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The Arrow Effect under Competitive R&D

Guido Cozzi*

*University of Macerata, guidocozzi@durham.ac.uk

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The Arrow Effect under Competitive R&D*

Guido Cozzi

Abstract

This paper shows that standard Schumpeterian theory does not imply that the incumbent monopolist has too little incentive to carry out R&D aimed at displacing its own product. If the patent holder is rational as is any other R&D investor, she will know that in equilibrium her patent’s obsolescence shall not be affected by her own R&D investment, because all the R&D firms operate under perfect competition and constant returns to scale at the private level. This reconciles Schumpeterian theory with the empirical evidence on innovation by incumbents. It is proved that the usual macroeconomic implications maintain their validity.

KEYWORDS: Arrow effect, basic Schumpeterian model, R&D and growth, innovation by incumbents

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1. Introduction

In virtually all Schumpeterian growth models with vertical innovation it is claimed that an outsider research and development (R&D) firm has a higher incentive to undertake quality improving R&D than the current patent holder. In fact, when considering R&D investment, the incumbent monopolist would internalize its monopoly right’s obsolescence and therefore would subtract its current value from the payoff of successful innovation\(^1\). As a result, the successful patent holder that temporarily monopolizes each product line will never be the same firm: the incumbent monopolists rest on their laurels in order to avoid to cannibalize themselves.

It is well known that such a drastic theoretical implication is at odd with real world evidence, in which firms tend to possess patent rights on products that rendered obsolete their previous products: this prediction is traditionally viewed as the main flaw of Schumpeterian growth theory (Aghion and Howitt 1992, Grossman and Helpman 1991, Segerstrom 1998, etc.), otherwise rich in important insights on the macroeconomics of growth, general equilibrium innovation, and the political economy of international competition. For example, Malerba and Orsenigo (1999), study the patents granted to firms in 1978-1991 in important countries, showing that the percentage of patents granted to firms that had already innovated within their sector was 70% in Germany, 60% in France, 57% in Great Britain, 39% in Italy, 68% in the USA, and 62% in Japan. According to the authors, the percentage of patents granted to firms that innovated for the first time in the sector was 15% in Germany, 24% in France, 24% in Great Britain, 42% in Italy, 18.4% in the USA, and 16% in Japan. The remaining fractions of patents granted in the period in each country accrued to the (often large) firms which had already innovated in other sectors (lateral diversification). Hence, the evidence points to a quite mixed scenario, in which the incumbents seem to dominate innovation in a seemingly irregular way. This and similar evidence seem strongly at odds with the clearcut implication - no R&D by incumbents - traditionally ascribed to the basic Schumpeterian theory.

Several important models - e.g. Barro and Sala–I-Martin (1995), Stein (1997), Segerstrom and Zolnierek (1999), Denicolo\(^2\) (2001), Etro\(^2\) (2004) and Segerstrom (2006) - have been introducing additional assumptions into the

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\(^2\)Which refers to a working paper version of this article (p. 298, footnote 22) and recognizes the validity of its result under constant returns to scale R&D technologies. Section 3 will show how my result continues to hold with decreasing returns to scale in the innovation.
Schumpeterian growth framework in order to predict positive equilibrium R&D by the incumbents. This line of research cast light on important industrial features (incumbent’s distribution channels, better R&D experience/ability by the previous patent holder, incumbent’s visibility and Stackelberg R&D leadership) that considerably extended Schumpeterian growth theory. Clearly, in order to reconcile the theoretical predictions with a seemingly patternless evidence, one would need to introduce a lot of heterogeneity in the assumed leaders’ cost advantages and visibility.

In this paper I claim that even the original Aghion and Howitt’s (1992) basic Schumpeterian economy was not at odd with the available empirical evidence because it did not predict that in a dynamic general equilibrium with perfectly competitive R&D sectors no innovation should be carried out by the incumbent. The reason is very simple and applies to all Schumpeterian equilibrium growth models whereby product quality, respectively unit production cost, undertake upward, respectively downward, jumps as a consequence of purposeful R&D by profit seeking firms in perfectly competitive R&D markets. If any outsider can run the R&D technology at the same efficiency level as the incumbent and if there are increasing marginal costs of R&D at the sectorial level but constant returns at the individual level, then a perfectly competitive incumbent is a price taker, not a quantity taker: outsiders’ R&D level is not known before the incumbent’s. Theoretically speaking, as each R&D firm size is indeterminate, the Walrasian auctioneer will select prices only for consistent individual participations to the unique amount of total R&D that clears the labor market. Hence given everybody’s optimal strategy, at the equilibrium prices the incumbent is indifferent on its own R&D investment.

According to my analysis, the basic Schumpeterian model laid down by Aghion and Howitt (1992) exactly replicates the most irregular evidence: if leaders are indifferent, we should expect for their participation to R&D all sorts of patterns. Paradoxically, as a consequence of the interpretation of the Arrow effect suggested by this paper, it is if leaders have research and distribution advantages that we should make more drastic predictions, perhaps conflicting with the data.

Our result can be readily extended to a multisector framework. Instead, it does not apply to an alternative framework such as Aghion et al (2001) in which, as in Reinganum (1983), R&D inputs are firm specific. In CRS cases where all R&D inputs are freely tradeable and there are industry-specific limitational factors, the potential entrants uniquely pin down the amount of R&D investment targeting any particular product line, thereby pegging the sectors and price (i.e. perfect) competition.

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incumbent’s obsolescence. The incumbent’s R&D choice then amounts to the choice of how many tickets to buy of a fair lottery whose odds are independent of its R&D decision.

2. The Arrow Effect and Basic Schumpeterian Growth

To help dissipate almost two decades of incorrect interpretation of the Arrow effect in general equilibrium models with perfectly competitive R&D, I think it is useful to the reader to put my argument in the simplest possible perspective. For this reason, the reader is invited to tolerate the following "prelude" that tries to cast a bridge between the elementary microeconomic theory of perfectly competitive industries and Schumpeterian R&D and growth theory.

2.1. Prelude: a Simple Perfectly Competitive Industry

Let us assume an industry having a perfectly competitive structure. Each firm, in continuous time, at any instant $\tau \geq 0$ can produce an amount $\lambda > 0$ of output flow by employing one unit of flow labor under constant returns to scale. There is free entry and exit. The market price of each unit of output is constant and denoted $p \equiv \frac{V_{t+1}}{A_{t+1}}$. Moreover, assume that, the unique input employed, labor, is offered - in amounts $n \geq 0$ - to this industry as an increasing function

$$n = S(\omega) = L - \bar{x}(\omega)$$

of wage $\omega > 0$, with $\bar{x}'(\omega) < 0$. This would generate the upward sloping labor supply curve depicted in Figure 1.

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3 For reasons to be clarified later.
Figure 1

Labor Market Equilibrium in a Perfectly Competitive Industry
We can invert the labor supply function by writing \( \omega = S^{(-1)}(n) \), with \( S^{(-1)\prime}(n) > 0 \) at all levels of \( n \). As shown in Figure 1, the equilibrium employment level will be equal the unique number, \( n_t \), such that the value of the marginal product of labor is equal to wage, that is:

\[
\lambda V_{t+1} = S^{(-1)}(n_t) \equiv \omega_t. \tag{2.1}
\]

Hence the unique industry-wide equilibrium employment level is equal to \( n_t = S(\lambda V_{t+1}) \equiv \bar{L}_{\lambda V_{t+1}}(\lambda V_{t+1}) \).

This of course leaves the firm size indeterminate, as well as the number of firms. Let \( N \) denote the equilibrium number of operating firms and \( n^i, i = 1, 2, ..., N \) their positive employment levels, with \( \sum_{i=1}^{N} n^i = n_t \). What would happen if one of these firms, say firm 1, decides to hire fewer workers? Would this lead to a decrease in aggregate equilibrium employment? Microeconomic theory tells us that it will not, otherwise the real wage would become lower than the value of the marginal product of labor. What would happen if one of the firm, say firm 1, decides to hire more workers? Would this lead to an increase in aggregate equilibrium employment? Microeconomic theory tells us that it will not - unless firm 1 hires more than \( n_t \) labor units - because otherwise the real wage would become higher than the value of the marginal product of labor.

Let us now assume that, for some reason, the owner of one and only one of the firms, say firm 1, dislikes this industry’s total employment: for example, assume that \( B(n) \) is the pecuniary equivalent of its disutility and that its expected profit can be written as \( \pi^1(n^1, n) = \left( \lambda \frac{V_{t+1}}{A_{t+1}} - \omega_t \right) n^1 - B(n) \), with \( B'(n) < 0 \) for all \( n \geq 0 \). Notice that \( n^1 \) is firm 1’s choice variable, whereas \( n \) is the aggregate employment level in this industry, capturing the negative externality of industry-wide employment on firm 1. In this environment, firm 1’s optimal quantity decision on \( n^1 \) is taken at the equilibrium prices of all outputs and inputs, that is knowing that \( \omega = \omega_t \) and therefore that \( n = n_t \), unless it decides to gain monopsonistic power, which happens if it decides to drive the other firms out of the market by employing \( n > n_t \). Let us remind the reader that \( n_t \) is a real number, not a variable, for all choices of \( n^1 \leq n_t \). Therefore the industry remains perfectly competitive as long as \( n \leq n_t \). In such a perfectly competitive industry, by its choice of \( n^1 \) firm 1 knows it would
not affect $n_t$ and thereby $B(n_t)$. Therefore we can write:

$$\frac{d}{dn^t} \pi^1(n^1, n) = \begin{cases} 
\frac{V_{t+1}}{A_{t+1}} - \omega_t - B'(n_t) \frac{dn_t}{dn^t} = \frac{\lambda V_{t+1}}{A_{t+1}} - S(-1)(n_t) = 0, & \text{if } n^1 \leq n_t \\
\frac{V_{t+1}}{A_{t+1}} - S(-1)(n^1) - S(-1)(n^1)n^1 - B'(n^1) < 0, & \text{if } n^1 > n_t.
\end{cases}$$

As a consequence:

Lemma 2.1. a. The unique perfectly competitive equilibrium will prevail with wage $\omega_t$ and employment level $n_t$ satisfying eq. (2.1).

b. Firm 1 gains zero profits in equilibrium, by optimally employing any amount of flow labor $n^1 \in [0, n_t]$.

Remark. Notice that this result is valid no matter how "large" firm 1 is in other markets - provided it holds no monopsonistic power on the same input - and regardless of how "large" firm 1 might wish to be in this industry. Also notice that $\omega_t$ is viewed by firm 1 as a constant as long as $n^1 \leq n_t$, just because $\omega_t = S(-1)(n_t)$, and $n = n_t$ for $n^1 \leq n_t$. This suggests that perceived does not allow firm 1 to internalize the negative externality $B(n)$ as well as, for any perfectly competing firm in this industry, the negative pecuniary externality $S(-1)(n)$.

Do we really need the constant returns to scale assumption for our externality irrelevance result to hold? What if instead we had deviations from constant returns, but they were entirely external to the firms? That would amount to assuming that the private productivity $\lambda$ depended on aggregate $n$. If $\lambda(n)$ were decreasing or relatively slightly increasing, the equilibrium industry employment level would still be given, implicitly, by

$$n_t = L - \bar{x} \left[ \lambda(n_t) \frac{V_{t+1}}{A_{t+1}} \right].$$

Therefore, decreasing returns to scale at the industry level or sufficiently mild increasing returns, provided their are external to the firm - a necessary requirement of perfect competition - would not alter our result.

2.2. Introducing Incumbent’s R&D

In the simplest version of the pioneering Aghion and Howitt’s (1992 and 1998) basic Schumpeterian model with drastic innovations, an infinite number of

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perfectly competitive firms produce, at each instant of a continuous and unbounded time horizon, a unique non-storable final consumer output flow, $y_t$, according to the following production function:

$$y_t = A_t x_t^\alpha M^{1-\alpha}$$

where $x_t$ is the flow of generation $t = 0, 1, 2, ...$ intermediate product, which is sold monopolistically by the holder of its patent, who produces it on a one-to-one basis from skilled labor, whose inelastically supplied aggregate amount is equal to $L$. Moreover, $M$ denotes manufacturing specific unskilled labor, whose aggregate amount is normalized to 1. Index $A_t$ is the productivity level associated to the $t$-th intermediate good, and $0 < \alpha < 1$.

Perfectly competitive R&D firms hire skilled labor flow and produce a probability intensity of inventing the $t+1$st generation of the intermediate good, with associated productivity parameter $A_{t+1} = \gamma A_t$, with $\gamma > 1$. The assumed constant returns to scale (CRS) R&D technology transforms one flow unit of research labor into $\lambda > 0$ flow probability of innovation. As soon as a new good is invented it gets immediately patented and produced by the unique patent holder, who then enjoys unconstrained monopoly profits; in fact the quality jump $\gamma$ is here assumed high enough to render inconvenient for the previous patent holder to produce when the top quality patent holder sets profit maximizing prices$^4$.

Standard arguments prove that the intermediate good monopolist’s profit flows are $\pi_t = A_t \pi (w_t/A_t)$, where $w_t$ is the wage rate of the skilled labor and $\pi (\cdot)$ gives productivity-adjusted profit, $\pi_t/A_t$, as a decreasing function of productivity-adjusted wage $w_t/A_t \equiv \omega_t$. Optimal intermediate output is $x_t = \bar{\omega}(\omega_t)$, with $\bar{\omega}' < 0$. This implies that the skilled labor market clearing condition is:

$$L = \bar{x}(\omega_t) + n_t, \quad \text{(L)}$$

where $n_t$ is the mass of labor employed in the R&D and $L$ is the total supply of skilled labor.

The financial arbitrage between consumer loans and firm bonds and equities, viewed as perfect substitutes implies that the annuity value of the firm stock, $V_t$, in case of liquidation be equal to the dividend per unit time plus

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$^4$This is a case of "drastic innovation". For low enough quality jumps innovation is "non-drastic". It is easy to prove that our results are valid also in such a case.

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expected capital gains or losses\(^5\) per unit time, that is
\[
rV_t = A_t \pi (\omega_t) - \lambda n_t V_t,
\]
that is:
\[
V_t = \frac{A_t \pi (\omega_t)}{r + \lambda n_t}.
\]

When an R&D firm hires a flow labor unit its output will be a flow probability \(\lambda > 0\) of discovering the \(t + 1\)st patent, which is worth
\[
V_{t+1} = \frac{A_{t+1} \pi (\omega_{t+1})}{r + \lambda n_{t+1}}
\]

This framework fits the previous subsection’s perfectly competitive industry, after interpreting labor average and marginal product of labor, \(\lambda\), as the usual output flow in patent race models: the probability per unit time of inventing the relevant patentable idea (an immaterial good). The market value of each unit of that probability flow is exactly \(V_{t+1}\), that is the present expected value of the monopolistic rents generated by the intellectual property of the \(t + 1\)st intermediate good. Moreover, firm 1 is the incumbent monopolist - i.e. the owner of the patent on intermediate good \(t\) - and \(B(n)\) is the expected loss of value of its current patent in productivity adjusted terms, that is \(\lambda n \frac{V_t}{A_{t+1}}\).

The previous subsection’s analysis applies and therefore we can say that perfect competition in the research sector yields Aghion and Howitt’s (1992) and (1998, ch. 2) well known R&D arbitrage condition:
\[
\omega_t = \lambda \frac{V_{t+1}}{A_{t+1}} = \frac{\lambda \gamma \pi (\omega_{t+1})}{r + \lambda n_t}, \tag{A}
\]
where \(r > 0\) is the real rate of time preference in the linear instantaneous utility functional of the consumer.

The labor market clearing condition and the R&D arbitrage condition allow to completely describe the dynamics of this economy.

The R&D arbitrage condition shall hold because each skilled labor unit - owned by each zero measure individual - has the option of working in the manufacturing sector or of self-employing herself in the R&D sector. In fact, the constant returns to scale R&D technology mapping flow labor into the flow

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\(^5\)In this model with infinite intertemporal elasticity of substitution it can be proved that there cannot exist equilibria with non-zero deterministic capital appreciation or depreciation. This is why only obsolescence is considered.
probabilities of the development of the next generation of the intermediate good in proportion to $\lambda > 0$ is available to everybody. Hence there is no way for any firm to prevent the R&D workers from appropriating the full value of their marginal products.

Given productivity-adjusted real wage $\omega_t > \bar{x}^{-1}(L)^{\delta}$, the amount of skilled labor will be inelastically supplied to the R&D sector.

The R&D labor demand side is guided by an indeterminate number of potential employers: since R&D technology can be operated at zero entrepreneurial cost every agent - consumer, worker, non-human organization unit - can run a "firm", raise funds, and hire the available skilled workers. Hence each such firm must earn zero profits.

The perfect competition assumption rules out the possibility of any agreement between market participants - be they R&D firms, skilled workers, or financial intermediaries - to affect aggregate employment and wage. Moreover, there exist no firm-specific factors: all relevant factors are freely tradable within the industry. If non-tradable firm-specific factors were assumed, our results would change, as in Aghion et al.'s (2001) investigation of the effects of large firms on growth.

What would happen in this framework if the current monopolist hired some R&D workers? In particular what would happen if a positive mass $n^M_t$ no larger than $n_t$ of skilled workers were hired by the intermediate good firm that monopolizes the production of the leading-edge intermediate good? This paper claims that equations (L) and (A) would simply have to be rewritten as

$$L = \bar{x}(\omega_t) + n^O_t + n^M_t,$$

and

$$\omega_t = \frac{\lambda \gamma \pi (\omega_{t+1})}{r + \lambda (n^O_{t+1} + n^M_{t+1})},$$

where $n^O_t$ denotes the mass of skilled workers hired by the outsider R&D firms.

It is important to remark the aspects of the basic Schumpeterian model that are crucial to my argument. The research sector and the capital markets are perfectly competitive: therefore the current intermediate good monopolist has no market power in the R&D labor and financial markets. This implies that neither the intermediate monopolist firm nor any of its shareholders perceive any ability to affect aggregate R&D. Consequently the current leading-edge product obsolescence is taken as given and as independent of any individual

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$^{\delta}$To get non-trivial results we will always assume this to hold.
agent’s action. The fact that the intermediate good producer - or the final goods producers depending on the particular model of this class - has market power in its own market may have led the growth literature to adopt the logic of patent race models with oligopolistic R&D markets (e.g. Reinganum 1983). However, when one distinguishes between markets in which firms have the power to influence aggregates - and hence prices - and markets in which they cannot, the situation becomes more similar to the deterministic framework of Gilbert and Newbury (1982).7

However, it is interesting to observe that all the macroeconomic and comparative statics implications of Aghion and Howitt (1992) remain valid: interestingly, the Arrow effect indirectly continue to affect the aggregate dynamics in the case of drastic innovations though not showing up at the industry level.

In a steady state with constant R&D employment, \( n_{t+1} = n_t = \hat{n} \), and productivity-adjusted wages, \( \omega_{t+1} = \omega_t = \hat{\omega} \). The equilibrium conditions become:

\[
L = \bar{x}(\hat{\omega}) + \hat{n}, \quad (L')
\]

and

\[
\hat{\omega} = \frac{\lambda \gamma \pi (\hat{\omega})}{r + \lambda \hat{n}}, \quad (A')
\]

Equations \( (L') \) and \( (A') \) uniquely determine the steady state value of all the endogenous variables. The steady state pair \((\hat{\omega}, \hat{n})\) can be reached immediately in the economy of Aghion and Howitt (1992), and gives a stochastic growth process with expected growth rate \( \lambda \hat{n} \log \gamma \).

By allowing the incumbent to undertake R&D, equations \( (L') \) and \( (A') \) change into:

\[
L = \bar{x}(\hat{\omega}) + n^O + n^M, \quad (L'')
\]

and

\[
\hat{\omega} = \frac{\lambda \gamma \pi (\hat{\omega})}{r + \lambda (n^O + n^M)}, \quad (A'')
\]

where steady state R&D employment by outsiders, \( n^O \), and (successive) incumbents, \( n^M \), are considered.

From \( (L'') \) and \( (A'') \) the unique steady state mass of skilled workers hired

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7 Obviously, not because innovation is non-stochastic, but because - unlike Reinganum’s (1982) industry - no R&D firm perceives it can profitably modify the aggregate flow probability of the next innovation.
by the competitive fringe of R&D firms would be \( n^O = \hat{n} - n^M \geq 0 \), with no change in the aggregate amount of R&D employment and therefore in the expected growth rate.

### 2.3. Perfect Competitive R&D and Arrow Effect

In this section - with no loss of generality - we will restrict the analysis to the steady state. In what follows we prove that under the competitive assumptions of Aghion and Howitt (1992) the unique steady state has to satisfy the previous equations \((L'')\) and \((A'')\), leaving the incumbent’s participation to R&D indetermined.

The most important question is the following: is the monopolist’s R&D employment \( n^M \in [0, \hat{n}] \) convenient at the equilibrium wage and prices? In a simultaneous game such as our competitive market with externality this depends on what all other market participants are doing. Let us remind that the incumbent’s stock market value, \( V_{t+1} \), is determined by the current expectations about the future R&D employment levels\(^8\). Since each of an infinite number of potential entrants has constant returns to scale their R&D labor demand correspondence is equal to zero for \( w_t > \omega A_t \), to infinity if the reverse inequality holds, and to any amount for \( w_t = \omega A_t \). Under perfect competition it is as if every firm’s R&D labor demand correspondence and worker’s labor supply correspondence were simultaneously communicated to the market maker - that is to the frictionless theoretical Walrasian auctioneer - who would then find a wage level for which there exist a distribution of quantity demanded and supplied belonging to the corresponding set values of the demand and supply correspondences, and that clear the market. Given its knowledge of the market clearing mechanism and of the strategies of all the other market participants, the monopolist’s best response would be to prescribe a labor demand equal to zero for \( w_t > \omega A_t \) and to any amount in \([0, \hat{n}]\) for \( w_t = \omega A_t \). The reason is that it recognizes that given the outsiders’ optimal strategies and the Walrasian market mechanism it will never be able to reduce the flow probability of its product \( t \) obsolescence for wage levels \( w_t \) lower than or equal to \( \omega A_t \).

Notice that for the competitive labor market to drive wages to full employment \((L')\) and the perfectly competitive capital markets to exhaust all profitable R&D investment opportunities until arbitrage condition \((A')\) is satisfied it is not necessary that the outsiders can observe in advance the action of

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\(^8\)Which will directly affect future obsolescence and indirectly affect future monopoly profits.
the monopolist: the Walrasian auctioneer shall know it and re-assign outsiders’
quantities.

In such a perfectly competitive market for R&D labor with an externality
on the intermediate good monopolist the total amount of R&D labor employed
is dictated by the unique equilibrium and at the corresponding wage level the
monopolist’s net expected revenue from investing $n^M \omega A_t$ is $n^M \lambda V_{t+1}$ for all
$n^M \in [0, \widehat{n}]$. Hence the monopolist is as indifferent as any R&D firm in its
optimal R&D investment: the identity of the R&D firms to which the real or
fictional market maker will eventually assign the R&D labor - thereby resolv-
ing the indeterminacy inherent in the constant returns framework - does not
alter aggregate obsolesce but only individual participation to the gains from
the innovative lottery. As in all competitive markets with constant returns to
scale, the size of operation by each firm is indeterminate: the market mak-
ers’ activity - stylized by the theoretical Walrasian auctioneer device - plays
exactly the role of assigning this scale arbitrarily without violating agents’ de-
mand/supply correspondences. Hence the incumbent cannot take other firms’
R&D quantities as given before an equilibrium is reached: it can only take
prices as given. Therefore:

**Proposition 2.2.** In the unique positive steady state of the basic Schump-
eterian model with drastic innovations equations (L") and (A") hold, and
the mass $n^M$ of R&D labor the incumbent intermediate good monopolist hires
in the unique steady state can be any number in $[0, \widehat{n}]$.

**Corollary 2.3.** Aghion and Howitt’s (1992) macroeconomic analysis holds
with no modification in its implications about the aggregate growth rate, the
aggregate R&D employment, wages, prices, and profits, and all other aggregate
variables. Moreover, it is robust to any evidence of incumbents’ undertaking
R&D up to $\widehat{n}$.

Notice that it is not even necessary that $n^M$ stays constant over time,
provided that it never exceed $\widehat{n}$.

Key to understanding our result is a correct interpretation of the tricky
aspects of the perfectly competitive R&D sector assumed in the basic Schump-
eterian economy. In fact, given a real wage level the total supply of R&D
labor is given at the sectoral level; when an R&D firm demands one more flow
unit of R&D labor at the equilibrium wage its demand will be satisfied if and
only if that unit of labor does not get an alternative research job within the
same sector. Otherwise the wage rate would rise and the firm would not de-
mand it any more. Hence at the equilibrium wage an individual firm’s hiring
one more R&D labor unit does not create an additional supply of R&D labor for the whole R&D sector: it simply forces a particular allocation of a given amount of aggregate R&D employment. But by not increasing aggregate R&D employment in that sector it cannot increase the obsolescence of the current incumbent. Therefore its ability to hire more of a given pool of R&D labor only increases its own chances of making the innovation and of appropriating its fruits: this is equivalent to buying one more ticket in a lottery with a predetermined success probability.

Redistributing R&D employment across firms is like redistributing the ownership of the success probabilities, but the sum of these probabilities is determined only by the aggregate supply of R&D labor at the equilibrium wage.

The common argument proving the Arrow effect typically says that the incumbent by buying a marginal unit of R&D labor increases its chances of winning the next patent race and of losing its current patent value. However, whenever there is only one level of aggregate R&D labor employment for each wage rate, though the incumbent’s marginal chances of being the winner really depend on its own marginal R&D employment decision, the probability of its current patent’s becoming obsolete only depends on the total amount of R&D labor supplied to the sector - and pinned down by the real wage.

2.4. Is Arrow Effect Anywhere?

It is useful to comment on the previous result and in particular on its relationship with the so called Arrow effect. Actually, we can say that the Arrow effect is still at work, but not in the sense usually intended by the literature. Of course, the common belief that the incumbent should do no research turns out to be incorrect: in light of the previous analysis of the basic Schumpeterian model, we should not be surprised to observe that in the real world the incumbent monopolists do a lot of R&D and hold sequential patents. However it is absolutely true that the aggregate variables behave as if R&D were undertaken only by outsider R&D firms aiming to challenge the current product so as to displace the current incumbent. This paper suggests that it is in this sense that the Arrow effect should be interpreted.

Moreover our result is similar in the spirit to the industrial organization classic result on contestable markets. In fact it is the potential competition of free entrants that suffices to discipline the monopolist’s R&D policy. Were the R&D market not open to outsider firms, the monopolist would be the only one to invest and its investment decision would internalize its business stealing. Hence restricting entry into R&D may severely reduce R&D investment and
innovation. However, it is important to remark that it is the potential entry that matters for innovation. Similarly, more efficient capital markets and venture capitalists may have important beneficial effects on growth even if their activity in financing start up R&D firms is only potential.

3. Multisector Extensions

In the multisector extensions of the basic Schumpeterian model (e.g. Grossman and Helpman 1991a and 1991b, Segerstrom 1998, Howit 1999, Li 2003), firm-specific R&D intensities $I_i$ add up to the industry-wide R&D intensity $I$ due to the assumption that the returns to R&D intensities are independent across firms within the same industry, across industries and over time. It is most common to solve quality ladders models by imposing symmetric R&D efforts across product lines. However this is only a simplified methodology used when there are no decreasing returns to R&D at the sectorial level, that allow to uniquely determine the R&D levels and therefore to solve the one period indeterminacy inherent in these models with a linear cross-sector R&D transformation function. In such a setting the supply of R&D labor per-sector is not uniquely pinned down by the wage rate. Therefore it is not per se incorrect to assume that any R&D firm’s marginal hiring can affect that sector’s R&D labor. However, with a more general and realistic microfoundation leading to a strictly concave cross-sector R&D transformation function our argument applies, whenever R&D is perfectly competitive.

For example, by assuming a sectorial fixed factor that renders the single firm’s success probability of R&D in a particular sector strictly decreasing in the aggregate R&D in the sector - as e.g. in Dinopoulos and Segerstrom (1999) - we can easily rule out the industrial organization implications of the Arrow effect along the lines of the previous section. More specifically, we could express the instantaneous success probability of R&D $I(\omega, t)$ in sector $\omega \in [0; 1]$ at time $t > 0$ as in Dinopoulos and Segerstrom’s (1999) eq. (4)

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9 An analysis of the growth consequences of such an indeterminacy in the quality ladders model is provided by Cozzi (2006).

10 Of course, models with a linear transformation function are as useful as more complicated models in analyzing Schumpeterian growth. Hence there is no lack of generality and realism in so long as their macroeconomic focus is concerned. What I am trying to emphasize here is that some of their macroeconomic implications do not hold when more generality and realism is included. Instead of weakening it, this paper shows that incumbent’s R&D involvement shall not be taken as evidence against Schumpeterian growth theory.
\[
I(\omega, t) \equiv \frac{AL_I(\omega, t)^\alpha h_I(\omega, t)^{1-\alpha}}{X_I(\omega, t)}
\]

where \(h_I(\omega, t)\) is the amount of "workers with specialized R&D skills" in sector \(\omega\) (Dinopoulos and Segerstrom, 1999), and where \(L_I(\omega, t)\) are "workers with general R&D skills". Alternatively, following Segerstrom et al. (1990), we could assume a negative sectorial externality of R&D and derive the unique steady state R&D employment level from the zero profit condition.

As the reader can easily see, these routes allow us to uniquely pin down each sector’s R&D intensity in a formally similar way as in Aghion and Howitt’s (1992) steady state. Of course, in their model the upward sloping aggregate supply of R&D workers was due to general equilibrium effects through manufacturing decreasing marginal returns to skilled labor, whereas here it is the general-skill R&D labor aggregate demand that slopes down due to partial equilibrium effects (decreasing returns in own sector’s R&D). However, given the individual firms’ infinitely elastic labor demands at the wage rate solving the arbitrage condition any R&D firm’s - including the incumbent manufacturing firm’s - marginal decision to hire general-skill R&D labor cannot increase its total employment in the sector. More employment by any firm can only be coupled with less employment by other firms, but never with more employment by the whole.

With non-drastic innovations in a Grossman and Helpman’s (1991a and b) economy, the quality advantage (which determines prices) of a leader who innovated twice may not become \(\lambda^2\) because "potential entrants can, via inspection of the goods on the market, learn enough about the state of knowledge to mount their own research efforts, even if the patent laws […] prevent them from manufacturing the current generation products" (Grossman and Helpman 1991a, p. 47). Alternatively, as often assumed, the previous leading-edge technology becomes prior knowledge and the patent on this technology expires. In both cases the quality difference between the leader and the followers is given by \(\lambda\), i.e. by the size of one innovation. Thus, product prices and monopoly profits do not change.

4. Conclusions

In this paper I have shown that there is no reason to expect that the Schumpeterian growth model with vertical innovations imply that the monopolist firm or its shareholders should not undertake R&D. This contrasts the traditional interpretation of the Arrow effect, though the Arrow effect is vindicated.
at the macroeconomic level. In this sense the existing empirical evidence of sequential patenting by the same firms or individuals should not be taken as evidence against the basic Schumpeterian growth theory.

The usual empirical critique to the Schumpeterian models of quality ladders crucially hinges on an interpretation of their R&D sector that employs a peculiar game-theoretic model of investment (e.g., each firm chooses its investment, taking that of its rivals as given). Thus each firm has a small but positive impact on aggregate investment; in this case, the incumbent invests nothing. This impact becomes increasingly small as the number of firms grows, but the limit is taken along this equilibrium path, where the incumbent invests zero for each value of $n$. Instead by literally following the Schumpeterian model’s explicit assumption of perfectly competitive R&D, it is correct to say that a perfectly-competitive model yields an indeterminate investment for the incumbent, thereby predicting that incumbents should invest randomly.

The quantity game interpretation of the perfectly competitive R&D assumption may have been inspired by an analogy with the classic industrial organization’s statement\footnote{Tirole (1988, p. 220-221) and Motta (2004, p. 559).} that as the number of firms in a Cournot oligopoly tends to infinity, with each firm’s market share tending to zero, the equilibrium price tends to the perfectly competitive equilibrium price. However, this quantitative coincidence does not mean that Cournot behavior (i.e. quantity taking behavior) is the same as the perfectly competitive (i.e. price taking) behavior: when the Schumpeterian models explicitly assume that R&D is perfectly competitive they are not conceiving of R&D being carried out by a lot of Cournot competitors, which renders quantity game equilibrium limit arguments not only mis-leading but inappropriate.

Are incumbent monopolists ‘small’ firms? This is left to empirical studies, which can confirm or reject this hypothesis depending on time, sector or country. Since this paper’s arguments are not restricted to the assumption that industry leaders are small - namely, size does not matter in our propositions just because equilibrium R&D firm size is indeterminate - I hope to have clarified that the original Schumpeterian theory cannot be contradicted empirically also on these grounds, but that instead it was consistent from its very beginning with incumbents either doing little R&D or a lot of R&D aimed at cannibalizing their own patent portfolios. The reason why the incumbents can be doing any amount of R&D from zero to the potentially very large amount we would expect the outsider actual and potential entrants to the research sector to carry out is that the perfectly competitive market clearing mechanism and the private CRS R&D technology renders the incumbent’s
R&D decision uninfluential for extending her patent’s expected life. Free entry and constant returns to scale, as is well known in IO, are consistent both with quantity and price competition. The empirical implication of quantity competition are known (Barro and Sala-I-Martin 2004), whereas those of price competition are shown in this paper. The controversial interpretation of the basic Schumpeterian theory stems from its initially stated assumption (price competition in the R&D) and its derivation in terms of quantity competition. The available empirical evidence does not falsify the interpretation in terms of price competition.

I wish to emphasize that such an acquittal12 of the original Schumpeterian theory13 does not preclude new explanations, and in a sense this long period of misunderstanding of its original properties had the virtue of stimulating the researchers to explore potentially useful alternatives to the assumption of free entry into a simultaneous R&D activities by outsiders having equal opportunities. For example, once one observes empirically14 that incumbents are not small, and brings enough evidence that it be more realistic to assume that the quality leader has some market power also in the R&D market and thus can act as a Stackelberg leader in its own industry, then, we will find the models of Barro and Sala-i-Martin, 2004, Denicolò, 2001, Etro (2004), and Segerstrom (2006) more appropriate.

References


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12For insufficiency of empirical evidence against it.

13As well as of its more recent scale effect free evolution, such as Segerstrom (1998), Howitt (1999), Li (2003), etc., which inherited its R&D industry microstructure.

14For example, Segerstrom (2006, p. 9, footnote. 19) notes that “follower firms appear to have negligible market value compared to industry leaders”, which can be empirically verified. Barro and Sala-i-Martin (2004, pp. 333-334) discuss the possibility of the industry leader to engage in R&D without having a cost advantage relative to quality followers. They distinguish between a Cournot-Nash equilibrium, where all firms take as given the R&D effort of all other firms, and a Stackelberg equilibrium, where the leader moves first and can commit to a specific R&D investment. They argue that the former equilibrium is consistent with zero R&D expenditures of the industry leader, whereas the latter equilibrium may be more reasonable since “the leader is entrenched in production and can make various types of visible investments” (p. 334).


