Scheduling Infrastructure Renewal for Railway Networks

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ABSTRACT

The pressing necessity to renew infrastructure assets in developed railway systems leads to an increased number of activities to be scheduled annually. Scheduling of renewal activities for a railway network is a critical task since these activities often require a significant amount of time and create a capacity conflict in operation scheduling. This paper discusses economic and technological aspects, opportunities, and constraints in the renewals of multiple rail infrastructure components at several locations in a railway network. We address and model a challenging situation that there are inter-relationships between different track lines, and thus, possession of a track line can have impacts on the other track lines and prevent renewal works on them. A mathematical formulation for the railway infrastructure renewal scheduling problem in the network context is presented to minimize the total renewal and unavailability costs. A method based on a triple-prioritization rule and an optimal sharing of renewal times allocated for different types of rail infrastructure components in a possession is proposed to solve the problem. The method is applied to a real case of a regional railway network in Northern Netherlands and it is shown that up to 13% of total costs can be saved compared to the current scheduling practice.
INTRODUCTION

Railway infrastructure represents an important backbone of our modern society. Keeping its performance at reliable and safe levels is thus of utmost importance for the services it provides to the economy and social life. However, many railway assets have reached the end of their end-of-life time and need to be replaced or substantially renewed. In recent years, large investments have been taken to deal with the problem of aging assets. According to the European Rail Market Monitoring, the rail infrastructure expenditure in Europe reached more than 44 billion in 2014 (Rail Market Monitoring 2016), only 24% of which (10.6 billion) was for regular maintenance and the majority (33.8 billion or 76%) was for renewal of existing infrastructure and upgrading or construction of new infrastructure to improve the overall system performance. A recent report similarly reveals that the US spends approximately $27 billion on freight rail and around $11 billion on passenger rail annually to ensure the networks good condition (American Society of Civil Engineers 2017), the planning of renewal activities has become a challenging task for railway infrastructure agencies.

Railway maintenance and renewal works are performed in a possession and the possession duration depends on the type of work to be executed. While regular preventive and minor corrective maintenance activities can be performed in short possessions, e.g. a few hours at night, and do not cause large traffic disruptions, renewal works require long hours of working and often block track lines from train services (Lidén 2015; Lake et al. 2002). Longer possessions on the railway network also result in nuisances for train customers since the tracks are not available for train service and an alternative way of transport such as bus replacement is required with longer travel time. The more possessions are requested for renewing railway assets, the larger the capacity conflict with train operation is experienced. Therefore, possessions for renewal work are typically planned several months in advance as part of a negotiating process between infrastructure managers and train operators. It often requires from infrastructure managers an intensive effort to establish annual renewal schedules for a railway network that limits the capacity reduction for train services while keeping renewal costs within budget (Gorman and Kanet 2010).

In literature, there are quite a few studies addressing the problem of rail infrastructure main-
tenance and renewal scheduling. Many researchers focus on a single type of component, e.g. ballast, sleeper, and overhead line, or a single type of activity where the optimal maintenance or renewal intervals for the component are identified (Zhao et al. 2006; Andrade and Teixeira 2011; Vale et al. 2012; Zhao et al. 2007; Zorita A. L. et al. 2010; Santos and Teixeira 2012). Others scholars investigate a single track line with different types of component to determine the optimal maintenance schedule to be applied to the line (Budai et al. 2006; Pouryousef et al. 2010; Pargar et al. 2017; Caetano and Teixeira 2013; Caetano and Teixeira 2015; Zhao et al. 2009; Dao et al. 2018; Higgins 1998; Burkhalter et al. 2018). Some recent studies (Peralta D. et al. 2018; Sharma et al. 2018) employ deterioration models and geometry measurement data for the maintenance problem at track level. There has been little research on scheduling railway maintenance and renewal on the network level with multiple track lines. An example is the work of Zhang et al. (2013) who study the problem of assigning limited maintenance teams to perform maintenance activities at several track segments in a railway network by using an enhanced genetic algorithm approach. Similarly, Peng et al. (2011) suggest an iterative heuristic solution approach to minimize the travel costs of maintenance teams as well as the impact of renewal projects on train operation for large-scale railway networks. Another example is the mixed integer linear programming (MILP) model for scheduling the renewal of rails, ballast, and sleepers in a network context developed by Caetano and Teixeira (2016). A special study with an application on a metro rail transit network by Argyropoulou et al. (2019) focuses on scheduling urgent corrective maintenance activities and presents an integer linear programming (ILP) optimization model to minimize the impacts of these maintenance activities on passenger delay.

Despite the network perspective and unavailability consideration, previous studies do not address the inter-relationship between track lines within a network for the scheduling problem. This inter-relationship between different track lines is critical to ensure a continuous traffic flow in the network during scheduled renewal work. For example, if a track line is blocked due to a renewal possession, other works on divert routes in the network are not allowed since a certain part of the railway network then becomes isolated and is no longer accessible for train services. Another
constraint can occur when different track lines are part of the same railway corridor. Simultaneous renewal of several track lines of the same corridor can confront travelers with multiple rail replacement transport during their journey.

The scheduling process becomes more challenging and complex in the network context, especially due to a large number of different infrastructure components and the numerous constraints to be fulfilled. In a recent proof of concept for an automatic job scheduling system in railway maintenance (Durazo-Cardenas et al. 2018), the problem in the network context is considered as a complex and data-rich problem as it involves a large number of components and maintenance jobs with complex interactions, several cost structures, and huge economic impacts.

In order to reduce the impact of track possessions on regular train operation, infrastructure managers often attempt to cluster maintenance and renewal works (Su et al. 2017). However, whether clustering is beneficial depends on several factors such as the importance of a track for train services and the network-wide consequences of track unavailability. This also includes the technological possibility of combining work for different infrastructure components and the economy of scale effect gained through the combination of work for the same type of infrastructure component. Although previous research could already show that grouping, or clustering, of activities can result in cost savings for possession and maintenance work on single tracks (Budai et al. 2006; Pargar et al. 2017), the benefits of clustering on the network level have not been investigated. In addition, if clustering is considered, it is assumed that activities can be either fully combined or mutually exclusive (Peng et al. 2011). The extent to which different types of infrastructure components can possibly be renewed in the same possession and the extent to which economies of scale can be realized through clustering of activities for the same type of infrastructure component have not been addressed.

In this paper, we study the renewal scheduling of multiple railway infrastructure components on the network level. We advance previous research by discussing the joint-renewal of a similar type and different types of components. Two economy of scale mechanisms that practically apply for a similar type of rail infrastructure components are presented, and the joint-renewal possibility for
combining different types of components is modeled. We also consider the case where there is a limitation on the possession time on each location of the railway network. In addition to the renewal cost, the unavailability cost is estimated as an economic representation of the time when certain track links are not available for train services. The railway infrastructure scheduling problem in the network context is formulated as a non-linear optimization model, and a solution method based on a triple-prioritization rule with a nested linear programming model for maximizing the total renewal time for different types of activity is proposed. It is noted that the current study complements a previous study by the same authors (Dao et al. 2018). While the previous study investigates the railway maintenance scheduling problem for a single track line, the current paper investigates the problem at a network level. The two problems are not similar in terms of complexity and in this paper additional constraints and further exploration on the joint renewal of components are discussed; the details of problem modeling and solution approach are also different. The model and solution method are applied to a real-life case concerning the renewal of track components (rails, ballast, sleepers), switches, and level crossings in the region of Northern Netherlands. Our study shows that up to 13% of total costs can be saved using the proposed method compared to the current practice at the railway agency in the Netherlands.

The remaining part of this paper is organized as follows. The general description of the railway infrastructure renewal scheduling problem is provided in the next section. Possible economy of scale mechanisms when renewing several similar-type components and the possibility of joint renewing several types of components are also discussed. Then, in the section of Model formulation, we present a formulation of the renewal scheduling problem in the network context to minimize the total renewal and unavailability cost. An algorithm to best allocate the time for different types of components in a possession and to obtain a solution for the problem is presented in the Solution approach. The Case study illustrates the benefits of the method by applying it to the case of track, switch, and level crossing renewals in a regional railway network in Northern Netherlands. The final section provides conclusions of this research.

RAILWAY INFRASTRUCTURE RENEWAL SCHEDULING IN NETWORK CONTEXT
Problem descriptions

Unlike the problem for a single railway track line, the rail infrastructure renewal scheduling for a network includes various locations where renewal works are needed. A network location can represent a railway line or a railway station and at each location, there can be several infrastructure components of different types and multiple components of the same type that need to be renewed in a finite planning horizon. Since the planning horizon at railway agencies is typically shorter than the lifetime of rail infrastructure components, we assume that each component is only renewed once within a planning period. The focus of this paper is on the railway infrastructure renewal scheduling problem and its complexity in the network context. The track deterioration process is out of the scope of this paper. Instead, it is assumed that renewal activities and their due-dates are given input data. The renewal due-dates can be the outcome of life expectancy estimations or track degradation prediction models of railways infrastructure assets. The renewal of each type of infrastructure component also comes with individual cost and duration. Information such as network topology, components locations, due-date, and individual cost and time are generally provided.

In this study, we consider economy of scale effects in terms of both cost and duration for renewing multiple components of the same type in one possession. This combination can reduce the average renewal cost and duration per component. The same holds, in principle, for the renewal of infrastructure components of different types in one possession. However, clustering of activities for different types of components can be restricted due to technological reasons. The details on the clustering of several renewal activities are discussed in the following sub-sections. A renewal activity may affect the availability of its associated location for regular train operation, i.e. a renewal stops trains from operation. The renewal of a component may have an impact on the availability of single or multiple track lines in the railway network. Depending on the impact, there is an unavailability cost per location per unit of time when the line is not available for train services.

Another distinct feature of the model in this paper is the network constraint and the available possession time constraint. The network constraint refers to situations where renewal activities in a track line prevent components in another line from being renewed, in order to (partly) ensure
train services in the network. The possession time constraint reflects the situation that there is a
restriction on the available possession time at a location due to train operation capacity requirement,
that is, the total renewal time in a possession must be less than a specified threshold and the number
of possessions in a year is limited.

The aim of the railway infrastructure renewal scheduling problem on the network level is to
determine at which time in a planning horizon each renewal of an infrastructure component should
be performed and to estimate the total renewal and unavailability costs that are associated with the
implementation of the schedule. Inputs of the railway infrastructure renewal scheduling problem
include:

- Railway network topology
- Components to be renewed and due-date for renewing
- Locations and renewal impact on availability
- Individual renewal cost and time
- Economy of scale and possibility of joint renewal
- Available possession time
- Unavailability cost of each location

In this paper, the renewal of several rail infrastructure components is investigated: track com-
ponents (rails, ballast, sleepers), switches, and level crossings. Depending on the renewal char-
acteristics, they can be classified into two groups. In the first group, the renewal is measured
by an integer number of components to be renewed and include switches and level crossings. In
the second group, the renewal is measured by the length in meters of the track segment to be
renewed. Specifically, the renewal of components such as rails, ballast, sleepers, and components
of the fastening system are all measured by length. In the proposed model, these components are
combined in the same group of track component, which implies that if there are more than one
type of components in the same segment, e.g. if rails and ballast are renewed, they are presumably
grouped. This assumption is reasonable since the combination of components in the same segment
increases efficiency due to shared setup time and renewal machinery (Caetano and Teixeira 2016).

By grouping these components, the modeling of the renewal cost and time for track component is simpler, still technically correct, and practically relevant.

**Economy of scale in rail infrastructure renewal**

Economy of scale in rail infrastructure renewal reflects the fact that the average time and cost per unit decrease as the size of renewal work increases. It occurs when renewal activities of the same-type components are performed. In this section, we present two saving mechanisms based on the number of components and the duration/length of the segment to be renewed, respectively.

The first economy of scale mechanism measures the economical advantage by the number of components to be renewed. Fig. 1 presents examples of the economy of scale factors for cost and time when renewing multiple switches at the same location.

*Fig. 1. Economy of scale in switch renewal*

Let $c_0$ be the cost of renewing a switch individually, the economy of scale factor in cost $f_c(n_s)$ when renewing switches together is the coefficient to estimate the average renewal cost of a switch, $c_s$, as shown in Equation (1).

$$c_s = f_c(n_s) \times c_0 \quad (1)$$

Similarly, the average time for renewing a switch, $t_s$, can be estimated by defining $t_0$ and $f_t(n_s)$ as the individual renewal time and the time economy of scale factor respectively (see Equation 2).

$$t_s = f_t(n_s) \times t_0 \quad (2)$$

In these representations, $f_c(.) = f_t(.) = 1$ when there is only one component, i.e. $n_s = 1$. These factors decrease and approach stable values as the number of components reaches a certain maximum. As seen in Figure 1, the average renewal cost (time) per component is a discrete function as we can only renew an integer number of switches.

The second economy of scale mechanism specifically applies to track components and is
represented by how fast the renewal is conducted. When the renewal time is long enough, e.g., longer than 8 hrs, or the length of the required track section is greater than a certain threshold, a renewal train can be used for track renewal. The performance of the renewal train is higher as the renewal time is longer. The following formula can be used to represent the renewal speed, \( v \), of track components:

\[
v = v_0 - a \tau^{-b},
\]

(3)

where \( a \) and \( b \) are positive coefficients; \( v_0 \) is the limit renewal speed (meters per hour) of the renewal train; and \( \tau \) is the renewal time (hours). The renewal length, \( s \), in meters can be determined if the renewal time is known.

\[
s = v \tau = v_0 \tau - a \tau^{1-b}
\]

(4)

The renewal train can be used for any length of a track section that is greater than its usage limit. Thus, the relationship between renewal time and average renewal speed is a continuous function. Fig. 2 shows a possible track renewal speed depending on the available renewal time.

Fig. 2. Economy of scale in track renewal

In this figure, the largest improvement in track renewal speed occurs when the renewal time is between 10 and 60 hours. The renewal speed still increases beyond 60 hours, but at a slower rate. When the available time for a possession or the track length is too short, e.g. less than 8 hours (see Figure 2), the use of a renewal train is not desirable and track renewal is performed manually with no significant economy of scale.

In addition to the track renewal speed, renewal costs can be reduced if the renewal train is used for a longer duration. Equation (5) shows an example of a step function representing the relationship between the nominal track renewal cost per meter \( c_f \) and the renewal time depending on renewal time being less or greater than a threshold \( \tau_0 \).
\[ c_t(\tau) = \begin{cases} 
    c_t & \text{if } \tau < \tau_0 \\
    e(\tau)c_t & \text{otherwise}
\end{cases} \]  \hspace{1cm} (5)

where \( e(\tau) \) is the cost efficiency factor that is usually a positive value less than 1. This efficiency factor may vary and be smaller when the renewal time/length increases.

**Joint renewal of different types of components**

Several renewal activities can be performed in a long possession and it is possible to schedule different types of components in the same possession. Generally, the total cost of renewing several types of components is a summation of the costs of renewing each type. However, their renewals can be, to some extent, done at the same time and therefore the total renewal time will be less than the sum of individual renewal times. In this paper, we use a probability \( p_{ij} \) to represent the joint renewal possibility of component \( i \) and component \( j \). For several components, the joint renewal probabilities can be combined in a table (Table 1).

**Table 1. Joint renewal probability for different types of component**

Each probability \( p_{ij} \) in Table 1 represents the overlap percentage between two types of components with respect to the duration of the shorter renewal activity. It is obvious that \( p_{ij} = p_{ji}, 0 \leq p_{ij} \leq 1 \), and the diagonal elements are the joint-renewal of the same-type components as presented in the previous section. When \( p_{ij} = 0 \), no overlap between two types of activities is possible and when \( p_{ij} = 1 \), the two types of activities can be fully executed in parallel. If the number of activities is known, this table and the data on individual renewal time of each component can be used to calculate the total renewal time.

Figure 3 shows an example of combining renewal activities of 3 types of components with probabilities: \( p_{12} = 0.75 \) and \( p_{23} = 0.25 \).

**Fig. 3. Example of activities combination**

In Figure 3, the longest activity (component 1) is put on top. Renewal time of component 2 is 8 hours, of which 6 hours (75%) is the overlap with component 1. For component 3 the renewal time is 4 hours, of which 1 hour (25%) is the overlap with component 2. The total renewal time is
16 + 2 + 3 = 21 hours.

MODEL FORMULATION

In this section, the renewal scheduling problem for multiple components in a railway network is modeled as an optimization problem. Assume that we need to schedule renewal activities for \( N \) components at \( L \) locations in a discrete and finite planning horizon from period \( t = 1 \) to \( T \).

In the network, exact identification of a component can be determined by a set of three indexes, including location \( l \), type \( k \), and an ordinal index number \( i \). There are \( K \) types of components to be renewed, of which \( K - 1 \) types of component renewal can be measured by the number of components and one type of component renewal is measured by the length of track segment. Without losing the generality, we can assume that component types 1 to \( K - 1 \) are measured by the number of components to be renewed and component type \( K \) is measured by the length of the track segment to be renewed.

Renewal cost and time

For component types 1 to \( K - 1 \), let's define \( x_{i,k,l,t} \) as a binary variable representing whether component \( i \), type \( k \), at location \( l \) is renewed in period \( t \) or not. For component type \( K \), let \( s_{i,K,l,t} \) be a non-negative real variable representing a length of segment of component \( i \), type \( k \), at location \( l \) to be renewed in period \( t \). The total renewal cost of all components in the network can be calculated using Equation (6).

\[
C_R = \sum_{t=1}^{T} \sum_{l=1}^{L} \sum_{k=1}^{K-1} \sum_{i=1}^{N_{k,l}} f^k_e(x_{i,k,l,t})c_{i,k,l}x_{i,k,l,t} + \sum_{t=1}^{T} \sum_{l=1}^{L} \sum_{i=1}^{N_{K,l}} c_{i,K,l}e(s_{i,K,l,t})s_{i,K,l,t} 
\]

(6)

In Equation (6), \( c_{i,k,l} \) is the cost of renewing a unit of component \( i \), type \( k \), at location \( l \); \( c_{i,k,l} \) represents the individual cost for component type \( k \), \( k = 1, 2, ..., K - 1 \), or the unit cost per meter for component type \( K \); \( N_{k,l} \) is the number of component type \( k \), \( k = 1, 2, ..., K \), at location \( l \). The first summation in (6) is the total renewal cost of component types 1 to \( K - 1 \) and the second summation represent the total renewal cost of component type \( K \). The economy of scale for both groups of components is taken into account in this equation. The economy of scale factor \( f^k_e(\cdot), k = 1, 2, ..., K - 1 \)
and the cost efficiency \( e(.) \) are both functions of decision variables.

Similar to the renewal cost, let \( t_{i,k,l} \) be the time of renewing a unit of component \( i \), type \( k \), at location \( l \). The total renewal time of all components type \( k \), at location \( l \) in period \( t \) is shown in Equations (7).

\[
T_{k,l,t} = \begin{cases} 
  f_t^k(x_{i,k,l,t}) t_{i,k,l,t} \sum_{i=1}^{N_{k,l}} x_{i,k,l,t} & \text{for } k = 1, 2, \ldots, K - 1 \\
  \sum_{i=1}^{N_{k,l}} s_{i,k,l,t} \sum_{v_{i,k,l,t}} & \text{for } k = K
\end{cases}
\] (7)

When different types of components are renewed separately, the total renewal time of all types of components at location \( l \) in period \( t \), \( T_{l,t} \), is the summation of all \( T_{k,l,t} \) for \( k = 1, 2, \ldots, K \), as shown in Equation (8).

\[
T_{l,t} = \sum_{k=1}^{K} T_{k,l,t} = \sum_{k=1}^{K-1} \sum_{i=1}^{N_{k,l}} f_t^k(x_{i,k,l,t}) t_{i,k,l,t} x_{i,k,l,t} + \sum_{i=1}^{N_{K,l}} \sum_{v_{i,K,l,t}} s_{i,K,l,t} \] (8)

When different types of activities are clustered, the total renewal time in each period is a function of \( T_{k,l,t} \) and the combination matrix \( P \). The total renewal time of all types of components at location \( l \) in period \( t \) is calculated as in Equation (9).

\[
T_{l,t} = \sum_{k=1}^{K} P_{g,k} \overset{\to}{\otimes} T_{i,k,l}
\] (9)

In this equation, we define an order multiplication operator, \( \overset{\to}{\otimes} \), between an element in vector \([T_{k,l,t}]\) and an element in \( P \). To implement this operator, we need to order the time vector to a non-ascending order and find the corresponding element \( P_{g,k} \), where the renewal of component type \( k \) begins subsequently to the start of renewing component type \( g \) (see an illustration in Figure 3). Further discussions and a procedure for calculating the total renewal time in each location for each period are presented in the section of Solution Approach.

**Unavailability cost**

When a possession is required at a location in the network, the railway system can still operate at a lower service level as passengers can either use a divert train (longer travel time) or choose other
modes of transportation. In any case, there is a loss due to the possession since paid passengers should be offered alternative transportation without any additional fee. In our model, this loss is valued by a given unavailability cost per unit time of the possession location in periods of high and low service demand $c^h_t$ and $c^u_t$. The two types of unavailability cost related to periods of high and low service demand are practical since the unavailability of train services during a weekend day cause less nuisance for customers than during a normal working day. The unavailability cost per location per unit time includes all the costs related to additional services required for customers and also the indirect cost such as a decrease in customer satisfaction and losses of future customers. Generally, the unavailability cost per unit time depends on location and the expected number of customers in the possession period. However, this paper does not focus on how to calculate the unavailability cost per unit time; readers can refer to (Dao et al. 2018) for a method to estimate this cost.

The total unavailability cost for all locations in the entire planning horizon can be estimated using Equation (10).

$$C_U = \sum_{l=1}^{T} \sum_{t=1}^{L} (c^u_t h^u_{l,t} + c^b_t h^b_{l,t})$$

(10)

where $h^u_{l,t}$ and $h^b_{l,t}$ are the possession times allocated in periods of low and high service demand respectively; $c^u_t$ and $c^b_t$ represent the unavailability cost per unit of time in periods of low and high service demand. The allocated possession times for two options of cost calculation can be evaluated using the total renewal time, $[T_{l,t}]$, with an assumption that the renewal activities are scheduled in periods of low service demand first. The following equations show the relationship between $h^u_{l,t}$, $h^b_{l,t}$ and $[T_{l,t}]$.

$$h^u_{l,t} = \begin{cases} \left\lceil \frac{T_{l,t}}{H_u} \right\rceil & \text{if } T_{l,t} < d_{u,t} H_u \\ d_{u,t} & \text{otherwise} \end{cases}$$

(11)
In Equations (11) and (12), \( d_{u,t} \) is the maximum number of time units for periods of low service demand in period \( t \), e.g. number of days of low service demand is 2 weekend days; \( H_u \) is the number of hours in a time unit for a low service demand period that can be used for renewal activities. Equation (12) implies that the possession time allocated in periods of high service demand is 0 when the total (required) possession time is less than the maximum time of low service demand. The two Equations (11) and (12) are designed for the renewal activities to be scheduled in the period of low service demand (lower unavailability costs) first before utilizing the period of high service demand (higher unavailability costs).

In the proposed model, the service demand, possession location, unavailability cost per unit time, and total possession hours have been considered in the calculation of total unavailability cost. In addition, this cost is aggregated for all locations in the network in the entire planning horizon. Thus, the loss of capacity at the network level if a number of tracks is not available for train operation has been taken into consideration.

**Constraints in rail infrastructure renewal**

We distinguish between three major types of constraints for the rail infrastructure renewal scheduling on the network level: (1) the due-date of a component, (2) the available possession time at a location, and (3) the restriction when components at multiple locations in the network are renewed.

- **Type 1 due-date constraint**: This constraint ensures that the renewal of a component is done on or before its latest possible date.
- **Type 2 - available possession time constraint**: In each period, the available time to occupy a location for renewal work is limited and the total time of scheduled activities for a location may not exceed this time limitation of a possession. This also includes a limitation of the
number of possessions in a year at each location.

- Type 3 - network constraint: Some locations cannot be possessed at the same time if that causes a severe interruption of train services or isolates a part of the network from the train service. For example, if a location is on a divert route of another location, renewal activities cannot be performed at both locations at the same time.

The detailed formulations of these constraints are presented in the next subsection

**Optimization model**

The following model can be formulated for the proposed renewal scheduling problem in a railway network.

**Model 1:**

\[
\begin{align*}
\text{Min } C &= \sum_{t=1}^{T} \sum_{l=1}^{L} \sum_{k=1}^{K-1} \sum_{i=1}^{N_{k,l}} x_{i,k,l,t} c_{i,k,l,t} + \sum_{t=1}^{T} \sum_{l=1}^{L} \sum_{i=1}^{N_{k,l}} \sum_{c_{i,k,l,t} \in (S_{i,k,l,t})} s_{i,k,l,t} \\
&+ \sum_{t=1}^{T} \sum_{l=1}^{L} (c_{i}^{u} h_{i,l}^{u} + c_{i}^{b} h_{i,l}^{b})
\end{align*}
\]

Subject to:

\[
\begin{align*}
\sum_{t=1}^{T_{i,l}} x_{i,k,l,t} &= 1, \forall i, l; \forall k = 1, 2, \ldots, K - 1 \\
\sum_{t=1}^{T} s_{i,k,l,t} &= S_{i,k,l}, \forall i, l \tag{14}
\end{align*}
\]

\[
T_{i,l} \leq T_{i,l}^{0}, \forall l, t \tag{16}
\]

\[
\sum_{t=1+52(y-1)}^{52y} \delta_{l,t} \leq NP_{l}, \forall l, \forall y = 1, 2, \ldots, Y_{\text{max}} \tag{17}
\]

\[
\delta_{l,t} + \delta_{l,t} \leq 1, \forall t, \forall l_{1} \in \overline{C}(l_{2}); \forall l_{2} \in \overline{C}(l_{1}) \tag{18}
\]

\[
\delta_{l,t} = \begin{cases} 
1 & \text{if } \sum_{k=1}^{K-1} \sum_{i=1}^{N_{k,l}} x_{i,k,l,t} + \sum_{l=1}^{N_{k,l}} s_{i,k,l,t} > 0 \\
0 & \text{if } \sum_{k=1}^{K-1} \sum_{i=1}^{N_{k,l}} x_{i,k,l,t} + \sum_{l=1}^{N_{k,l}} s_{i,k,l,t} \leq 0
\end{cases}, \forall l, t \tag{19}
\]
The objective of the renewal optimization is to minimize the total renewal cost and unavailability cost, which are explained at the beginning of this section. The first two sets of constraint guarantee that the renewal of a component has to be performed prior to its due-date, \( \tau_{i,k,l} \), and this type of constraint is separately modeled for the two introduced types of component that correspond to two types of economy of scale. In Equation (15), \( S_{i,K,l} \) is the total required renewal length of track component \( i \) at location \( l \). Constraint (16) implies that all renewal activities are executed within the available possession time in each period, \( T_{0,l,t} \). Constraint (17) limits the number of possessions at each location in a year \( y, y = 1, 2, \ldots, Y_{\text{max}} \), by a maximum number of possessions at location \( l, NP_{l} \), where \( Y_{\text{max}} \) is the maximum year in the planning horizon. The network constraint (18) ensures that renewal activities cannot be performed at two locations \( l_1 \) and \( l_2 \) if they belong to a set of locations, \( C(,.) \), that cannot be combined with each other. Constraint (19) defines a zero-one indicator variable \( \delta_{l,t} \) for the two previous constraints. This variable takes the value of 1 if a possession is needed, i.e. at least one renewal activity is scheduled, at location \( l \) in period \( t \). The last two variable constraints state that \( x_{i,k,l,t} \) is a binary variable for the first \( K - 1 \) types of components, and the renewal length of the component type \( K \) must be a non-negative value.

SOLUTION APPROACH

The renewal scheduling problem described above is usually applied as a large-scale optimization problem characterized by multiple locations, multiple types of components at the same location, and multiple components of each type at each location. It is a non-linear optimization problem with both renewal and unavailability costs being non-linear functions. In this section, we propose a solution method using a triple- prioritization rule and an optimal mechanism of allocating renewal time for several types of components within a possession.
Prioritization rule

The idea of introducing a prioritization rule is to identify the location, type, and component to schedule first. To describe the prioritization rule, we introduce three definitions as follows.

- **Critical location**: the location demanding most of the renewal activities compared to other locations in the network. In the scheduling process, the critical location should be given a priority if requests on several locations have to be fulfilled since there is limited time for performing the activities. In this paper, the location with a total renewal time of \( \max_l \{ \sum_{t=1}^{T} T_{it} \} \) is seen as the most critical location.

- **Critical type of component**: the type of component that requires the most renewal work compared to other types of component. At a location, the critical type of component is an important criterion to allocate the possession time. The most critical type of component is the type with a total renewal time of \( \max_k \{ \sum_{t=1}^{T} T_{k,lt} \} \) and it should be given a priority in possession time allocation. Further discussion on how to allocate time for each type of component can be found in the next subsection on the optimal allocation of renewal time for different types of components.

- **Critical component**: the component of the same type that is required to schedule first at a certain location. From several components of the same type, the most critical one can be defined as the component with the earliest due-date, i.e. \( \min_i \{ \tau_{i,k,l} \} \).

A 3-step prioritization rule is generated by identifying the criticality of location, type of component, and component consecutively and schedule the renewal activities based on the identified criticality. Three types of constraint are considered in the prioritization rule. The network constraint is addressed in the critical location identification. The available possession time constraint is dealt with in the possession time allocation for each type of component and the due-date constraint is considered while identifying the critical component. This triple-prioritization rule is integrated into an iterative algorithm to find a solution for the renewal scheduling problem. Further details on the iterative algorithm are presented in Figure 6.
Optimal allocation of renewal time for different types of components

For scheduling the renewal of multiple types of components at the same location, we need an approach of allocating the available possession time to the renewal of the different components if their renewal can be done in parallel to a certain extent. In this section, we focus on the allocation of time for each type of component at a location given a total possession time and the criticality of the component type. This sub-problem is called the time allocation problem.

In the time allocation problem, we have to find the renewal time for \( n \) types of components with \( X_1 \geq X_2 \geq ... \geq X_n \) in a total possession time of \( T^0 \) as shown in Figure 4. The allocation should fully utilize the available possession time for the total renewal time of all types of components.

**Fig. 4. Time allocation for different types of components**

The best allocation of time can be modelled as an optimization problem as in Model 2.

**Model 2:**

\[
\text{Max } \sum_{k=1}^{n} X_k
\]

Subject to:

\[
X_1 + \sum_{k=2}^{n} (1 - p_{k-1,k})X_k = T^0
\]

\[
X_1 \geq X_2 \geq ... \geq X_n
\]

In this optimization model, the objective of function (22) is to maximize the total allocated renewal time for all types of components. Constraint (23) shows the relationship between \( X_k, k = 1, 2, ..., n \) that can be developed from Figure 4. Constraint (24) indicates that the types of component are ordered using the type of component criticality as described in the second prioritization rule in this section. This is a linear programming (LP) optimization model and a solution can always be found using an LP solver package.

**Example:** Assume two types of components with renewal times of \( X_1 \) and \( X_2 \) hours, \( X_1 \geq X_2 \) and a combination percentage \( p = 0.75 \), the maximal possession time is 52 hours (see Figure 5).
The allocation of time problem can be modeled as in the following LP:

\[
\begin{align*}
\text{Max } & \quad X_1 + X_2 \\
\text{Subject to:} & \\
X_1 + 0.25X_k &= 52 \\
X_1 &\geq X_2
\end{align*}
\]  

The solution of this problem is \( X_1 = X_2 = T/(2 - p) = 41.6 \) hours.

This result indicates that if a possession of 52 hrs is available for 2 types of components, we can assign 41.6 hours for the renewal of each type. It is noted that we can only renew an integer number of components in the first \( K - 1 \) types of components (their renewal is measured by the number of components). Thus, the time for renewing each type of component may be a value near this ideal number, i.e. the more critical type would be allocated more time. In the scheduling practice, if there are components type \( K \) (their renewal is measured by the length of a segment), we will calculate the time for renewing an integer number of components first and the time for renewing type \( K \) components is calculated later using the relationship in (23).

**Renewal scheduling algorithm**

In this section, we will present an algorithm to schedule renewal activities in a railway network using the prioritization rule and the sub-optimization problem in Model 2. A brief diagram illustrating the algorithm is shown in Figure 6.

**Fig. 6. Procedure of renewal scheduling in network context**

The procedure is a closed loop starting with finding the most critical location for scheduling (first prioritization). The most critical location, i.e. the location with the maximum total expected renewal time of all types of components in the entire planning horizon, is selected for scheduling.
first. Then, the types of components, at the selected location, are ranked using their criticalities (second prioritization). At this step, the LP optimization (Model 2) is formulated with a specified $n$, $T^0$, and $p_{k-1,k}$. This model is then solved to find the optimal allocation time for renewing each type of component. In the next step, the renewal activities for each type of component at the selected location is scheduled using the following principles:

- At a location, the most critical type of component is scheduled first, and
- Within each type of component, the most critical component is scheduled first.

It should be noted that there is a loop when scheduling activities at the same location. When renewing type $k$ components, $k = 1, 2, K - 1$, we can only renew an integer number of components, and thus, the components are scheduled sequentially until the total renewal time is: i. A nearest value over the optimal allocation time if type $k$, $k = 1, 2, K - 1$, is the most critical type of component; or ii. A nearest value under the optimal allocation time if type $k$, $k = 1, 2, K - 1$, is not the most critical type of component.

The calculation of the allocation time for renewing the remaining types of components can be reformulated using a similar LP (Model 2) but with n-1 types of components and less available renewal time. This loop continues until there is only one type of component left with the remaining available renewal time.

After scheduling activities at the selected location, we need to update the scheduling time of the selected location as well as the following:

- The renewal cost of the scheduled activities using Equation (6);
- The unavailability cost of the current location using Equation (10);
- The remaining activities by removing the scheduled activities from the next scheduling step and re-estimate the expected renewal time of the remaining activities;
- The scheduling time for other related locations which have a network requirement with the selected location.
The algorithm finishes when all activities at all locations have been scheduled.

CASE STUDY

In this section, we present a case study with data of track components, switches, and level crossings in a regional railway network in Northern Netherlands (Figure 7). The data are provided by the railway agency responsible for this regional network. There are a total of more than 540 components and track segments located at 16 locations (10 track links and 6 stations) that need to be renewed within a planning horizon of 7 years from 2019 to 2025.

Fig. 7. Network topology in the region of Northern Netherlands (ProRail 2017)

The time unit \( t \) for scheduling activities is in week, i.e. we need to determine the week at which each component is to be renewed in the entire planning horizon. A summary of the total number of renewal activities and an estimation of total renewal time needed for each type of component are shown in Figure 8.

Fig. 8. Summary of the total renewal requirements

In Figure 8, the number of components is shown for switches and level crossings whereas the number of segments refers to tracks. The estimated hours are initial estimations by adding all individual renewal times of all components without taking the combination possibility into consideration. It can be seen that a massive amount of renewal work is required in the region, especially for track components with more than 280 track segments corresponding to over 200 kilometres of track to be renewed.

For three types of components under investigation, it is assumed that the economy of scale can be gained for switch and track renewals, but not for level crossing renewal. The possibility of joint renewal between each pair of activities is set to 0.75, which is a typical estimate at the Dutch railway agency practice for the considered types of components. The economy of scale factors for switch renewals are shown in Table 2.

Table 2. Economy of scale factors for switch renewal

In addition, the required renewals of track segments vary in length (size) and type. There are data on the individual renewal cost and time of each component/segment and the unavailability cost...
for each location, however, we only present the average cost and time data as in Table 3 because of a confidentiality reason.

Table 3. Other input data for the rail infrastructure renewal scheduling problem

Network and available possession time constraints

In this region, the network constraints apply when possessions at two locations in the same period cause a severe interruption of train services or make the network not accessible for train operation. First, possessions of any two out of the three lines Mp-Gn, Mp-Lw, and Lw-Gn are not allowed since that would isolate a part of the region. These lines also represent the divert routes for each other, e.g. a passenger can go from Mp to Lw by a direct train or by going from Mp to Gn, and then, to Lw. Therefore possessions of any two lines at the same time will cause some locations in the network unreachable. Second, for a joint station with multiple lines, possessions of two lines or more are not allowed since that may cause severe interruptions to the train service. For example, if there are renewal activities at two out of the four lines Gn-Mp, Gn-Lw, Gn-Zui, Gn-Swd at the same time, the transportation within the network would be severely interrupted around the Gn area and that is not allowed.

In this network, the limitations on the available possession time are given. The available possession time of a location in orange color (see Figure 7) is up to two weekend days per possession, four possessions a year, and the available possession time of a location in green color is up to a week, one possession a year. The maximum number of hours for a weekend possession is 52 hours and the maximum number of hours for a week possession is 168 hours.

Renewal and unavailability costs estimation

The renewal and unavailability costs for the entire Northern Netherlands network are estimated based on the renewal schedules generated by the proposed algorithm. In the proposed method, three different types of components are combined, and components of the same type are clustered together as presented in the Solution Approach section. Figure 9 shows the different cost elements of the proposed renewal schedule.

Fig. 9. Breakdown of total costs
For this network, approximately 61.64% of the total costs are dedicated to track renewal, followed by switch renewal (22.68%), track unavailability (9.84%), and level crossing renewal (only 5.84%). To evaluate the effectiveness of the proposed method, we compare it with two scheduling strategies applied at the railway agency.

- Strategy 1: Renewals of several components of the same type are scheduled sequentially in a possession without economy of scale considerations.
- Strategy 2: Renewals of several components of the same type are scheduled together to achieve economies of scale, but only one type of component is allowed per possession.

Although current renewal scheduling practice is often a mixture of strategy 1 and strategy 2, i.e. the scheduling method is moving from the individual renewal of each component towards combining several components of the same type in the one possession, we compare both strategies separately with our method to particularly reveal the cost advantages resulting from clustering. The cost comparison of the three strategies is shown in Figure 10.

Fig. 10. Cost comparison of three scheduling strategies

The results indicate that there are advantages in both renewal and unavailability costs when clustering several components of the same type and combining the renewal of different types of component in one possession. Strategy 1 is the least desirable strategy with the highest renewal costs for tracks and switches as well as the highest unavailability costs since no economic advantage through combination is utilized. Only the renewal costs for level crossings are identical in all three strategies since the economy of scale effects cannot be realized for this component. When renewing several components of the same type in one possession, but not combining different types of activities (strategy 2), the renewal costs drop for track components (5.5 million less) and switches (4.5 million less). The clustering of components also leads to lower unavailability costs compared to strategy 1 (8.1 million less). Further savings in unavailability costs are observed for the proposed method. Here, the unavailability costs are approximately 4.5 million less than for strategy 2 and 13.7 million less than for strategy 1. The renewal costs for the proposed method are slightly higher.
than for strategy 2 (0.2 million for tracks and 0.4 million for switches) which results from the
stronger economy of scale effect for clustering components of the same type. This effect is partly
lost when clustering also involves different types of component. However, the clustering now leads
to considerable savings in unavailability costs and makes the proposed method the most preferable
strategy when it comes to total costs. The total costs of the proposed strategy are approximately 4
million (2.57%) less than strategy 2 and 22 million (13%) less than strategy 1.

**Total unavailability time**

The unavailability time of a track line is the time during which the line is not available for train
service. The total unavailability time of track lines within a network can be seen as a measure of the
extent to which renewal activities are clustered. A scheduling strategy with less total unavailability
time than an alternative strategy indicates that more activities could be clustered. Here, we further
discuss the total unavailability time for the investigated case as presented in Figure 11.

*Fig. 11. Total unavailability time for three strategies*

It is apparent from Figure 11 that the proposed method results in the least total unavailability
time in the network, which also means more activities are clustered, compared to the other two
strategies. This also explains the unavailability cost savings and indicates a high utilization of
available possession time. In Figure 11, we also present the number of extra possessions required
to schedule all activities in the planning horizon. Under the available possession time constraints
for the regional network, the renewal schedule of the proposed method does not require any extra
possession, while the other two strategies need 10 and 32 extra possessions respectively to schedule
all activities as demanded in the entire planning horizon.

*Fig. 12. Unavailability time per location*

When breaking down the unavailability time to each location of the network, the critical
locations can be identified (Figure 12). In this region, Mp-Gn is the most critical location in
terms of unavailability, which implies that we need to pay more attention to this location when
planning renewal activities. Mp-Lw and Lw-Gn are the other two locations with relatively high
unavailability time. The high unavailability times can be explained by the high demand for renewal
activities for these locations. In addition, the proposed method yields the least unavailability time in all locations. The unavailability time difference of the three strategies increases with the required renewal activities at a location since the more activities need to be scheduled the more the clustering effect plays out.

**Sensitivity analysis of the combination possibility**

The combination possibility represents the overlap percentage between the renewals of different types of components in the same possession. It measures the degree of combination within the renewal plan that could affect both of the renewal and unavailability costs. In this section, for further understanding of the combination possibility impacts, a sensitivity analysis of the costs over combination possibilities is performed and the results are presented in Figure 13.

A general trend of decreasing in both renewal and unavailability costs are observed when the combination possibility increases. On average, the unavailability cost decreases approximately 0.75 million per 0.1 increments in combination possibility, which is slightly higher than the renewal cost decreasing rate of 0.66 million per 0.1 increments in combination possibility. This can be intuitively explained as the more the different types of components to be clustered together, the less total renewal time to be spent in the entire planning horizon. Besides, in the optimization model 2, if the combination possibility increases, the optimal solution of allocation time for each type of component increases. Therefore, more components are renewed in a possession, and that not only brings the total possession time down but also allows higher the economy of scale factors in cost and time. Consequently, both unavailability cost and renewal cost decrease when the combination possibility increases. The higher rate of decreasing unavailability cost emphasizes the impact of renewal activities on regular train operation in the network context in the presented case study.

**CONCLUSIONS**

The aged railway infrastructure stock in many countries requires from railway agencies large investments every year to keep the performance of the railway system at a desired working level. Scheduling the renewal of multiple railway components in a network is a challenging task because
of the large number of components in a network and several restrictions for executing renewal activities. In this paper, we have discussed the possibility of clustering several renewal activities for same types and different types of component in the network context. The renewal cost, unavailability cost, renewal time, and network constraints are formulated in a non-linear optimization model with an objective of minimizing the total cost incurred in a finite planning horizon. We propose a method which enables the clustering of renewal activities for components of the same type and optimizing the allocation of time for different types of components within a possession. The proposed method is applied to a regional railway network in Northern Netherlands for scheduling track, switch, and level crossing renewal in a 7-year planning horizon. Benefits in both total and unavailability costs, as well as shortened unavailability hours are observed in the results compared to the current practice at the Dutch railway agency.

From this research, a few future research directions are identified. First, this paper focuses on the scheduling of renewal activities, and it is worth to integrate and schedule other types of railway activities such as repetitive regular maintenance and new construction activities in the railway network to have an overall asset management plan. To do this, additional modeling techniques such as introducing new constraints and modified scheduling rules may be needed. Second, this paper does not consider component degradation models for component failure and renewal time prediction. Instead, renewal activities and due-dates are assumed to be known in advance, and thus, a model integrating the component degradation model into the maintenance scheduling problem in network context would be an essential future research direction. Last but not least, the formulated renewal scheduling problem in this paper is a non-linear optimization model characterized by a large number of variables. The proposed solution technique is based on a prioritization rule and optimization of renewal time which can stimulate clustering of renewal activities. In order to further improve the outcome of the model and solution method, new solution techniques such as evolutionary algorithms are recommended for future study.
**Acknowledgments**

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REFERENCES


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**TABLE 1.** Joint renewal probability for different types of component

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<thead>
<tr>
<th>Component type</th>
<th>1</th>
<th>2</th>
<th>...</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>−</td>
<td>$p_{12}$</td>
<td>...</td>
<td>$p_{1n}$</td>
</tr>
<tr>
<td>2</td>
<td>$p_{21}$</td>
<td>−</td>
<td>...</td>
<td>$p_{2n}$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>n</td>
<td>$p_{n1}$</td>
<td>$p_{n2}$</td>
<td>...</td>
<td>−</td>
</tr>
<tr>
<td>Number of switches renewed in a possession</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>------------------------------------------</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Cost factor</td>
<td>1</td>
<td>0.94</td>
<td>0.93</td>
<td>0.91</td>
</tr>
<tr>
<td>Time factor</td>
<td>1</td>
<td>0.75</td>
<td>0.72</td>
<td>0.65</td>
</tr>
</tbody>
</table>
### TABLE 3. Other input data for the rail infrastructure renewal scheduling problem

<table>
<thead>
<tr>
<th>Switch</th>
<th>Level crossing</th>
<th>Ave. renewal time per component (hrs)</th>
<th>Ave. renewal cost per component (€1,000)</th>
<th>Ave. unavailability cost per location (€1,000) per week day</th>
<th>Ave. unavailability cost per location (€1,000) per weekend day</th>
<th>Track renewal efficiency factor</th>
<th>Maximum track renewal speed (m/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>18.07</td>
<td>7.78</td>
<td>262.02</td>
<td>60.33</td>
<td>41.17</td>
<td>0.9</td>
</tr>
</tbody>
</table>

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Fig. 11. Total unavailability time for three strategies
Fig. 12. Unavailability time per location
Fig. 13. Sensitivity analysis of the combination possibility
Possession time, $T^\prime$ hrs

- Type 1, $X_1$ hrs
- Type 2, $X_2$ hrs
- ... 
- Type $n$, $X_n$ hrs

$(1-p_{12})X_2

(1-p_{n-1,n})X_n$
Schedule activities at the current available time using the component criticality

Update cost, remaining components, and current time schedule

All activities are scheduled?

Yes

End

Find and select the most critical location for scheduling

Rank the type of component & determine the optimal renewal time allocation

Start
Figure 7: Map with lines indicating "Up to 168 hrs of possession" and "Up to 52 hrs of possession."
Figure 10
Figure 11