Whispering gallery polaritons in cylindrical cavities

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Strong coupling of the whispering gallery modes with bulk excitons in semiconductor cylinders is demonstrated theoretically. The strong-coupling threshold is governed by the radiative decay rate of optical modes, which we analyze as a function of the radius of the cylinder. Interestingly, the value of the Rabi splitting is found to be almost independent of the radius of the cylinder in the large limit. This is a consequence of the strong confinement of the whispering gallery modes.

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I. INTRODUCTION

Soon after the first realization of Bose-Einstein condensation (BEC) in dilute atomic gases in 1995,[1,2] the "atom chip" emerged as one of the most promising device concepts for production and manipulation of matter wave systems not only for basic studies but also for practical applications.[3,4] To date, the miniaturization of atom chips remains challenging, with technological issues associated with small-scale electronic, optical, and vacuum instrumentation.[5] However, recently Kasprzak *et al.*[6] have demonstrated BEC at 20 K for a gas of polaritons in a CdTe-based quantum well (QW) microcavity.[6] Polaritons are mixed light-matter bosonic quasiparticles which result from the strong coupling between photon modes confined in the microcavity and excitons confined in embedded quantum wells.[7,8] Their extremely light effective mass, about 10^{-5} times the free electron mass, is one of the key ingredients to achieve condensation at elevated temperatures. The other important ingredient is to have robust excitons at high carrier densities. In the CdTe-based microcavities studied by Kasprzak *et al.*,[6] the polariton BEC would only be possible below 60–80 K because the exciton binding energy is only 25 meV in CdTe QWs. Increasing the exciton binding energy with microcavities based on wider-band-gap semiconductors, such as ZnO or GaN, should extend operation up to room temperature or beyond.[9] Interestingly, BEC of polaritons was achieved with a strictly nonresonant optical excitation, which suggests that an electrical injection of free carriers would be just as effective. This opens up the prospect for a compact and practical "BEC on a chip" and electronically pumped polariton lasers.[8,9]

To ensure the so-called strong-coupling regime in the microcavity, the interaction between the excitons and the photon modes should be larger than their respective damping losses.[8] For photons, this means that their lifetime in the cavity should be long enough, typically 1 ps for standard semiconductors such as GaAs or CdTe. This would correspond to a cavity quality factor $Q > 1000$, which could be easily satisfied with planar Fabry-Perot cavities using GaAs or CdTe and related materials, since the growth of high-quality Bragg mirrors is well controlled in these systems. Unfortunately, this is not the case for the nitride and oxide systems, largely due to the large lattice mismatch combined with the low refractive index contrast between members of the same system (GaN and AlN, for example).

In this paper we consider the use of devices of cylindrical symmetry (such as microdisks or microrods) as resonant cavities to avoid the challenging task of growing high-quality Bragg mirrors. We study coupling of strongly localized whispering gallery modes with bulk excitons. The whispering gallery modes in spheres or cylinders are known to have quite long radiative lifetimes, which is highly advantageous from the point of view of their application in polariton devices. We show that indeed they can be strongly coupled with bulk excitons even if they have a relatively low oscillator strength as is the case in GaAs. We discuss the pros and cons of cylindrical structures for realization of polariton lasers and conclude that they have a number of advantages with respect to planar microcavities.

Cylindrical microcavities of various design have been investigated theoretically[10–13] and experimentally,[14–16] showing a wide range of interesting physical effects, including the strong exciton-light coupling of the low-orbital-index light modes with excitons.

The aim of this work to investigate conditions under which strong coupling between bulk excitons and whispering gallery modes can be achieved. We analyze the radiative decay rates of the whispering gallery modes and show that those modes which have lower decay rate can be strongly coupled with bulk excitons. We demonstrate that due to the high quality factor of the whispering gallery modes their strong coupling with excitons can indeed be observed. The strength of the coupling is essentially independent of the radius of the cylinder, while the finesse of the cavity strongly depends on it. We foresee high potentiality of the microcylinders and microdisks for realization of the polariton lasers.

II. RESULTS AND DISCUSSION

The optical eigenmodes of a single cylinder can be obtained by matching the components of the electromagnetic field at the surface of the cylinder using the usual Maxwell boundary conditions.[10] An electromagnetic wave propagating perpendicularly to the cylinder's axis of symmetry can be represented as a superposition of two independent polarized modes: the TM mode, for which the magnetic field is parallel to the cylindrical axis, and the TE mode, for which the electric field is parallel to the axis. For an infinite cylinder of radius ρ with refractive index n_1 surrounded by a medium of refractive index n_2, the frequencies of TE eigenmodes are given by the solution of

$$n_1 J_m'(n_1 K_0 \rho) H_m^{(1)}(n_2 K_0 \rho) - n_2 J_m(n_1 K_0 \rho) H_m^{(1)'}(n_2 K_0 \rho) = 0, \quad (1a)$$

where $K_0 = \omega / c$, $\omega = 2\pi f$, J_m and $H_m^{(1)}$ are Bessel and Hankel functions of the first kind, and m is the azimuthal number of the mode.

For the TM polarization, the eigenfrequencies can be obtained from

$$n_2 J_m'(n_1 K_0 \rho) H_m^{(1)}(n_2 K_0 \rho) - n_1 J_m(n_1 K_0 \rho) H_m^{(1)'}(n_2 K_0 \rho) = 0. \quad (1b)$$

Equations (1a) and (1b) have complex roots, the imaginary parts of which determine the decay rate of the electromagnetic fields of the eigenmodes.

If we consider a cylinder of refractive index n in air with radius ρ that is much larger than the wavelength of light in this material and neglect decay, we can obtain a simplified equation for the eigenfrequencies:[12]

$$n \omega_{opt} \rho = \frac{\pi c}{2}(2j + m \pm 1/2), \quad (2)$$

where the + (−) sign corresponds to the TM (TE) polarization and j is an integer radial quantum number.

One can see that the minimum radius required for the appearance of a mode of frequency ω with specific m (which will be referred to subsequently as the first mode) in the cylinder is given by

$$\rho = \frac{\pi c}{2 n \omega_{opt}}(m \pm 1/2). \quad (3)$$

Figure 1(a) shows the *radius of appearance* of the first modes for an energy 1.515 eV (which corresponds to the exciton resonance in GaAs at 4 K) in a cylinder made of GaAs for various values of azimuthal number m. One can see that the radius of appearance increases with increasing m.

For the strong-coupling regime, the optical mode of the cavity should have a decay rate that is less than the splitting between polariton modes. Figure 1(b) shows the radiative decay rate of the first mode for different values of azimuthal numbers m which is seen to fall exponentially with increasing m. Note that radiative decay remains finite since total internal reflection does not occur for a cylindrical interface.[11]

For realization of the polariton devices, it is desirable that only one optical mode interacts with the exciton, and it is apparent from Eqs. (2) and (3) that the larger m, the more modes with different m and j coexist in the close vicinity of the first mode (one having $j=0$) as Fig. 2 shows. The radiative decay of the mode rapidly increases with increasing radial quantum number j, as one can see from Fig. 2. The first mode has the highest finesse and represents the case of most interest for strong coupling with excitons and realization of polariton devices.

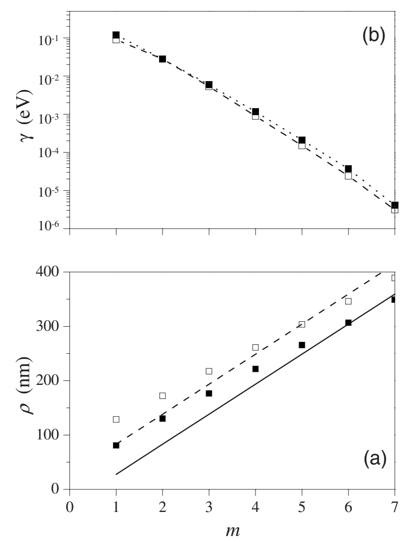

FIG. 1. (a) Dependence of the radius of the cylinder on the mode azimuthal number m when the first mode (with radial quantum number $j=0$) has a photon energy at 1.515 eV. The lines show the dependences obtained using the simplified Eq. (3) for eigenfrequencies for TM (dashed line) and TE (solid line) polarizations. (b) Dependence of the radiative decay rate on mode azimuthal number m of the first mode (with radial quantum number $j=0$) at photon energy 1.515 eV. Open (solid) symbols correspond to the TM (TE) polarization and are obtained using Eqs. (1).

The eigenfrequencies of exciton-polariton modes of the cylindrical microcavity can be obtained from Eqs. (1a) and (1b) by including the excitonic contribution[8] to the refractive index n_1 of the cylinder, so that

$$n_1^2 = \varepsilon = \varepsilon_b + \frac{\varepsilon_b \omega_{LT}}{\omega_{ex} - \omega - i\Gamma}, \quad (4)$$

where ε_b is the background dielectric constant, ω_{ex} is the exciton resonance frequency, ω_{LT} is the bulk value of

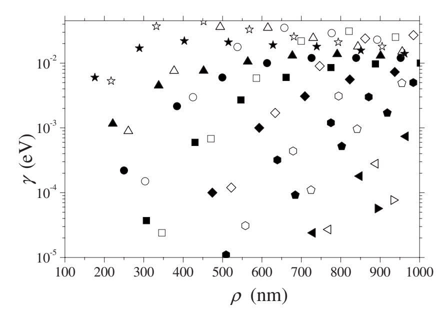

FIG. 2. Radiative decay rate of the optical eigenmodes of a cylinder as a function of radius for modes with the real parts of their photon energy equal to 1.515 eV for different azimuthal numbers $m=3$ (stars), 4 (up triangles), 5 (circles), 6 (squares), 7 (diamonds), 8 (hexagons), 9 (pentagons), 10 (left triangles), and 11 (right triangles). The open (solid) symbols correspond to TM (TE) polarization, respectively.

longitudinal-transverse splitting, and Γ is the nonradiative decay rate of the exciton. Here we have neglected the spatial dispersion of excitons, which is weak already in GaAs and even weaker in the wide-band-gap semiconductors, where the excitons are characterized by heavy effective masses.

Let us estimate analytically the value Δ of the Rabi splitting of the polariton modes. Consider a cavity with dielectric constant ε_b. If the optical mode is tuned to the exciton resonance ($\omega_{ex}=\omega_{opt}$) Eq. (2) for the bare cavity mode can be rewritten in the form

$$\varepsilon_b \omega_{ex}^2 \rho^2 = (\pi c/2)^2 (2j + m + 1/2)^2. \tag{5}$$

Neglecting nonradiative decay of the exciton and assuming $\varepsilon \approx \varepsilon_b[1+\omega_{LT}/(\omega_{ex}-\omega)]$, the polariton eigenmodes in a cylinder containing an exciton can be written as

$$\varepsilon_b\left(1 + \frac{\omega_{LT}}{\omega_{ex} - \omega}\right)\omega^2 \rho^2 = (\pi c/2)^2 (2j + m + 1/2)^2. \tag{6}$$

Taking into account Eq. (5) for any j and m, Eq. (6) can be reduced to

$$(\omega - \omega_{ex})(\omega + \omega_{ex}) - \frac{\omega^2 \omega_{LT}}{\omega - \omega_{ex}} = 0. \tag{7}$$

Assuming that $\omega - \omega_{ex} \ll \omega_{ex}$, we obtain

$$(\omega - \omega_{ex})^2 \approx \frac{\omega_{ex}\omega_{LT}}{2}. \tag{8}$$

Thus the splitting of polariton modes (or Rabisplitting) is

$$\Delta = \sqrt{2\omega_{ex}\omega_{LT}}. \tag{9a}$$

Note that the value of the splitting in a cylindrical cavity given by Eq. (9a) is larger than splitting in a planar microcavity[17] with cavity length L_c and Bragg mirrors characterized by penetration depth L_{DBR},

$$\Delta = \sqrt{\frac{2\omega_{ex}\omega_{LT}L_c}{L_{DBR} + L_c}}. \tag{9b}$$

For GaAs cylindrical cavity, when $\hbar\omega_{ex}=1.515$ eV and $\hbar\omega_{LT}=0.08$ meV, the value of the splitting is $\Delta \approx 16$ meV, while for planar GaAs-based microcavity typical value of the Rabi splitting is about 5 meV. One can see that, in the adopted approximation, the value of the Rabi splitting in the cylinder is equal to the bulk exciton-polariton Rabi splitting and does not depend on the indices j and m or on the radius of the cylinder. This result looks surprising, compared to the well-known case of planar microcavities with quantum wells. However, it has a simple physical reason: the whispering gallery modes are strongly confined in the cylinder, so that the penetration of the light field outside the cylinder is quite small. Equations (2) and (5) are obtained assuming full confinement of the optical modes, which is an accurate assumption in the case of high j and m, as our numerical modeling shows. The overlap integral of the cavity mode and the bulk exciton is close to 1, as in an infinite bulk semiconductor, which is why the coupling constant in the cylinder appears to be equal to that of a bulk semiconductor. In planar microcavities containing quantum wells, this is not the case, as the overlap of the cavity mode with the quantum well is always less than 1. Note that in the case of GaN-based cylindrical microcavity, the value of the Rabi splitting predicted by Eq. (9a) would be about 70 meV, while for ZnO Δ could be as large as 150 meV, which exceeds substantially the thermal energy corresponding to room temperature.

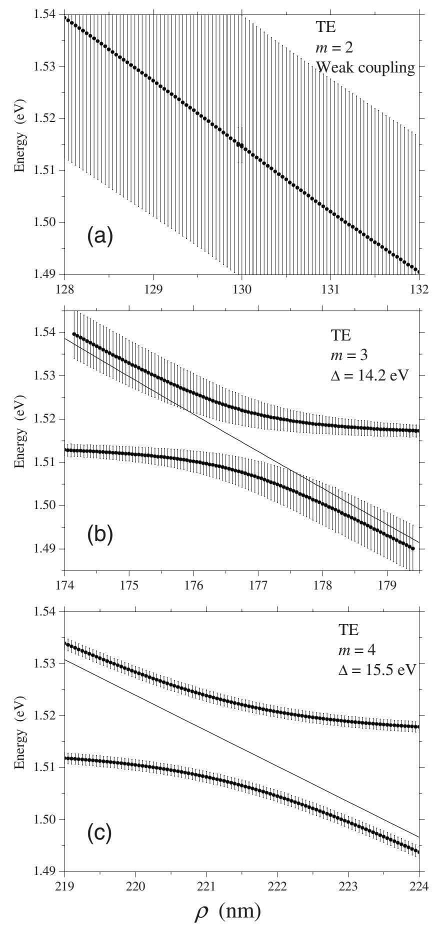

FIG. 3. Real part of photon energy of the first polariton modes of a cylinder versus the radius (circles) for different azimuthal numbers m=(a) 2; (b) 3; (c) 4. The vertical bars show the decay rate of the modes. The solid line shows the photon energy of the optical mode of the cylinder in the absence of the exciton.

Written in the standard form for the two-coupled-oscillator model, the equation for exciton-polariton eigenfrequencies can be written as

$$(\omega - \omega_{ex})(\omega - \omega_{opt}) - (\Delta/2)^2 = 0. \tag{10}$$

Figure 3(a) shows the dependence of the eigenmode frequency on the radius of the cylinder in the vicinity of the crossing point of the first mode with $m=2$ with the exciton resonance. The decay rate of the optical mode in this case is $\gamma=0.028$ eV, which is twice as large as the value of the Rabi splitting Δ, and hence the weak-coupling regime is obtained.

The decay rate of the first mode with $m=3$ is $\gamma=0.005$ eV, which is less than the value of the Rabi splitting Δ. In this case the system is in the strong-coupling regime as can be seen in Fig. 3(b).

The decay rate of the first mode with $m=4$ is $\gamma \approx 0.001$ eV, which is comparable with Γ and much less than the splitting Δ. The anticrossing of the exciton and the optical mode characteristic for the strong-coupling regime is clearly seen in Fig. 3(c). For cylinders of larger radii, similar mode behavior can be obtained for larger radial and azimuthal mode numbers. However, as the finesse of the optical modes at a given frequency decreases with increase of the radius of the cylinder, the single anticrossing seen in Fig. 3(c) would be replaced by multiple anticrossings between the exciton resonance and closely lying photon modes in this case.

Until now we have considered an infinitely long cylinder. Realistic systems having a finite length can be divided into two classes: microrods, whose length is much more than the diameter, and misrodisks, whose thickness is less than the diameter.

In the case of a microrod, motion of a polariton along the axis of the rod is allowed. This provides the opportunity to tune the optical mode. In particular, in a photoluminescence experiment, the frequency of photoluminescence will depend on the angle between the axis of the rod and the direction of detection. For a microrod the frequencies of the optical eigen-modes having a zero wave vector along the axis of the rod are given by Eqs. (1a) and (1b) with good accuracy. The dispersion of the modes along the axis of the rod is parabolic, with an effective mass dependent on the radius of the rod.

In the case of a microdisk, the background refractive index of the cylinder should be replaced in the calculation by the effective refractive index n_{eff} of the mode of a planar waveguide whose thickness is equal to that of the microdisk. Alternatively, the values ρ_{disk} of the radius of the microdisk required to observe the polariton anticrossing can be obtained by using the scaling relation

$$\rho_{disk} n_{eff} = \rho n_1, \tag{11}$$

where ρ are the radii obtained in the infinite cylinder model. Also, in the case of a microdisk, there will be coupling of TE and TM modes, though this effect will be negligible in the case of microdisks of large radius.

For both systems, the weak-strong coupling transition takes place in accordance with the formalism developed above, while for the microdisk an additional channel of radiative decay along the axis should be accounted for in calculating the damping γ. The advantage of microdisks over microrods for realization of polariton lasers comes from the fact that the wave vector along the axis of the cylinder in no longer a good quantum number in microdisks. Namely, it may not be conserved during the scattering of polaritons with acoustic phonons, so that the phonon bottleneck effect, which makes observation of polariton BEC in planar microcavities more difficult, should not play any role in microdisks.

III. CONCLUSIONS

In conclusion, we have demonstrated that the strong-coupling regime for bulk exciton-polariton coupled to the whispering gallery modes in GaAs microcylinders is indeed possible. This opens the way to observation of polariton lasing in GaAs-based microsystems. We expect that microdisks of wider-band-gap semiconductor materials, like GaN or ZnO, would allow for realization of room-temperature polariton lasers.

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[1] M. N. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Science **269**, 198 (1995).
[2] K. B. Davis, M.-O. Mewes, M. R. Andrews, N. J. van Druten, D. S. Durfee, D. M. Kurn, and W. Ketterle, Phys. Rev. Lett. **75**, 3969 (1995).
[3] W. Hänsel, P. Hommelhoff, T. W. Hänsch, and J. Reichel, Nature (London) **413**, 498 (2001).
[4] H. Ott, J. Fortagh, G. Schlotterbeck, A. Grossmann, and C. Zimmermann, Phys. Rev. Lett. **87**, 230401 (2001).
[5] Shengwang Du, M. B. Squires, Y. Imai, L. Czaia, R. A. Saravanan, V. Bright, J. Reichel, T. W. Hänsch, and D. Z. Anderson, Phys. Rev. A **70**, 053606 (2004).
[6] J. Kasprzak, M. Richard, S. Kundermann, A. Baas, P. Jeambrun, J. Keeling, F. M. Marchetti, M. H. Szymańska, R. André, J. L. Staehli, V. Savona, P. B. Littlewood, B. Deveaud, and Le Si Dang, Nature (London) **443**, 409 (2006).
[7] C. Weisbuch, M. Nishioka, A. Ishikawa, and Y. Arakawa, Phys. Rev. Lett. **69**, 3314 (1992).
[8] A. Kavokin and G. Malpuech, *Cavity Polaritons* (Elsevier, Amsterdam, 2003).
[9] G. Malpuech, A. Di Carlo, A. Kavokin, J. J. Baumberg, M. Zamfirescu, and P. Lugli, Appl. Phys. Lett. **81**, 412 (2002).
[10] A. Ishimaru, *Wave Propagation and Scattering in Random Media* (Academic Press, New York, 1978).
[11] G. Panzarini and L. C. Andreani, Phys. Rev. B **60**, 16799 (1999).
[12] M. A. Kaliteevski, R. A. Abram, V. V. Nikolaev, and G. S. Sokolovski, J. Mod. Opt. **46**, 875 (1999); M. A. Kaliteevski, R. A. Abram, and V. V. Nikolaev, *ibid.* **47**, 677 (2000).
[13] M. A. Kaliteevski, S. Brand, R. A. Abram, V. V. Nikolaev, M. V. Maximov, N. N. Ledentsov, C. M. Sotomayor Torres, and A. V. Kavokin, Phys. Rev. B **61**, 13791 (2000).
[14] J. M. Gérard, B. Sermage, B. Gayral, B. Legrand, E. Costard, and V. Thierry-Mieg, Phys. Rev. Lett. **81**, 1110 (1998).
[15] M. Fujita, R. Ushigome, and T. Baba, Electron. Lett. **36**, 790 (2000).
[16] J. P. Reithmaier, G. Sek, A. Löffler, C. Hofmann, S. Kuhn, S. Reitzenstein, L. V. Keldysh, V. D. Kulakovskii, T. L. Reinecke, and A. Forchel, Nature (London) **432**, 197 (2004).
[17] M. R. Vladimirova, A. V. Kavokin, and M. A. Kaliteevski, Phys. Rev. B **54**, 14566 (1996).