

## Durham Research Online

---

**Deposited in DRO:**

08 January 2013

**Version of attached file:**

Published Version

**Peer-review status of attached file:**

Peer-reviewed

**Citation for published item:**

Abel, S. and Buican, M. and Komargodski, Z. (2011) 'Mapping anomalous currents in supersymmetric dualities.', *Physical review D : particles and fields.*, 84 (4). 045005.

**Further information on publisher's website:**

<http://dx.doi.org/10.1103/PhysRevD.84.045005>

**Publisher's copyright statement:**

© 2011 American Physical Society

**Additional information:**

### Use policy

---

The full-text may be used and/or reproduced, and given to third parties in any format or medium, without prior permission or charge, for personal research or study, educational, or not-for-profit purposes provided that:

- a full bibliographic reference is made to the original source
- a [link](#) is made to the metadata record in DRO
- the full-text is not changed in any way

The full-text must not be sold in any format or medium without the formal permission of the copyright holders.

Please consult the [full DRO policy](#) for further details.

**Mapping anomalous currents in supersymmetric dualities**Steven Abel,<sup>1,2</sup> Matthew Buican,<sup>2</sup> and Zohar Komargodski<sup>3,4</sup><sup>1</sup>*Institute for Particle Physics Phenomenology, Durham University, United Kingdom*<sup>2</sup>*Department of Physics, CERN Theory Division, CH-1211 Geneva 23, Switzerland*<sup>3</sup>*Institute for Advanced Study, Princeton, New Jersey 08540, USA*<sup>4</sup>*Weizmann Institute of Science, Rehovot 76100, Israel*

(Received 15 June 2011; published 3 August 2011)

In many strongly coupled systems, the infrared dynamics is described by different degrees of freedom from the ultraviolet. It is then natural to ask how operators written in terms of the microscopic variables are mapped to operators composed of the macroscopic ones. Certain types of operators, like conserved currents, are simple to map, and in supersymmetric theories one can also follow the chiral ring. In this paper, we consider supersymmetric theories and extend the mapping to anomalous currents (and gaugino bilinears). Our technique is completely independent of subtleties associated with the renormalization group, thereby shedding new light on previous approaches to the problem. We demonstrate the UV/IR mapping in several examples with different types of dynamics, emphasizing the uniformity and simplicity of the approach. Natural applications of these ideas include the effects of soft breaking on the dynamics of various theories and new models of electroweak symmetry breaking.

DOI: [10.1103/PhysRevD.84.045005](https://doi.org/10.1103/PhysRevD.84.045005)

PACS numbers: 11.30.Pb, 11.40.-q

**I. INTRODUCTION**

At low energies, many strongly coupled field theories can be described in terms of emergent degrees of freedom—often markedly different from those used to define the theory at short distances. The most well-known example where this phenomenon occurs is QCD, which, in the chiral limit, flows from a theory described purely in terms of fermions and gauge fields to a free theory of massless pions.

Given this picture, an important question that arises is how to express long-distance correlation functions, written in terms of the fundamental quarks and color gauge fields, as correlation functions written in terms of mesons. (Of course, in order for this question to be well defined, one must only consider gauge-invariant correlation functions in the UV.)

For instance, one may consider correlation functions of conserved currents, which in QCD are associated with the symmetries  $SU(N_f)_L \times SU(N_f)_R \times U(1)_B$ , and attempt to rewrite the corresponding conserved currents in terms of the pions. This procedure is fairly straightforward for the non-Abelian currents, but some interesting complications arise for the baryonic current (for a general treatment see [1]).

Another interesting set of quark bilinears in QCD are the  $\psi\tilde{\psi}$  operators. In this case, one can use an  $SU(N_f)_L \times SU(N_f)_R$  spurion analysis and find that they map to  $U = e^{i\pi^a T^a}$ . Unfortunately, the coefficient in this mapping is incalculable. One can also consider the quark bilinear corresponding to the anomalous axial current,  $U(1)_A$ . However, we are not aware of any systematic procedure of mapping this operator to the IR.<sup>1</sup>

<sup>1</sup>The situation might be better in the large  $N$  limit of QCD; there one can imagine including the light  $\eta'$  particle [2].

In this paper we will discuss related questions in the context of supersymmetric (SUSY) theories. In SUSY theories it is relatively straightforward to follow the flows of two broad classes of operators—elements of the chiral ring and the (nonchiral) conserved current multiplets. The mapping of the conserved currents follows the same rules as in non-SUSY theories. We use either 't Hooft anomaly matching or Goldstone's theorem to realize various conserved currents in the IR.<sup>2</sup> The mapping of the chiral ring is, of course, possible due to the strong constraints imposed by holomorphy.

Generalizing the above ideas to nonconserved currents (and objects that vanish in the chiral ring) is more difficult. However, understanding their flow is crucial for many applications, such as the mapping of soft nonholomorphic mass terms, which, at weak coupling, can be thought of as the lowest components of (non)conserved current multiplets. Studying these questions is the chief goal of this paper.

The main utility of supersymmetry in this context is as follows. Consider a current broken explicitly by an anomaly. It satisfies the Adler-Bardeen equation (it is actually not important for us to work in a scheme where the anomaly is one-loop exact)

$$\partial^\mu j_\mu \sim F\tilde{F}. \quad (1.1)$$

However, supersymmetry relates  $F\tilde{F}$  with  $F^2$  since they together form the complex  $\theta^2$  component of  $W_\alpha^2$ . The final crucial ingredient is that  $F^2$  is related to the stress tensor via the usual trace anomaly

<sup>2</sup>Complications, as for baryon number in QCD, can arise too, although they do not arise in the simplest examples. See [3] for interesting discussions of closely related matters.

$$T_{\mu}^{\mu} \sim F^2. \quad (1.2)$$

Even if the theory goes through strong coupling, the *conserved* energy-momentum tensor is known at the end points of the flow (as long as there is a description in terms of weakly coupled degrees of freedom there). From this discussion, we see that we can follow  $F\tilde{F}$  and learn something about the flow of anomalous currents.

The problem can be simplified even further if there is a conserved  $R$  symmetry. Indeed, the corresponding  $R$  current is related to the energy-momentum tensor by SUSY. Being a conserved current, the  $R$  current is easily followed along the flow. Therefore, in a heuristic sense, supersymmetry extends the simplicity of the flow of the conserved  $R$  current to the flow of the anomalous (non- $R$ ) current.

The question of following nonholomorphic operators, at least in the guise of soft-SUSY-breaking masses, to the IR is not new. Indeed, there is significant literature on the subject; see e.g. [4–12] and references therein. While our paper has many new concrete results, its main purpose is to advocate several new points of view on the subject. These perspectives allow us to solve the problem while completely avoiding discussions of subtleties associated with renormalization group (RG)-invariant versus RG-noninvariant (i.e. scheme-independent versus scheme-dependent) quantities. Most importantly, our tools provide us with general results that are valid uniformly in all theories considered, including ones that have not been addressed before.

The plan of this paper is as follows. In Sec. II we describe in detail the procedure outlined above. In Secs. III, IV, and V we discuss three examples in which one can map some nonchiral operators to a weakly coupled dual description. These examples demonstrate slightly different aspects and nuances of the general procedure. In Sec. VI we consider theories that flow to an interacting superconformal field theory (SCFT) at long distances. In Sec. VII we discuss the remaining open questions, most importantly, emphasizing possible applications to electro-weak symmetry breaking (EWSB), and conclude.

## II. THE AXIAL ANOMALY AND THE ENERGY-MOMENTUM TENSOR

In this section we review the well-known connection between the axial anomaly, the energy-momentum tensor, and the  $R$ -symmetry current (if the latter exists). To that end, we first note that the anticommutation relations  $\{Q, \bar{Q}\} \sim P$  imply that the supercurrent and the energy-momentum tensor sit in the same multiplet. The question then becomes how to write an irreducible representation of SUSY containing the supercurrent and the energy-momentum tensor. The simplest solution is the Ferrara-Zumino (FZ) multiplet [13]<sup>3</sup>

<sup>3</sup>We adopt the following conventions:  $\ell_{\alpha\dot{\alpha}} = -2\sigma_{\alpha\dot{\alpha}}^{\mu}\ell_{\mu}$ ,  $\ell_{\mu} = \frac{1}{4}\bar{\sigma}_{\mu}^{\dot{\alpha}\alpha}\ell_{\alpha\dot{\alpha}}$ .

$$\bar{D}^{\dot{\alpha}}\mathcal{J}_{\alpha\dot{\alpha}} = D_{\alpha}X, \quad (2.1)$$

where  $X$  is chiral and  $\mathcal{J}_{\mu}$  is real. In some cases, which are not relevant to this paper, the FZ multiplet does not exist [14,15].

Writing out the solution to (2.1), we find that the  $\theta^2$  component of  $X$  contains the trace of the energy-momentum tensor as well as the divergence of the bottom component of  $\mathcal{J}_{\mu}$ . In pure super Yang-Mills theory,  $X$  is proportional to  $W_{\alpha}^2$ . The trace of the energy-momentum tensor is proportional to  $F_{\mu\nu}F^{\mu\nu}$ , while the Adler-Bell-Jackiw (ABJ) equation relates the divergence of the bottom component of  $\mathcal{J}_{\mu}$  to  $F\tilde{F}$ .<sup>4</sup>

Intuitively, it is this connection between the anomaly and the energy-momentum tensor that allows one to say more than usual about the flow of the anomalous current. It is relatively easy to identify the energy-momentum tensor in the IR of a complicated flow (as long as we know what the, possibly emergent, degrees of freedom are there). This discussion also suggests that coupling the rigid theory to supergravity, as in [10], might shed some light on the mapping of anomalous currents. We will not consider supergravity in this paper.

When the theory under consideration has an exact  $R$  symmetry, there is a more natural representation of the supercurrent which, as we will see below, leads to a simpler description of the physics. In such a case, the conserved  $R$  current transforms as the bottom component of a supercurrent multiplet that is defined by

$$\bar{D}^{\dot{\alpha}}\mathcal{R}_{\alpha\dot{\alpha}} = \chi_{\alpha}. \quad (2.2)$$

Here  $\chi_{\alpha}$  is chiral and satisfies the usual Bianchi identity  $D\chi = \bar{D}\bar{\chi}$ , and  $\mathcal{R}_{\mu}$  is real. In the systems of interest to us, both the FZ multiplet (2.1) and the  $\mathcal{R}$  multiplet (2.2) exist. From this statement, it follows that the Bianchi identity for  $\chi_{\alpha}$  can be solved in terms of a well-defined real superfield  $U$ , and so

$$\bar{D}^{\dot{\alpha}}\mathcal{R}_{\alpha\dot{\alpha}} = \bar{D}^2D_{\alpha}U. \quad (2.3)$$

The general picture of what happens to  $U$  along a flow is simple to understand. In the asymptotically free theories we will study below,  $\mathcal{R}$  and  $U$  start out in the UV as bilinears in the various weakly coupled superfields (with appropriate contributions of  $\sim \frac{1}{g^2}W_{\alpha}\bar{W}_{\dot{\alpha}}$  to  $\mathcal{R}$  and appropriate factors of  $e^V$  to render  $\mathcal{R}$  and  $U$  gauge invariant; we neglect these terms for simplicity—a more detailed recent discussion of many of the issues discussed here can be

<sup>4</sup>Incidentally, this relation leads to the famous “anomaly puzzle” (see [16–19], and many references therein, for a more detailed discussion of this puzzle) since one would expect the anomaly to naturally be one-loop exact while the beta function has contributions from all loop orders. This apparent paradox will not affect our discussion below in any way.

found in [15,20–26]). Indeed, solving (2.3), one finds that for the matter superfields  $\Phi_i$ , with  $R$  charges  $r_i$ , the expressions for  $\mathcal{R}_{\alpha\dot{\alpha}}$  and  $U$  take the form

$$\mathcal{R}_{\alpha\dot{\alpha}} = \sum_i (2D_\alpha \Phi_i \bar{D}_{\dot{\alpha}} \bar{\Phi}^i - r_i [D_\alpha, \bar{D}_{\dot{\alpha}}] \Phi_i \bar{\Phi}^i), \quad (2.4)$$

$$U = \sum_i \left(1 - \frac{3r_i}{2}\right) \bar{\Phi}^i \Phi_i. \quad (2.5)$$

Note that the contributions to  $U$  of fields with  $r_i = 2/3$  vanish because this is the superconformal  $R$  charge for fields at the Gaussian UV fixed point.

We should elaborate on what it means to solve for  $U$ . Equation (2.3) does not fix  $U$  uniquely, but only fixes  $\bar{D}^2 D_\alpha U$ . This leads to the usual supergauge ambiguity,  $U \rightarrow U + \Omega + \bar{\Omega}$ , where  $\Omega$  is chiral. In writing (2.5) we have discarded all such holomorphic terms. Indeed, in most cases they can be completely ignored by symmetry arguments. However, we will see cases where even if such terms are not included in the UV, adding such terms in the IR is forced on us by consistency.

It is also due to such ambiguities in solving superspace equations that we opt to use the  $\mathcal{R}$  multiplet rather than the FZ multiplet. Indeed, in the latter case one can show that ambiguities arise not only from purely holomorphic terms, but also from conserved currents (which generically exist and render the analysis harder).

Now, as we flow to the IR, we can use the  $\mathcal{R}$  multiplet to follow  $U$ . The IR is described by some SCFT, and  $U$  can be described as  $U \sim \Lambda^{2-d} \mathcal{O}$ , for some real operator  $\mathcal{O}$  of dimension  $d \geq 2$ , and some scale  $\Lambda$ . We can assume the dimension of the real operator  $\mathcal{O}$  is  $\geq 2$  by unitarity and by the fact that we can remove holomorphic plus antiholomorphic contributions.

In the case that  $d > 2$ ,  $U$  formally vanishes at the IR fixed point (i.e. deep in the IR). This means that the bottom component of  $\mathcal{R}_{\alpha\dot{\alpha}}$  becomes the superconformal  $R$  symmetry.<sup>5</sup> In other words, the  $R$  symmetry we have chosen in the UV becomes the superconformal one in the infrared. But we know this is not always the case. There could be multiple choices for the  $R$  symmetry in the UV, and there can also be accidental symmetries in the IR. When the  $R$  current we follow does not flow to the IR superconformal one, then  $U$  is *nonzero* in the IR and it flows to a certain current of dimension 2,

$$U \rightarrow \frac{3}{2}J. \quad (2.6)$$

<sup>5</sup>A special case which is slightly more subtle is when the IR SCFT is approached by a marginally irrelevant operator. This can be represented by  $U = \gamma J$ , where  $J$  is some dimension-2 operator in the IR SCFT and  $\gamma$  is an anomalous dimension that goes to zero in the deep IR, as required. The general construction of this  $J$  and the calculation of  $\gamma$  are presented in the framework of [27]. We thank D. Green and N. Seiberg for helpful conversations on the matter.

This conserved current,  $J$ , may be a conserved current of the full theory, or it may correspond to an accidental symmetry of the IR fixed point.<sup>6</sup>

Formally,  $J$  has a simple description. It is just the conserved current which parametrizes the difference between the superconformal  $R$  symmetry and the one in the multiplet we are following along the flow. This can be shown by recalling that the IR superconformal theory admits the superconformal multiplet  $\mathcal{R}_\mu^{\text{CFT}}$  (i.e., the multiplet for which  $\bar{D}^{\dot{\alpha}} \mathcal{R}_{\alpha\dot{\alpha}}^{\text{CFT}} = 0$  at the IR fixed point). We can write this multiplet in terms of the IR limit of (2.3) and the (perhaps accidentally) conserved current multiplet  $J$  as follows

$$\mathcal{R}_{\alpha\dot{\alpha}}^{\text{CFT}} = \mathcal{R}_{\alpha\dot{\alpha}}^{\text{IR}} - [D_\alpha, \bar{D}_{\dot{\alpha}}]J, \quad U^{\text{CFT}} = U^{\text{IR}} - \frac{3}{2}J = 0. \quad (2.7)$$

Here  $U^{\text{IR}}$  is the deep IR limit of  $U$ , and  $J$  is the multiplet for the symmetry that mixes with the  $R$  charge corresponding to  $\mathcal{R}_{\alpha\dot{\alpha}}$  to create the superconformal  $R$  symmetry.  $\mathcal{R}_{\alpha\dot{\alpha}}^{\text{IR}}$  and  $\mathcal{R}_{\alpha\dot{\alpha}}^{\text{CFT}}$  are related via improvement transformations for the supercurrent and stress tensor.<sup>7</sup>

We see that being able to follow the axial current relies crucially on being able to identify the superconformal  $R$  symmetry. In many examples this is fixed by duality. Additionally, we have the powerful tools of [33].

In many theories, there are free magnetic phases, where the IR is a Gaussian fixed point. Then the abstract discussion above takes a very simple form, since the superconformal  $R$  charge is  $2/3$  for all the chiral fields.  $U^{\text{IR}}$  is then fixed by the IR analog of (2.5), namely,

$$U^{\text{IR}} = \sum_i \left(1 - \frac{3r_i}{2}\right) \bar{\phi}^i \phi_i, \quad (2.8)$$

where the  $\phi_i$  are the “emergent” chiral superfields at low energies and  $r_i$  are their  $R$  charges. The simplest example of such a theory is SQCD in the free magnetic phase, which we will now discuss in much greater detail.

### III. THE ANOMALOUS CURRENT OF SQCD

In this section we will consider  $SU(N_c) \mathcal{N} = 1$  SQCD with  $N_f$  in the free magnetic range, i.e.  $N_c + 1 < N_f \leq 3N_c/2$ . Recall the matter content of the electric UV theory,

<sup>6</sup>The above discussion relies on the assumption that the fixed points are conformal in addition to being scale invariant. Whether this is always true is an open question (see [28,29] for some aspects of the problem). However, in many cases of interest, like supersymmetric QCD (SQCD) and various simple generalizations, conformality is strongly suggested by the discussion in [30] and various related works. This picture has been given further reinforcement recently in [31].

<sup>7</sup>More general studies of improvements of supercurrent multiplets can be found in [15,32].

$$\begin{array}{cccc}
SU(N_c) & SU(N_f) \times SU(N_f) & U(1)_R & U(1)_B \\
Q & \mathbf{N}_c & \mathbf{N}_f \times \mathbf{1} & 1 - \frac{N_c}{N_f} & 1 \\
\tilde{Q} & \bar{\mathbf{N}}_c & \mathbf{1} \times \bar{\mathbf{N}}_f & 1 - \frac{N_c}{N_f} & -1
\end{array} \quad (3.1)$$

A particularly interesting set of operators to try and follow is given by all the possible nonholomorphic bilinears

$$c_i^j Q^i Q_j^\dagger + \tilde{c}_i^j \tilde{Q}^i \tilde{Q}_j^\dagger, \quad (3.2)$$

where  $c_i^j, \tilde{c}_i^j$  represent some arbitrary real numbers, and  $i, j = 1, \dots, N_f$ .

The theory (3.1) is understood in the IR via Seiberg duality [30]. The low energy degrees of freedom consist of a dual IR-free  $SU(N_f - N_c)$  gauge group,  $N_f$  dual quark superfields  $q, \tilde{q}$  in the fundamental-antifundamental representations of  $SU(N_f - N_c)$ , and a gauge singlet meson  $N_f \times N_f$  matrix,  $M$ . We summarize this matter content in the following table:

$$\begin{array}{cccc}
SU(N_f - N_c) & SU(N_f) \times SU(N_f) & U(1)_R & U(1)_B \\
q & \mathbf{N}_f - \mathbf{N}_c & \bar{\mathbf{N}}_f \times \mathbf{1} & \frac{N_c}{N_f} & \frac{N_c}{N_f - N_c} \\
\tilde{q} & \bar{\mathbf{N}}_f - \bar{\mathbf{N}}_c & \mathbf{1} \times \mathbf{N}_f & \frac{N_c}{N_f} & -\frac{N_c}{N_f - N_c} \\
M & \mathbf{1} & \mathbf{N}_f \times \mathbf{N}_f & 2 - 2\frac{N_c}{N_f} & 0
\end{array} \quad (3.3)$$

Since the theory is IR free, the natural normalization of these dual fields is to choose their kinetic terms to be canonical.

Given this picture, we would like to know how the operators in (3.2) are realized in the dual theory of (3.3). It is difficult to answer this question exactly, since the result depends on incalculable corrections to the Kähler potential of the IR degrees of freedom. However, here we are only interested in knowing what the operators in (3.2) flow to in the deep IR, where all such corrections are irrelevant.

As mentioned in the Introduction, it is extremely easy to follow to the IR operators of the form (3.2) that correspond to conserved currents. For example, consider  $Q^i Q_i^\dagger - \tilde{Q}^i \tilde{Q}_i^\dagger$ . This operator can be identified with the bottom component of the conserved baryon superfield

$$\bar{D}^2(Q^i Q_i^\dagger - \tilde{Q}^i \tilde{Q}_i^\dagger) = 0. \quad (3.4)$$

We can immediately conclude that in the deep IR it should be matched to the baryon number current of the magnetic theory. In other words,

$$QQ^\dagger - \tilde{Q}\tilde{Q}^\dagger \rightarrow \frac{N_c}{N_f - N_c} (|q|^2 - |\tilde{q}|^2). \quad (3.5)$$

The numerical factor on the right-hand side of this equation follows from the well-known baryon charge of the magnetic quarks (see Eq. (3.3)).

It is just as easy to follow some other special bilinears in the squark superfields. Indeed, all the bilinears given by

linear combinations of  $QT^a Q^\dagger$  and  $\tilde{Q}T^a \tilde{Q}^\dagger$  (with traceless, Hermitian,  $T^a$ ) can be thought of as the bottom components of the non-Abelian currents associated with  $SU(N_f)_L \times SU(N_f)_R$  and can thus be directly mapped to the IR [this is done via the action of these symmetries on the magnetic degrees of freedom (3.3)].

In the space of all bilinears (3.2) there is, however, one linearly independent combination which is nontrivial to map to the IR. Without loss of generality, this linear combination can be chosen to be

$$J_A = QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger. \quad (3.6)$$

This is not the bottom component of any conserved current. In fact, it is the bottom component of the anomalous axial current

$$\bar{D}^2 J_A \sim W_\alpha^2. \quad (3.7)$$

As we have explained in the previous sections, following anomalous currents is nontrivial. We will now see that supersymmetry helps us bypass this problem in a simple manner.

We note that the theory (3.1) has a nonanomalous  $R$  symmetry, and so we can associate an  $\mathcal{R}$  multiplet to this  $R$  symmetry along the flow. Using the formula (2.5) and the table in Eq. (3.1), we can identify  $U$  in the far UV in terms of the electric quarks as

$$U^{\text{UV}} = \left(-\frac{1}{2} + \frac{3N_c}{2N_f}\right)(QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger), \quad (3.8)$$

and in the IR we can express  $U$  in terms of the magnetic degrees of freedom using (2.8) and the  $R$  charges in Eq. (3.3),

$$U^{\text{IR}} = \left(1 - \frac{3N_c}{2N_f}\right)(qq^\dagger + \tilde{q}\tilde{q}^\dagger) - \left(2 - \frac{3N_c}{N_f}\right)MM^\dagger. \quad (3.9)$$

This shows that the operator (3.6) undergoes the following flow:

$$QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger \rightarrow \frac{2N_f - 3N_c}{3N_c - N_f}(qq^\dagger + \tilde{q}\tilde{q}^\dagger - 2MM^\dagger). \quad (3.10)$$

This is an exact result. In this formula (3.10) we have chosen the mesons and magnetic quarks to be canonically normalized.<sup>8</sup>

One interesting consequence of the above discussion is that, upon acting with  $\bar{D}^2$  on both sides of the mapping in (3.10), we find the physical relation between the electric and magnetic field strengths,

<sup>8</sup>Note that if one interprets (3.10) as the action of the anomalous axial current on the IR degrees of freedom, we find that the cubic superpotential of the magnetic theory,  $W_{\text{mag}} = qM\tilde{q}$ , is invariant.

$$W_{\alpha,\text{el}}^2 \rightarrow \frac{2N_f - 3N_c}{3N_c - N_f} W_{\alpha,\text{mag}}^2. \quad (3.11)$$

This is again an exact result.<sup>9</sup>

### Soft-SUSY breaking

We can immediately apply the results in (3.10) and (3.11) to study the mapping of soft terms in the electric theory to soft terms in the magnetic theory. To that end, consider deforming the UV Lagrangian by adding the bottom components of the current in (3.6) and the electric field-strength bilinear in (3.11) so that we give small squark and gaugino soft masses to the electric fields,

$$\begin{aligned} \delta \mathcal{L}_{\text{el}} &= -m^2 J_A | - m_\lambda (W_{\alpha,\text{el}}^2 + \text{c.c.}) \\ &= -m^2 (QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger) + m_\lambda (\lambda_{\text{el}}^2 + \text{c.c.}), \end{aligned} \quad (3.12)$$

where we take  $m^2$  and  $m_\lambda$  positive with  $m, m_\lambda \ll \Lambda_{\text{el,mag}}$ , and  $\Lambda_{\text{el,mag}}$  are the dynamical scales of the electric and magnetic theories, respectively.

Since the soft deformations in (3.12) are small (compared to  $\Lambda_{\text{el,mag}}$ ), we can treat the underlying dynamics of the theory as supersymmetric and work in the ‘‘probe approximation,’’ where the subleading  $\mathcal{O}(m/\Lambda)$  and  $\mathcal{O}(m_\lambda/\Lambda)$  corrections to the IR soft masses are neglected.<sup>10</sup> For simplicity, we also neglect possible contributions to scalar masses squared scaling like  $m_\lambda^2$ . These may be important for phenomenological applications, but we will not discuss them here.

In this approximation, we see from (3.10) and (3.11) that the magnetic deformation corresponding to (3.12) is

<sup>9</sup>The reader may wonder how this relates to the claim  $W_{\alpha,\text{el}}^2 \rightarrow -W_{\alpha,\text{mag}}^2$  made in [34] and elsewhere. The main point is that this mapping is derived by using the holomorphic scale matching relation, which means that  $W_{\alpha,\text{el}}^2 \rightarrow -W_{\alpha,\text{mag}}^2$  is only valid modulo trivial elements of the chiral ring. By the ABJ equation, the squares of the field strengths are themselves trivial in the chiral ring of the undeformed theory, and so the mapping  $W_{\alpha,\text{el}}^2 \rightarrow -W_{\alpha,\text{mag}}^2$  carries the same information as  $0 = 0$ . (The relation  $W_{\alpha,\text{el}}^2 \rightarrow -W_{\alpha,\text{mag}}^2$  has nontrivial content if the theory is deformed.) On the other hand, our result in (3.11) gives the physical normalization which can be measured, for instance, by studying correlation functions at long distances. Note that the exact result also has a sign flip in the free magnetic phase, so the interpretation of one coupling growing while the other decreases remains.

<sup>10</sup>In QCD this is what one does to follow quark masses in the chiral Lagrangian. In the chiral Lagrangian, however, there is an in calculable overall coefficient in the mapping. This in calculable coefficient can be expressed in terms of the mass of the physical pion, and the probe approximation amounts to expanding in  $\frac{m_\pi}{f_\pi}$ . In our case, since we solved for the mapping exactly, no in calculable coefficients arise.

$$\begin{aligned} \delta \mathcal{L}_{\text{mag}} &= -m^2 \cdot \frac{2N_f - 3N_c}{3N_c - N_f} (qq^\dagger + \tilde{q}\tilde{q}^\dagger - 2MM^\dagger) \\ &\quad + m_\lambda \cdot \frac{2N_f - 3N_c}{3N_c - N_f} (\lambda_{\text{mag}}^2 + \text{c.c.}). \end{aligned} \quad (3.13)$$

These results agree with [7,8,10]. Our derivation shows that the ability to map soft terms follows from the simple mapping of the electric and magnetic  $R$  symmetry.

Note that if all the masses in the UV are positive, then, in the IR, the magnetic squarks are tachyonic (we are in the free magnetic phase and so  $2N_f - 3N_c < 0$ ). It turns out that even the magnetic  $D$  terms and superpotential do not help to stabilize the magnetic squarks; for example, there is an instability along the direction  $q \sim 1, \tilde{q} = 0, M = 0$ .<sup>11</sup> Our approximation does not allow one to know where the theory settles.

However, it is interesting to note that we can stabilize the dynamics by considering a simple deformation of SQCD. To see this, consider weakly gauging baryon number with some small gauge coupling,  $g_B$ . Then, it is easy to prove that there are no instabilities which take us out of the calculable regime (as long as  $g_B$  is not too small). Indeed, one finds a vacuum with  $q \sim \frac{m}{g_B} \mathbb{1}, \tilde{q} = 0, M = 0$  and of course a similar vacuum with  $q$  interchanged with  $\tilde{q}$ . Therefore, all we need for calculability is that  $g_B$  is much larger than  $m/\Lambda$  but sufficiently smaller than all the other couplings in the theory. This vacuum breaks the magnetic gauge symmetry and Higgses baryon number, too. The remaining non-Abelian flavor symmetry is  $SU(N_f - N_c) \times SU(N_c) \times SU(N_f)$ . (Note the color-flavor locking phenomenon. Ideas along these lines thus present an opportunity for extending various recent studies such as [35–38] into the nonsupersymmetric domain.) If, on the other hand, the gauge coupling  $g_B$  is sufficiently large compared to the gauge and Yukawa couplings of the theory, a different stable vacuum appears, where  $q \sim \tilde{q} \sim m$  and  $M = 0$ . Both of these vacua will be mentioned again briefly in the last section, motivated by some possible phenomenological applications.

Finally, from our discussion above it is clear that we can consider the most general set of nonholomorphic soft terms in the UV by adding (3.2) and decomposing it into the soft terms associated with the conserved currents and the anomalous current we have discussed at length.

## IV. THE DEFORMED MODULI SPACE

When some of the symmetries of the short-distance theory are broken spontaneously, there are interesting subtleties in the flow of the  $U$  operator (2.5). In particular,

<sup>11</sup>By the equation  $q \sim \mathbb{1}$ , we mean that we choose the upper left  $(N_f - N_c) \times (N_f - N_c)$  block to be proportional to the unit matrix, and the rest of the entries to be zero. The same comment applies everywhere below.

the holomorphic plus antiholomorphic pieces of the type discussed in Sec. II appear.

The deformed moduli space of  $N_f = N_c$  SQCD [39] is a simple arena in which to study these ideas. Indeed, the quantum dynamics of SQCD with  $N_f = N_c > 2$  deforms the moduli space so that it is parametrized by baryons and mesons subject to

$$\det M - B\tilde{B} = \Lambda^{2N_c}. \quad (4.1)$$

Hence, some of the UV symmetries are necessarily spontaneously broken.

We will see below that  $U$  receives contributions from the corresponding Goldstone multiplets and that requiring invariance of  $U$  under the resulting nonlinearly realized symmetries both necessitates the inclusion of holomorphic plus antiholomorphic corrections to  $U$  that are quadratic in the Goldstone multiplets and, simultaneously, fixes their mixing with  $U$  exactly. We will also see a vacuum in which this ambiguity is not fixed by symmetries.

Even though the global symmetries are spontaneously broken, it is still straightforward to follow conserved currents to the IR.<sup>12</sup>

The anomalous current, is, of course, harder to follow. To proceed, we consider the following highly symmetric vacuum satisfying (4.1),

$$M = 0, \quad B = \tilde{B} = \Lambda^{N_c}. \quad (4.2)$$

This vacuum breaks the symmetry according to  $SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_R \hookrightarrow SU(N_f)_L \times SU(N_f)_R \times U(1)_R$ . The massless fluctuations in this vacuum are the meson matrix  $\delta M$  and the Goldstone superfield  $\delta b$  associated with  $U(1)_B$  breaking.

We are interested in finding the low energy limit of the axial current,  $J_A = QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger$ . Noting that all the chiral fields have vanishing  $R$  charge, we use (2.8), and immediately find that, up to holomorphic plus antiholomorphic pieces,  $U = \delta M \delta M^\dagger + \delta b \delta b^\dagger$ . Note, however, that this operator is not invariant under the nonlinear imaginary shift symmetry of  $\delta b$ . Therefore, we must replace  $\delta b \delta b^\dagger \rightarrow \frac{1}{2}(\delta b + \delta b^\dagger)^2$ , and we conclude that

$$QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger \rightarrow \text{Tr}(\delta M \delta M^\dagger) + \frac{1}{2}(\delta b + \delta b^\dagger)^2. \quad (4.3)$$

We see that the addition of a purely holomorphic and antiholomorphic piece quadratic in the Goldstone multiplet is forced on us. The answer (4.3) is exact in the deep IR; in particular, there are no further holomorphic ambiguities.<sup>13</sup>

<sup>12</sup>There could, however, be some complications. In addition to the one already mentioned, analogous to the complication in following the baryon current in QCD, there are also exotic cases when the ordinary linear multiplets are not globally well defined; see [24] and references therein.

<sup>13</sup>The  $Z_2$  interchange symmetry acting on the UV degrees of freedom as  $Q \leftrightarrow \tilde{Q}$ , with an appropriate action on the vector superfield, rules out the appearance of the linear term  $\delta b + \delta b^\dagger$ . The remaining linearly realized symmetries also force holomorphic contributions in  $\delta M$  to vanish.

Unlike the discussion of the previous section, just adding a soft deformation in the UV,  $\delta \mathcal{L} = -m^2(QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger)$ , is enough to end up with a stable vacuum in the IR. Equation (4.3) shows that all the meson fluctuations are massive, the real part of the baryon is massive as well, and the imaginary part is the ordinary Goldstone boson for  $U(1)_B$  breaking in this non-SUSY vacuum.

One can also consider adding a soft gaugino mass in the UV. Even though there are no gauge fields in the IR, this affects the infrared in a nontrivial way at leading order in the gaugino mass. We can hit (4.3) from both sides with  $\bar{D}^2$ . On the left-hand side we get the usual field strength squared operator from the ABJ equation, while on the right-hand side most terms vanish by the free equations of motion (neglecting irrelevant corrections from the Kähler potential). However, when we hit  $(\delta b^\dagger)^2$ , we get  $\sim \bar{\psi}_b \psi_b$ . In other words, up to an order one coefficient,  $\frac{1}{8\pi^2} W_\alpha^2 \rightarrow \bar{D}_\alpha b^\dagger \bar{D}^\alpha b^\dagger$ . As a result, a gaugino mass in the UV manifests itself in the IR via a mass term for the fermionic partner of the Goldstone boson.<sup>14</sup>

The deformed moduli space has another vacuum with an enhanced symmetry,

$$M = \Lambda \mathbb{1}, \quad B = \tilde{B} = 0. \quad (4.4)$$

The symmetry breaking here is  $SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_R \hookrightarrow SU(N_f)_V \times U(1)_B \times U(1)_R$ . In this vacuum, the massless fluctuations are the traceless mesons  $\delta M$  in the  $Adj_{(0,0)}$  representation, and the baryons  $\delta B$  and  $\delta \tilde{B}$  in the  $0_{(\pm N_c, 0)}$  representation.

Repeating the mapping of the axial current, we again find that some holomorphic terms in the mesons are necessarily induced with known coefficients. However, we now have an ambiguous chiral singlet operator of the form  $\delta B \delta \tilde{B}$ , whose mixing with  $U$  we cannot fix. We therefore add it with an unknown coefficient  $c$ ,

$$QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger \rightarrow \frac{1}{2} \text{Tr}(\delta M + \delta M^\dagger)^2 + \delta B \delta B^\dagger + \delta \tilde{B} \delta \tilde{B}^\dagger + c(\delta B \delta \tilde{B} + \text{c.c.}). \quad (4.5)$$

This ambiguity prevents us from making exact statements about the nature of this vacuum when we softly deform the theory in the UV.

The case of  $N_f = N_c = 2$  might be interesting for model building, so we comment on it, too. In the most symmetric vacuum, one finds the symmetry breaking pattern  $SO(6) \hookrightarrow SO(5)$ . The fluctuations are in two five-dimensional representations of  $SO(5)$ . This symmetry precludes any linear terms in the fluctuations from appearing in the map and so the symmetric point remains an

<sup>14</sup>Note that this fermionic mass term is enhanced by a loop factor compared to the gaugino mass. This could have some interesting phenomenological applications, because it is usually hard to generate large fermionic masses compared to scalars in the same multiplet. One possible connection to phenomenology could thus be through the problems revolving around the  $\mu$  term.

extremum upon softly deforming the theory in the UV by the bottom component of the axial current. With tools identical to those we have used above, one also finds that all the partners of the Goldstone bosons are stabilized and hence the most symmetric point is a local minimum. There is no ambiguity in quadratic holomorphic terms, and all the masses are calculable, as in all the examples we have studied besides (4.4).

## V. KUTASOV DUALITY

In the above sections we considered theories with only one nonconserved current in the UV—the current corresponding to the anomalous symmetry. In this section, we will analyze theories with more nonconserved currents in the UV. A simple example is given by adjoint SQCD with a superpotential for the adjoint,  $X$ . In such a case there are two independent nonconserved currents in the UV—the one corresponding to  $U$ , which sits in the same multiplet as the (unique) nonanomalous  $R$  symmetry, and a nonanomalous current which is explicitly broken by the superpotential.

The theories we will discuss in this section were studied in [40–42] and have the following particle content and symmetries:

	$SU(N_c)$	$SU(N_f) \times SU(N_f)$	$U(1)_R$	$U(1)_B$
$Q$	$\mathbf{N}_c$	$\mathbf{N}_f \times \mathbf{1}$	$1 - \frac{2}{k+1} \frac{N_c}{N_f}$	1
$\tilde{Q}$	$\bar{\mathbf{N}}_c$	$\mathbf{1} \times \bar{\mathbf{N}}_f$	$1 - \frac{2}{k+1} \frac{N_c}{N_f}$	-1
$X$	$\mathbf{N}_c^2 - \mathbf{1}$	$\mathbf{1} \times \mathbf{1}$	$\frac{2}{k+1}$	0

(5.1)

The superpotential for the adjoint has the form  $W = s_0 \text{Tr}(X^{k+1})$  and breaks the symmetry associated with the nonanomalous current

$$J_X = \frac{N_c}{N_f} (QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger) - XX^\dagger, \quad (5.2)$$

where  $\bar{D}^2 J_X \sim s_0 \text{Tr}(X^{k+1})$ . The other nonconserved current is just the anomalous (and, for  $k > 2$ , broken by the superpotential) current  $U$  associated with the  $\mathcal{R}$  multiplet,

$$U^{\text{UV}} = \left(-\frac{1}{2} + \frac{3}{k+1} \frac{N_c}{N_f}\right) (QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger) + \left(1 - \frac{3}{k+1}\right) XX^\dagger. \quad (5.3)$$

In what follows, we will focus mostly on the free magnetic phase ( $\frac{N_c}{k} < N_f < \frac{2N_c}{2k-1}$ ), where the dual description is a weakly coupled theory with the following massless fields:

	$SU(kN_f - N_c)$	$SU(N_f) \times SU(N_f)$	$U(1)_R$	$U(1)_B$
$q$	$\mathbf{kN}_f - \mathbf{N}_c$	$\bar{\mathbf{N}}_f \times \mathbf{1}$	$1 - \frac{2}{k+1} \frac{kN_f - N_c}{N_f}$	$\frac{N_c}{kN_f - N_c}$
$\tilde{q}$	$\overline{\mathbf{kN}_f - \mathbf{N}_c}$	$\mathbf{1} \times \mathbf{N}_f$	$1 - \frac{2}{k+1} \frac{kN_f - N_c}{N_f}$	$-\frac{N_c}{kN_f - N_c}$
$Y$	$(\mathbf{kN}_f - \mathbf{N}_c)^2 - \mathbf{1}$	$\mathbf{1} \times \mathbf{1}$	$\frac{2}{k+1}$	0
$M_j$	$\mathbf{1}$	$\mathbf{N}_f \times \bar{\mathbf{N}}_f$	$2 - \frac{4}{k+1} \frac{N_c}{N_f} + \frac{2}{k+1} (j-1)$	0

(5.4)

and the following superpotential:

$$W_{\text{mag}} = -\frac{s_0}{k+1} \text{Tr} Y^{k+1} + \frac{s_0}{\mu^2} \sum_{j=1}^k M_j \tilde{q} Y^{k-j} q. \quad (5.5)$$

Let us now consider the mapping of the currents of the theory to the IR. The mapping of the conserved currents proceeds trivially as before. The mapping of the  $U$  operator follows from our general discussion above with the non-trivial result that

$$\begin{aligned} U^{\text{IR}} = & \left(-\frac{1}{2} + \frac{3}{k+1} \frac{kN_f - N_c}{N_f}\right) (qq^\dagger + \tilde{q}\tilde{q}^\dagger) \\ & + \left(1 - \frac{3}{k+1}\right) YY^\dagger \\ & + \sum_j \left(-2 + \frac{6}{k+1} \frac{N_c}{N_f} - \frac{3(j-1)}{k+1}\right) M_j M_j^\dagger. \end{aligned} \quad (5.6)$$

While we are able to use our methods to map all the conserved currents and the nonconserved operator (5.3), there is one current whose mapping we cannot fix—namely, that of  $J_X$ . Being able to follow such an operator would amount, via  $\bar{D}^2 J_X \sim s_0 \text{Tr}(X^{k+1})$ , to following  $s_0 \text{Tr}(X^{k+1})$ , but since the latter vanishes in the chiral ring this is not straightforward (any formula obtained from chiral ring relations cannot be trusted since it contains the same information as  $0 = 0$ ).<sup>15</sup>

## VI. CONFORMAL THEORIES

In Sec. II we described the flow of  $U$  when the IR is given by some general SCFT, but so far we have focused mostly on theories with a free IR description. In this

<sup>15</sup>By matching chiral primaries, we can, however, show that the charge of  $Y$  under  $J_X$  is the same as that of  $X$ .

section we will briefly consider theories with an interacting IR fixed point.

Let us start from SQCD in the conformal window  $3N_c/2 \leq N_f < 3N_c$ . Now, the nonanomalous  $R$  symmetry of the ultraviolet (3.1) becomes the superconformal one in the IR, and so in the deep IR  $QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger$  flows to zero. But we would like to say a little more. At the onset of the conformal window  $N_f = 3N_c/2$ , the free fixed point in the IR is approached logarithmically, due to a marginally irrelevant operator. This means that  $QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger$  flows to zero in the deep IR logarithmically, too. [Indeed, the right-hand side of (3.10) vanishes upon substituting  $N_f = 3N_c/2$ .] However, in the bulk of the conformal window, the fixed points are approached by strictly irrelevant operators,<sup>16</sup> and so  $QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger$  flows to some operator of dimension  $>2$  in the IR SCFT, divided by an appropriate power of the strong scale,  $\Lambda$ .

Let us see what this implies for soft deformations of the theory. Suppose we softly deform the theory in the ultraviolet by  $\delta\mathcal{L} = -m^2(QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger)$ . Then, in the bulk of the conformal window, all the effects of this deformation in the infrared (say, at energy scales of order  $m$ ) are suppressed by powers of  $\Lambda$ , which can be thought of as an ultraviolet cutoff at low energies. Therefore, unlike the examples we have studied in the free magnetic phase, here the effects of a deformation at the scale  $m$  in the UV may become important only at much lower energy scales. For instance, this scale would be  $m^2/\Lambda$  if the first term appearing in  $U$  is a real operator in the SCFT of dimension 3 divided by  $\Lambda$ . This scenario can be thought of as a very close relative of the phenomenon that non-BPS operators obtain positive anomalous dimensions in SCFTs, which then lead to suppressed effects of non-SUSY deformations.<sup>17</sup>

A more interesting example to consider is adjoint SQCD without a superpotential. Some of the fields in the IR decouple and allow us to write an explicit expression for their contribution to  $U^{\text{IR}}$ . The matter content and representations of this theory are

	$SU(N_c)$	$SU(N_f) \times SU(N_f)$	$U(1)_R$	$U(1)'$	$U(1)_B$
$Q$	$\mathbf{N}_c$	$\mathbf{N}_f \times \mathbf{1}$	$1 - \frac{2N_c}{3N_f}$	1	1
$\tilde{Q}$	$\bar{\mathbf{N}}_c$	$\mathbf{1} \times \bar{\mathbf{N}}_f$	$1 - \frac{2N_c}{3N_f}$	1	-1
$X$	$\mathbf{N}_c^2 - \mathbf{1}$	$\mathbf{1} \times \mathbf{1}$	2/3	-1	0

(6.1)

<sup>16</sup>One can argue that this is true as follows. The conventional wisdom about the conformal window of SQCD is that, in the bulk of it, there are no accidental symmetries in the infrared. However, marginally irrelevant operators must violate some of the symmetries of the SCFT [27,43]. But since the RG flow preserves all the symmetries, we conclude that the conformal point is not approached via marginally irrelevant operators.

<sup>17</sup>Such a setup may even lead to accidental SUSY and may be of phenomenological interest. See the nice recent discussion in [44] and references therein.

Associated with the  $R$  symmetry in (6.1), one finds the axial anomaly operator in the UV,  $U^{\text{UV}} = (-\frac{1}{2} + \frac{N_c}{N_f}) \times (QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger)$ . We would like to find the IR end point of the flow for this operator.

The procedure summarized in (2.6) and (2.7) instructs us to identify the superconformal  $R$  symmetry in the IR, and, once this is done, the end point of the flow of  $(-\frac{1}{2} + \frac{N_c}{N_f}) \times (QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger)$  is determined: up to an overall factor of  $3/2$ , it is simply the global symmetry current operator that makes up for the difference between the  $R$  symmetry in (6.1) and the superconformal one (which can be determined from  $a$ -maximization in this case).

As has been discussed in great detail in [45,46], for small enough values of  $N_f/N_c$  there are also free fields in the low energy SCFT. For instance (at large  $N_c$ ), if  $N_f/N_c < (3 + \sqrt{7})^{-1}$ , then  $M_0 = Q\tilde{Q}$  becomes free. Upon lowering  $N_f/N_c$  further, more and more mesons of the form  $M_i = QX^i\tilde{Q}$  become free. Their superconformal  $R$  charge is therefore corrected to be  $2/3$ , and we can immediately use (2.6) to determine how they appear in the low energy expression of the operator  $(-\frac{1}{2} + \frac{N_c}{N_f})(QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger)$ ,

$$\begin{aligned} & \left(-\frac{1}{2} + \frac{N_c}{N_f}\right)(QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger) \\ & \rightarrow \sum_{j=0}^{P(N_f/N_c)} \left(1 - \frac{3R(M_j)}{2}\right) M_j M_j^\dagger + \dots \\ & = - \sum_{i=0}^{P(N_f/N_c)} \left(j + 2 - 2\frac{N_c}{N_f}\right) M_j M_j^\dagger + \dots \end{aligned} \quad (6.2)$$

Here the “...” stand for an operator [which is also a global current according to (2.6)] in the interacting SCFT module. We have only displayed the contributions from the free fields, because these are the ones that can be represented explicitly in terms of some well-defined degrees of freedom. Also,  $P(N_f/N_c)$  is defined to be the number of free fields for the given value of  $N_f/N_c$ . This function can be deduced from  $a$ -maximization.

Note that (6.2) implies that if we softly deform the UV theory by adding a mass squared for the electric scalars of the form  $\delta\mathcal{L} = -m_{\text{UV}}^2(QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger)$ , the free fields in the infrared acquire a leading-order mass squared of the form  $m_j^2 = (\frac{N_f}{N_c - N_f/2})(2N_c/N_f - 2 - j)m_{\text{UV}}^2$ . For instance, when  $M_0$  becomes free,  $N_f/N_c \leq (3 + \sqrt{7})^{-1}$ , such a soft deformation in the UV would stabilize it at the origin.

## VII. DISCUSSION AND OPEN QUESTIONS

In this paper, we have described a simple way to follow anomalous currents along the RG flow. In the context of Seiberg duality, this extends the map of operators to anomalous (nonchiral) current multiplets. We have also seen that there are some simple results for theories whose

low energy description is given in terms of an interacting SCFT. Beyond the general interest in understanding the maps of different operators under complicated RG flows, our study could be of phenomenological relevance in supersymmetric models of compositeness, and most obviously in models of composite electroweak symmetry breaking.

$$\begin{array}{cccc}
 & SU(N_c) & SU(2)_L & U(1)_Y \\
 H & \mathbf{N}_c & \mathbf{2} & \frac{1}{2} \\
 \tilde{H} & \bar{\mathbf{N}}_c & \mathbf{2} & -\frac{1}{2} \\
 \Phi_{i=1\dots N_c-1} & \mathbf{N}_c & \mathbf{1} & \frac{1}{2} \\
 \tilde{\Phi}_{i=1\dots N_c-1} & \bar{\mathbf{N}}_c & \mathbf{1} & -\frac{1}{2}
 \end{array} \quad (7.1)$$

It is worth presenting a simple example that illustrates how these results might be applied. Consider the model for a composite Higgs sector shown in Eq. (7.1), which consists of an SQCD theory with  $N_f = N_c + 1$  and  $N_c \geq 3$ , and  $U(1)_B$  being identified with hypercharge.

The confined phase of this theory has  $N_f = N_c + 1$  baryon/antibaryon pairs,  $B, \tilde{B} \equiv (h, \tilde{h}, \phi_{i=1\dots N_c-1}, \tilde{\phi}_{i=1\dots N_c-1})$ , and also mesons  $M$  of zero hypercharge, which can be identified as follows:

$$M \equiv \begin{cases} h_{i=1\dots N_c-1} = (H\tilde{\Phi}_i) \\ \tilde{h}_{i=1\dots N_c-1} = (\tilde{H}\Phi_i) \\ \mathcal{T} + \eta = (H\tilde{H}) \\ \eta_{ij} = (\Phi\tilde{\Phi}) \end{cases} \quad (7.2)$$

One also adds the usual tree-level superpotential

$$W^{(\text{conf})} = \tilde{B}MB - \Lambda^{3-N_f} \det M, \quad (7.3)$$

where we suppress flavor indices.

The nice feature of this model is that the hypercharge is identified with  $U(1)_B$ , which is therefore gauged. As we mentioned in Sec. III, when  $g_Y$  is of order unity and sufficiently large with respect to the Yukawa coupling, one obtains a minimum in which the baryons and antibaryons get vacuum expectation values (VEVs) of order  $m$ , but the mesons' VEVs are zero. This can naturally break  $SU(2)_L \times U(1)_Y \hookrightarrow U(1)_{\text{QED}}$ . Note that the  $SU(2)_L$  triplet does not obtain a VEV. This is phenomenologically desirable.

An obvious technical issue that needs to be taken care of is vacuum alignment, namely, forcing the theory to break the global symmetries in the required fashion, and avoiding other points on the Goldstone manifold. For this it is promising to consider explicit breaking of flavor in the SUSY-breaking operators. Then only the baryon/

antibaryon pair with the most negative mass squared in the IR gets a VEV, while the remaining modes are all massive. In particular, all the erstwhile Goldstone modes associated with the broken global flavor symmetries are stabilized (except for the  $R$  axion, which can be lifted by other means).

This kind of setup seems very advantageous. The scale of electroweak symmetry breaking is naturally of order the SUSY-breaking parameters. The stable EWSB minimum appears automatically, perhaps without some of the complications conventional minimal supersymmetric standard model electroweak symmetry breaking entails.

Note that there are many alternative possibilities: for example, by taking the minima with  $B \neq 0$  and  $\tilde{B} = M = 0$  that appear when  $g_B$  is smaller than the Yukawa coupling (up to some numerical constant), and embedding both  $h$  and  $\tilde{h}$  in the baryons  $B$  (and of course dropping the  $Y \equiv B$  identification), one can easily construct models without triplets. Development of these and similar ideas will be the subject of future work [47].

Our results may also have applications in the context of gauge mediation and more general supersymmetric technicolor model building as well. (See [48], and for some more modern work on the subject see [49–51] and references therein.) These mappings of operators could also be relevant in attempts to interpret the minimal supersymmetric standard model as the magnetic, low energy, theory of some completely different degrees of freedom. (This idea is due to [30], and a relation to coupling constant unification was pointed out recently in [52].) Perhaps, some of the applications for particle physics would require one to understand the map beyond the “probe approximation.” Moving beyond this approximation would also be an interesting theoretical question to investigate.

## ACKNOWLEDGMENTS

We would like to thank O. Aharony, I. Antoniadis, D. Green, and N. Seiberg for helpful conversations. S. A. A. acknowledges support from the Leverhulme Trust. M. B. would like to thank the high energy theory groups of the IFT at the Universidad Autónoma de Madrid, the École Normale Supérieure, and NORDITA for their wonderful hospitality during various stages of this project. The work of M. B. is funded by the ERC Advanced Grant No. 226371 and Contract No. PITN-GA-2009-237920. Z. K. gratefully acknowledges support from DOE Grant No. DE-FG02-90ER40542. Z. K. would also like to thank the theory group at CERN for its warm hospitality during the final stages of this project..

- [1] E. Witten, *Nucl. Phys.* **B223**, 422 (1983).
- [2] E. Witten, *Nucl. Phys.* **B156**, 269 (1979).
- [3] A. V. Manohar, *Phys. Rev. Lett.* **81**, 1558 (1998).
- [4] N. J. Evans, S. D. H. Hsu, and M. Schwetz, *Phys. Lett. B* **355**, 475 (1995).
- [5] O. Aharony, J. Sonnenschein, M. E. Peskin, and S. Yankielowicz, *Phys. Rev. D* **52**, 6157 (1995).
- [6] N. J. Evans, S. D. H. Hsu, M. Schwetz, and S. B. Selipsky, *Nucl. Phys.* **B456**, 205 (1995).
- [7] H.-C. Cheng and Y. Shadmi, *Nucl. Phys.* **B531**, 125 (1998).
- [8] N. Arkani-Hamed and R. Rattazzi, *Phys. Lett. B* **454**, 290 (1999).
- [9] A. Karch, T. Kobayashi, J. Kubo, and G. Zoupanos, *Phys. Lett. B* **441**, 235 (1998).
- [10] M. A. Luty and R. Rattazzi, *J. High Energy Phys.* 11 (1999) 001.
- [11] T. Kobayashi and K. Yoshioka, *Phys. Lett. B* **486**, 223 (2000).
- [12] A. E. Nelson and M. J. Strassler, *J. High Energy Phys.* 07 (2002) 021.
- [13] S. Ferrara and B. Zumino, *Nucl. Phys.* **B87**, 207 (1975).
- [14] Z. Komargodski and N. Seiberg, *J. High Energy Phys.* 06 (2009) 007.
- [15] Z. Komargodski and N. Seiberg, *J. High Energy Phys.* 07 (2010) 017.
- [16] M. A. Shifman and A. I. Vainshtein, *Nucl. Phys.* **B277**, 456 (1986).
- [17] X. Huang and L. Parker, *Eur. Phys. J. C* **71**, 1570 (2011).
- [18] N. Arkani-Hamed and H. Murayama, *J. High Energy Phys.* 06 (2000) 030.
- [19] K. Yonekura, *J. High Energy Phys.* 09 (2010) 049.
- [20] K. R. Dienes and B. Thomas, *Phys. Rev. D* **81**, 065023 (2010).
- [21] M. Magro, I. Sachs, and S. Wolf, *Ann. Phys. (N.Y.)* **298**, 123 (2002).
- [22] S. M. Kuzenko, *J. High Energy Phys.* 04 (2010) 022.
- [23] I. Antoniadis and M. Buican, *J. High Energy Phys.* 04 (2011) 101.
- [24] T. T. Dumitrescu, Z. Komargodski, and M. Sudano, *J. High Energy Phys.* 11 (2010) 052.
- [25] S. M. Kuzenko, *Eur. Phys. J. C* **71**, 1513 (2011).
- [26] D. Butter and S. M. Kuzenko, *J. High Energy Phys.* 12 (2010) 080.
- [27] D. Green, Z. Komargodski, N. Seiberg, Y. Tachikawa, and B. Wecht, *J. High Energy Phys.* 06 (2010) 106.
- [28] J. Polchinski, *Nucl. Phys.* **B303**, 226 (1988).
- [29] D. Dorigoni and V. S. Rychkov, arXiv:0910.1087.
- [30] N. Seiberg, *Nucl. Phys.* **B435**, 129 (1995).
- [31] I. Antoniadis and M. Buican, *Phys. Rev. D* **83**, 105011 (2011).
- [32] T. T. Dumitrescu and N. Seiberg (unpublished).
- [33] K. A. Intriligator and B. Wecht, *Nucl. Phys.* **B667**, 183 (2003).
- [34] K. A. Intriligator and N. Seiberg, *Nucl. Phys. B, Proc. Suppl.* **45**, 1 (1996).
- [35] Z. Komargodski, *J. High Energy Phys.* 02 (2011) 019.
- [36] M. Shifman and A. Yung, *Phys. Rev. D* **83**, 105021 (2011).
- [37] M. Shifman, W. Vinci, and A. Yung, *Phys. Rev. D* **83**, 125017 (2011).
- [38] M. Eto, T. Fujimori, M. Nitta, K. Ohashi, and N. Sakai, arXiv:1105.1547.
- [39] N. Seiberg, *Phys. Rev. D* **49**, 6857 (1994).
- [40] D. Kutasov, *Phys. Lett. B* **351**, 230 (1995).
- [41] D. Kutasov and A. Schwimmer, *Phys. Lett. B* **354**, 315 (1995).
- [42] D. Kutasov, A. Schwimmer, and N. Seiberg, *Nucl. Phys.* **B459**, 455 (1996).
- [43] B. Kol, *J. High Energy Phys.* 09 (2002) 046.
- [44] R. Sundrum, *J. High Energy Phys.* 01 (2011) 062.
- [45] D. Kutasov, A. Parnachev, and D. A. Sahakyan, *J. High Energy Phys.* 11 (2003) 013.
- [46] K. A. Intriligator and B. Wecht, *Nucl. Phys.* **B677**, 223 (2004).
- [47] S. Abel, M. Buican, and Z. Komargodski (work in progress).
- [48] M. Dine, W. Fischler, and M. Srednicki, *Nucl. Phys.* **B189**, 575 (1981).
- [49] B. A. Dobrescu, *Nucl. Phys.* **B449**, 462 (1995).
- [50] M. A. Luty, J. Terning, and A. K. Grant, *Phys. Rev. D* **63**, 075001 (2001).
- [51] H. Fukushima, R. Kitano, and M. Yamaguchi, *J. High Energy Phys.* 01 (2011) 111.
- [52] S. Abel and V. V. Khoze, *J. High Energy Phys.* 11 (2008) 024.