C(B, \varepsilon_{xx} = 0, \varepsilon_{yy} = 0)$. The tape and the probe were thermally cycled every time the applied strain was changed. This repeated thermal cycling risks damaging the sample and is very time-consuming. Future work will include biaxial strain measurements using equipment that can apply biaxial strains at low temperatures in-situ, as well as improving the strain range of the crossboard so the linear relationship in Fig. 2 can be verified for a larger strain range.

In order to obtain consistent fits to our data, we have allowed the scaling law fitting parameter $A$ (3) to vary between the three sets of strain data as well as between different field angles. More data are required to determine why this has been necessary. It is probably associated with plastic deformation in some components of the tape or the crossboard and is consistent with the relatively large variation in $I_C$ of up to ~10% at zero applied strain that we found throughout this experiment. For example, during the experiment, we noted that the strain state of the tape did not relax back to zero strain when we stopped applying stresses to the tape. We have also assumed that the single crystal

### Table II

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\theta = 90^\circ$</th>
<th>$\theta = 0^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$</td>
<td>0.49 ± 0.03</td>
<td></td>
</tr>
<tr>
<td>$g_A$ ($K_v^{-1}$)</td>
<td>1.8 ± 0.1</td>
<td></td>
</tr>
<tr>
<td>$g_B$ ($K_v^{-1}$)</td>
<td>$-1.3 ± 0.1$</td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>0.706 ± 0.006</td>
<td>0.674 ± 0.006</td>
</tr>
<tr>
<td>$q$</td>
<td>0 ± 0.2</td>
<td>5.8 ± 0.3</td>
</tr>
<tr>
<td>$A_{BDI}$ (MAm$^{-2}$T$^4$K$^{-2}$)</td>
<td>20.0 ± 0.5</td>
<td>23.2 ± 0.7</td>
</tr>
<tr>
<td>$A_{BDII}$ (MAm$^{-2}$T$^4$K$^{-2}$)</td>
<td>20.5 ± 0.5</td>
<td>23.2 ± 0.7</td>
</tr>
<tr>
<td>$A_{BDIII}$ (MAm$^{-2}$T$^4$K$^{-2}$)</td>
<td>21.5 ± 0.5</td>
<td>23.6 ± 0.7</td>
</tr>
</tbody>
</table>

In Fig. 2 maps a line on Fig. 5 that is parallel to the ridge and leads to the observed linear behavior for $I_C$. However, the $I_C$ data in Fig. 3 and Fig. 4 correspond to a range of strains that pass over the ridge and lead to the parabolic behavior observed.
variable strain data showing $T_C$ is linear in strain can be extrapolated to tensile strains and over the relatively large range of strains measured in these experiments. This assumption needs independent verification.

Single crystal measurements from Welp et al. generated values of $g_A$ and $g_B$ that were positive and negative respectively and of roughly equal magnitude [28]. Our parameterisation gives $g_A$ and $g_B$ opposite signs, consistent with Welp, however the magnitude of our $g_A$ and $g_B$ values are approximately 30% different. Experiments on detwinned (RE)BCO tapes have also shown that $g_A$ and $g_B$ can have different magnitudes [27]. Indeed we expect some variability in these parameters from tapes because these tapes are loaded with APC’s to increase $I_C$. However, these APC’s will also strain the (RE)BCO material and introduce disorder.

V. Conclusions

We have completed $I_C$ measurements on a single SuperPower SCS4050 APC tape sample at 77 K in fields up to 0.7 T using a crossboard and have parameterised the results using our model. The model for $I_C(B,E_{xx},E_{yy})$ treats the (RE)BCO layer as a 1D chain of A- and B- domains that have their a- and b-axes aligned with the length of the tape respectively. We have shown that the fraction of A-domains in the (RE)BCO layer is roughly the same as the B-domains (i.e. $f$ is 0.49) and that the strain sensitivities of the $T_C$’s of both domain types have values of 1.8 $\%/\%$ for strains along the a-axis and $-1.3$ $\%/\%$ for strains along the b-axis. By using biaxial strain measurements, we have observed and explained for the first time, a change between linear strain behavior for $I_C$ and parabolic strain behaviour for $I_C$ in the same tape. In the future, we aim to investigate parabolic and linear strain behaviour on tape samples from other manufacturers which have different twin boundary orientations.

Acknowledgment

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References


