The Slowdown in American Educational Attainment

Elisa Keller *

March 2014

Abstract

Relative to those for high school graduates, lifetime earnings for college graduates are higher for more recent cohorts. At the same time, across successive cohorts born after 1950, there is a stagnation in the fraction of high school graduates that go on to complete a college degree. What explains this phenomenon? I formulate a life-cycle model of human capital accumulation in college and on the job, where successive cohorts decide whether or not to acquire a college degree as well as the quality of their college education. Cohorts differ by the sequence of rental price per unit of human capital they face and by the distribution of initial human capital across individuals. My model reproduces the observed pattern in college attainment for the 1920 to 1970 birth cohorts. The stagnation in college attainment is due to the decrease in the growth rate of the rental price per unit of human capital commencing in the 1970s. My model also generates the increase in lifetime earnings for college graduates relative to those for high school graduates observed across cohorts.

JEL: I24, J2, J3.

Key words: Education. College attainment. Human capital. Earnings growth.

*Durham University Business School. Email: elisa.keller2@durham.ac.uk. I am indebted to B. Ravikumar and Guillaume Vandenbroucke for their continued guidance and support. Gustavo Ventura provided many beneficial suggestions. I thank Maria Canon, George-Levi Gayle, Limor Golan, Diego Restuccia, Todd Schoellman, Yongseok Shin, and Michael Sposi for their helpful comments. I also thank the seminar participants at the 2011 Midwest Macro Meetings, 2013 North American Summer Meetings of the Econometrics Society, Durham University, Universidad de Alicante, and the University of Iowa.
1 Introduction

Throughout American history, almost every generation has acquired substantially more education than its parental generation. This is no longer true. Figure 1 shows the fraction of white males with a high school diploma that went on to complete a four-year college degree (hereafter “college”) for the 1920 to 1970 cohorts, which are grouped by year of birth. The fraction for the 1950 cohort was nearly twice as large as that for the 1920 cohort. However, for cohorts born after 1950, the fraction of high school graduates that completed college remained flat. These trends have been documented by, among others, Altonji, Bharadwaj, and Lange (2008) and Goldin and Katz (2008). In this paper, I ask the following question: What accounts for the trend observed in college attainment of white males and, in particular, the slowdown in college attainment starting with the 1950s cohorts?

I argue that changes in the growth rate of the rental price per unit of human capital (hereafter “price growth”) are crucial for generating the observed pattern of college attainment. I illustrate this point with a simple back-of-the-envelope calculation. Consider the earnings \( E^S_t \) of full-time workers of education level \( S \) at time \( t \):

\[
E^S_t = w_t \times h^S_t.
\]

In this identity, \( h^S_t \) is the quantity of human capital for education level \( S \in \{H,C\} \) (\( H \) stands for high school and \( C \) stands for college) at time \( t \), and \( w_t \) is the price per unit of human capital at time \( t \). Suppose individuals live for two periods and college involves sacrificing current earnings for future human capital, \( h^C_{t+1} \). Lifetime earnings of a high school graduate are:

\[
LE^H = w_t \times h^H_t + w_{t+1} \times h^H_{t+1}.
\]

Lifetime earnings of a college graduate are:

\[
LE^C = w_{t+1} \times h^C_{t+1}.
\]

Individuals choose the option that yields the highest lifetime earnings. Thus, college is chosen if

\[
LE^C \geq LE^H, \ i.e., \ \frac{h^C_{t+1}}{h^H_{t+1}} \geq \frac{w_t \times h^H_t}{w_{t+1} \times h^H_{t+1}} + 1.
\]

Figure 2 plots college-graduate lifetime earnings relative to high school-graduate lifetime earnings (hereafter “college premium”) for the 1920 to 1970 cohorts and shows that the premium has steadily increased starting with the 1940

\[1\] College attainment also remained flat for cohorts of white males born after 1970, but is not shown in Figure 1 (see Appendix A). Differently from the case of males, college attainment for females rose throughout the century, with only one brief stall during the 1950s cohorts (see Appendix A). This increase, however, was arguably part of a more secular trend in both education and labor force participation influenced by reasons beyond the scope of this paper. Although this paper deals only with the college attainment of white males, general equilibrium price effects induced by the evolving college attainment of other demographic groups and influencing the college decisions of white males are taken into account within the quantitative strategy.

\[2\] I acknowledge that other potential explanations such as changes in idiosyncratic earnings risk, changes in the progressivity of taxation, changes in credit constraints, and changes in individuals’ uncertainty about their innate ability can be important and deserve a quantitative assessment. However, I abstract from these potential alternative explanations in this article.
cohort. The college premium corresponds to \( \frac{k_{C_{t+1}}}{k_{H_{t+1}}} \). Assuming \( \frac{w_t \times h_{H_{t+1}}}{w_{t+1} \times h_{H_{t+1}}} \) is constant over time, the inequality will grow larger, which implies that more people will go to college. This would contradict Figure 1 for those born after 1950. Previous studies find the flat college attainment of those born after 1950 puzzling since the college premium has been increasing (see, among others, Card and Lemieux, 2001, and Castro and Coen-Pirani, 2013). Figure 3 plots the reciprocal of \( \frac{w_t \times h_{H_{t+1}}}{w_{t+1} \times h_{H_{t+1}}} \), i.e., gross earnings growth of high school graduates. Earnings growth drops significantly after the 1970s, the same time that the 1950s cohorts graduated from high school. I investigate whether the drop in earnings growth reconciles Figure 2 and Figure 1 for those born after 1950.

As from the simple back-of-the-envelope calculation, earnings are the product of prices and quantities, which are both unobservable. To explore the quantitative role of price growth for the pattern of college attainment, I develop a model of human capital accumulation in college and on the job that identifies prices and in turn produces the observed pattern of US college attainment. The model takes the rental price of human capital \( w \) as exogenous, and

---

3The college premium for a cohort is constructed as the ratio of median earnings for individuals in that cohort that graduated from college relative to those that graduated from high school only. Earnings for cohorts are measured over one year when individuals in the cohort are between ages 31 and 40, depending on data availability. Figure 2 reports the college premium for cohorts grouped in six-year bins. Patterns similar to those in Figure 2 are reported for other measures of the returns to college. Heckman, Lochner, and Todd (2008) report that cohort-based returns to college increased continuously over time for white men entering the labor market between 1960 and 1985.
produces endogenous patterns of earnings growth and the college premium along with the pattern of college attainment.

My model builds on Heckman, Lochner, and Taber (1998). It features life-cycle human capital accumulation à la Ben-Porath (1967) and a college choice. Individuals start off with a high school degree and they differ by their innate ability and their initial human capital. Each individual decides whether or not to acquire college education as well as the quality of his college education. Once schooling is completed, individuals join the labor market and can accumulate human capital on the job. Accumulation of human capital in college requires both time and goods (that is, college quality) as inputs, while accumulation on the job requires only time. Cohorts differ by the sequence of the rental price per unit of human capital (hereafter “price sequence”) they face (a time effect) as well as by the distribution of initial human capital across individuals (a cohort effect). A decrease in price growth influences the college decision in two ways. First, it decreases the returns to human capital investment and, therefore, the returns to college. Second, it increases the opportunity cost of human capital accumulation in college relative to that on the job because of the lower relative price of time. These two effects decrease the incentives to go to college more for individuals with low innate ability.

I use earnings from the National Longitudinal Survey of Youth (NLSY) for the cohorts 1961 through 1964 to calibrate the structural parameters of the model. I calibrate the price sequence and the evolution of the distribution of initial human capital to Integrated Public
Use Microdata Series (IPUMS-USA) earnings data for the cohorts 1884 through 1970. The shape of the distribution of individuals' endowments determines the elasticity of college attainment to changes in the price. Since endowments cannot be directly measured, I follow the strategy in Huggett, Ventura, and Yaron (2006) and include the college premium as an additional source of discipline. I use the NLSY dataset as it has a fixed-panel structure and allows me to infer endowments from life-cycle earnings.

The model produces three main quantitative findings. First, in line with Bowlus and Robinson (2012), I find that the rate of growth of the rental price per unit of human capital declines commencing in the 1970s. Price growth declines from 1.6 percent per year before the 1970s to -0.1 percent per year after the 1970s. Second, the model reproduces the observed pattern in US college attainment. For the 1920-1950 cohorts, the fraction of high school graduates attaining college increases from 15 percent to 37 percent in the model and from 18 percent to 38 percent in the data. For the 1950-1970 cohorts, the fraction decreases of 4 percent and it remains constant at the level of the 1950 cohort in the data. The slowdown in college attainment is generated almost exclusively by the slowdown in price growth. Successive cohorts born after 1950 face diminished returns to human capital investment on the job and a flat profile of returns to college quality, as the rental price of human capital grows very slowly after 1970. Third, the model generates the increase in the college premium for the 1920-1970 cohorts. The increase is generated by the slowdown in price growth and an increased dispersion of the initial human capital for successive cohorts born after 1940. Pre-1970s price growth fuels the increase in college attainment for the cohorts 1920 to 1950, which has a significant selection effect on the average innate ability and average human capital associated with college and high school. The decline in price growth of the 1970s causes selection into college to depend more on an individual’s innate ability over time. As the rental price of human capital at high school graduation increases and its growth over the lifecycle decreases, initial human capital becomes less important for the college decision; and the college decision is ruled more by an individual’s innate ability. The increase in dispersion of the initial human capital for successive cohorts born after 1940 is central to the recent rise in the college premium: in its absence, the college premium increases of only 2 percent after the 1950 cohort.

A few papers study the slowdown in college attainment in the US. Gemici and Wiswall (2014) find an increase in tuitions costs discouraged college attendance for the 1950s to 1960s cohorts. I consider an alternative formulation of my model where some college expenditures
might be beyond the control of individuals. I divert from the baseline by assuming individuals
must pay a fixed cost to complete college and the relative price of college education is not
constant over time. Despite the model fit on college attainment improves for the 1940s
cohorts under this alternative formulation, the elasticity of college attainment to price growth
does not change substantially. Donovan and Herrington (2013) explain the slowdown in
college attainment with a version of myopic expectations on the rise of the college premium.
In an alternative exercise, I relax the assumption of perfect foresight and consider the simple
scenario of individuals expecting the price growth observed at high school graduation to
persist during their lifetime. The model implied timing of the slowdown in college attainment
aligns with the data under this alternative scenario. Lastly, Castro and Coen-Pirani (2013)
conduct a comprehensive analysis of the slowdown in college attainment by considering
multiple channels. They also conclude tuitions and expectations do not hold a primary role
on the flat college attainment observed for the post-1950 cohorts. However, they find a small
effect of price growth on college attainment. By endogenizing human capital accumulation
in college and on the job, my framework allows for price growth to influence the returns to
college quality and human capital accumulation on the job along with the returns to time
investment in schooling.

The papers that are the closest to mine are Restuccia and Vandenbroucke (2013), Heckman,
Lochner, and Taber (1998), and Guvenen and Kuruscu (2010). Restuccia and Vandenbroucke
(2013) study the rise of educational attainment and the evolution of relative earnings across
education groups. I consider both the rise and the flattening in college attainment. Heck-
man, Lochner, and Taber (1998) conduct a qualitative analysis of the dynamics of college
attainment and earnings inequality resulting from skill-biased technical change. Guvenen
and Kuruscu (2010) perform a quantitative study along the lines of Heckman, Lochner, and
Taber (1998). Their results are consistent with the evolution of earnings inequality, the col-
lege premium, and the rise in college attainment. My paper replicates the pattern in earnings
inequality, the college premium, and both the rise and the flattening of college attainment
shown in the data.

The rest of the paper is organized as follows. Section 2 outlines the model and section 3
calibrates it. Section 4 details the results of the quantitative experiment. Section 5 concludes.
2 Model

I extend the Ben-Porath (1967) framework to include an explicit college decision and to let the rental price per unit of human capital change over time. Time is discrete and runs from $t = 1, 2, \ldots, T$. The economy is populated by individuals who live for 20 periods. Each period corresponds to two years of calendar time. Individuals enter the model as high school graduates at age 19, which is age 1 in the model. I use $\tau$ to denote a cohort: cohort $\tau$ is composed of individuals of age 1 at time $t + 19$. I use $j$ to denote age. Within a cohort, individuals are heterogeneous with respect to their innate ability, $z \in \mathbb{R}^+$, and their level of initial human capital $h_1 \in \mathbb{R}^+$, where the subscript indicates model age. Innate ability represents an agent’s ability to learn and is fixed over his lifecycle.\footnote{I interpret innate ability to reflect both endowment at birth and the influence of family background up to age 19, as in Carneiro and Heckman (2002).} Endowments are distributed according to a cumulative distribution function, $\Gamma_\tau(z, h_1)$. This function varies across cohorts on the initial human capital dimension. The marginal distribution of innate ability is time-invariant. Individual types are pairs $b \equiv (z, h_1)$ on the set $B = \mathbb{R}_+^2$. I assume that individuals observe their type before any decision is made and that credit markets are complete and there is no uncertainty.\footnote{I assume a frictionless credit market and abstract from borrowing constraint for two reasons primarily: (i) to keep the model as tractable as possible in order to investigate the role of the rental price per unit of human capital on college attainment, and (ii) in consideration of the evidence that, once family background factors are taken into account, borrowing constraints play only a minor role in the college decisions of the 1957-1965 cohorts, which are among those cohorts experiencing a stagnation in college attainment (see Carneiro and Heckman, 2002).}

Individuals are endowed with one unit of time that can be spent either on working or on human capital accumulation. They can accumulate human capital in college and on the job. The college enrollment decision is made by cohorts at age 1. Individuals decide whether or not to attend college as well as the quality of their college education. After schooling is completed, human capital can be accumulated on the job by subtracting productive time to work. Human capital is homogeneous between and within schooling types. There is one price that clears the human capital market, $w$. The price grows exogenously at rate $g_t$.\footnote{The model is in partial equilibrium, as I study the college decision given the exogenous rental price per unit of human capital. My approach can be viewed as the reverse of Krusell, Ohanian, Ros-Rull, and Violante (2000)’s approach, who study the evolution of skill prices given exogenous college decisions.} I use $R$ to denote the gross interest rate that is exogenously given. Each cohort differs by the price sequence it faces and by the distribution of initial human capital.
2.1 No-college Path

Individuals who decide not to go to college join the labor market right after high school graduation at age 1. They maximize the present value of earnings over their working lifetime by dividing available time between human capital accumulation, $i$, and work $(1 - i)$. The problem for an individual of type $(z, h_1) \in \mathcal{B}$, born in cohort $\tau$, on the no-college path is given by

$$
\max_{\{i_j\}_{j=1}^{20}} \sum_{j=1}^{20} \left( \frac{1}{R} \right)^{j-1} E_j
$$

s.t. 

- $E_j = w_{\tau + 2(j-1)}h_j(1 - i_j)$
- $h_{j+1} = f(z, h_j, i_j \mid H) + \delta h_j$
- $i \in [0, 1], \hat{\tau} = \tau + 19$

given $h_1$.

An individual’s earnings at age $j$, $E_j$, equal the product of the amount of human capital accumulated up to age $j$, the price of human capital at age $j$, and the fraction of time allocated to market work at age $j$. The cost of human capital investment on the job is forgone earnings. Earnings are adjusted downward by the fraction of time spent in human capital investment. The return to human capital investment is higher future earnings. New human capital is produced by combining the existing stock of human capital with time and innate ability. Following Ben-Porath (1967),

$$
f(z, h, i \mid H) = z(hi)^{\beta_H}.
$$

The subscript $H$ denotes the no-college path. The elasticity of human capital investment on the job, $\beta_H \in (0, 1)$, determines the degree of diminishing marginal returns of human capital investment. The productivity of human capital investment depends on an individual’s innate ability. This specification is widely used in both the empirical literature and the human capital literature (see, for example, Mincer, 1997 and Kuruscu, 2006). Finally, notice that nothing is lost when studying human capital accumulation decisions by abstracting from consumption and saving decisions. In particular, the focus on lifetime earnings maximization does not require the assumption of risk neutrality: any concave utility function implies the same human capital investment behavior.
I formulate the problem in the language of dynamic programming. The value function, \( V_j(h; z, w \mid H) \), gives the maximum present value of earnings at age \( j \) from state \( h \) for an individual of innate ability \( z \) who faces the life-cycle price sequence \( w \). In its recursive formulation,

\[
V_j(h; z, w \mid H) = \max_{h', i \in [0, 1]} w_j h (1 - i) + R^{-1} V_{j+1}(h'; z, w \mid H)
\]

\[
\text{s.t. } h' = z(hi)^{\beta_H} + \delta h.
\]

(1)

For \( \beta_H \in (0, 1) \) the problem is concave. Standard methods can be used to solve for the value function and the policy function for time investment in human capital \( i \).

The first-order conditions for human capital investment and working time imply the Euler equation:

\[
w_j h_j \leq \frac{z \beta_H h_j^{\beta_H} i_j^{\beta_H - 1}}{\Delta \ln h} \frac{w_j (1 + g_j)}{R} \left( \sum_{u=0}^{19-j} \delta^u \prod_{k=1}^{u} \frac{1 + g_j+k}{R} \right),
\]

(2)

for \( g_j = \frac{w_{j+1}}{w_j} - 1 \). The equation holds with equality for \( i \in (0, 1) \), otherwise individuals spend their entire time on human capital accumulation. In eq. 2, the left-hand side is the marginal cost of human capital accumulation, that is, forgone earnings; the right-hand side is the marginal benefit of human capital accumulation, that is, the present discounted value of the future stream of earnings derived from a marginal increase in the time attributed to human capital accumulation. The amount of time spent accumulating human capital depends on individual characteristics and prices. Individuals with higher innate ability invest more time accumulating human capital on the job and therefore have steeper earnings profiles. Individuals with higher human capital at high school graduation spend less time accumulating human capital on the job and therefore have flatter earnings profiles. Time allocation decisions are independent from price units. Because the cost of human capital accumulation on the job is forgone earnings, multiplying the price sequence by a positive constant increases the marginal benefit of and the marginal cost of human capital accumulation equally, leaving the trade-off between the two unaffected. The time-allocation decision depends instead on the shape of the price sequence, that is, the rate of growth of the price over the lifecycle. Because human capital investment involves sacrificing earnings today for higher human capital tomorrow, an increase in price growth increases the marginal benefit of human capital accumulation, leaving the marginal cost unchanged. The forward-looking
nature of eq. 2 implies that the overall stream of future price growth throughout the lifecycle influences current human capital investment.\footnote{I relax the assumption of perfect foresight in Section 4.1.}

The value of the no-college path for an individual of type \((z, h_1) \in B\), born in cohort \(\tau\), is:

\[
V_H(h_1, z, w_\tau) = V_1(h_1; z, w_\tau \mid H) = \left(\frac{1}{R}\right)^{j-1} V_j(h_j; z, w_\tau \mid H) = \left(\frac{1}{R}\right)^{j-1} w_{\tau+2(j-1)} \left[ a_j h_j + b_H \frac{z^{\tau-\beta_H}}{\tau-\beta_H} \right],
\]

where \(j\) denotes the first age at which earnings are strictly positive. The constant \(a_j\) represents the discounted lifetime return at age \(j\) to renting out a unit of human capital net of depreciation. For the case of no full-time accumulation on the job, that is, \(j = 1\), the first addend denotes the contribution of initial human capital to lifetime earnings, while the second addend adjusts lifetime earnings for the new human capital accumulated on the job throughout the lifecycle. The constant \(b_H\) is indexed by schooling, \(H\), since it depends on the elasticity of on-the-job accumulation, \(\beta_H\). Both constants \(a\) and \(b_H\) have closed-form solutions when the parameter values satisfy some restrictions. See Appendix B for details.

### 2.2 College Path

Individuals on the college path stay in college for two periods and join the labor market at age 3. When they start college, they pick the quality of their college education. After graduation from college, they maximize the present value of earnings over their working lifetime by dividing time between work and human capital accumulation, as with the no-college path. The problem for an individual of type \((z, h_1) \in B\), born in cohort \(\tau\), on the
college path is given by

$$\max_{\{i_j\}_{j=3}^{20}} \sum_{j=3}^{20} \left( \frac{1}{R} \right)^{j-1} E_j - \left( 1 + \frac{1}{R} \right)e$$

s.t. \[ E_j = w_{z+2(j-1)}h_j(1 - i_j) \]

\[ h_{j+1} = q(z, h_j, e) + h_j, \quad j = 1, 2. \]

\[ h_{j+1} = f(z, h_j, i_j | C) + \delta h_j, \quad j \geq 3. \]

\[ i \in [0, 1], \quad \hat{\tau} = \tau + 19 \]

given \( h_1 \).

A college graduate’s on-the-job human capital accumulation technology differs from the no-college case by the value of the elasticity of human capital investment, \( \beta_C \). I assume that college requires full-time investment, therefore earnings are zero for the first two periods for those in college. Human capital accumulation in college requires innate ability, college quality \( e \), and human capital as inputs. Individuals who invest more on their college quality acquire more human capital while in college given their endowments. Investment in college quality represents all sorts of college expenditures, such as tuition and fees, as well as the disutility associated with putting a certain effort in learning. I assume that college quality is chosen once and for all at the beginning of college and corresponding expenditures are paid in two equal amounts each period while in college. The in-college human capital accumulation function is\(^8\)

\[ q(z, h, e) = zh^\eta e^{1-\eta}. \]

Given human capital at college graduation, \( h_3(h_1, e) \), the on-the-job human capital accumulation problem for the college path is identical to the one for the no-college path up to the elasticity of human capital investment on the job, \( \beta_C \). The college quality problem can be formulated as \( \max e V_3(h_3(h_1, e); z, w_\tau | C) - \left( 1 + \frac{1}{R} \right)e \), for \( V_3 \) as in eq. 1 with \( j = 3 \). The first-order conditions are

\[ \left( 1 + \frac{1}{R} \right) = \Delta \frac{\partial h_3}{\partial e} \left( \frac{1}{R} \right)^2 \frac{\partial V_3(h_3; z, w_\tau | C)}{\partial h_3}, \]

\[ \Delta \text{ in } h_3 \text{ lifetime return to a unit of } h_3 \]

---

\(^8\)Similar functional forms for the technology of human capital accumulation in school have been considered by, among others, Manuelli and Seshadri (2010) and Erosa, Koreshkova, and Restuccia (2010).
that is,
\[
\left(1 + \frac{1}{R}\right) = \frac{\partial h_3}{\partial e} \left(\frac{1}{R}\right)^{j-1} w_{t+2(j+2)} a_j \left[ \prod_{u=4}^{j} (A\beta C_h)^{\delta C_i-1} + \delta \right],
\]
where \( j \) and \( a \) are defined as for the no-college path. The left-hand side of eq. 3 is the marginal cost of increasing college quality — that is, the present value of additional expenses derived from a marginal increase in college quality. The right-hand side of eq. 3 is the marginal benefit of increasing college quality — that is, the present discounted value of the future stream of earnings derived from a marginal increase in college quality. Individuals with higher innate ability and higher initial human capital invest more on college quality. Both a higher initial level and a higher growth over the lifecycle of the price imply a higher optimal college quality. When the price at high school graduation increases, the return to college quality increases proportionally with it, while the cost of college quality remains unaltered. When price growth increases, the benefit of human capital accumulation on the job increases (\( a \) increases in price growth) and so does the return to college quality, while the cost of college quality remains unaltered once again.

The value of the college path for an individual of type \((z, h_1) \in B\), born in cohort \(\tau\), is the discounted value of lifetime earnings net of total expenditures on college quality:
\[
V_C(h_1, z, w_\tau) = V_3(h_C(h_1, e^*); z, w_\tau | C) - \left(1 + \frac{1}{R}\right) e^*.
\]
\[
= \left(\frac{1}{R}\right)^{j-1} w_{t+2(j+2)} a_j \left[ a_j h_j + b^C \frac{1}{2} z^{1-\gamma_C} \right] - \left(1 + \frac{1}{R}\right) e^*,
\]
where \( e^* \) denotes optimal college quality (from eq. 3) and \( b^C \) is indexed by schooling, \( C \), because it depends on the elasticity of on-the-job accumulation as for the no-college case.

### 2.3 College Decision

Individuals within a cohort choose their education level upon graduation from high school. They do so based on their type, \((z, h_1)\), and the price sequence observed during their lifetime, \(w\). A college education is pursued if and only if
\[
V_C(h_1, z, w) \geq V_H(h_1, z, w).
\]
Let the indicator function \( 1(h_1, z, w) \) take the value of 1 if an individual pursues a college education and 0 if he does not. Thus,

\[
1(h_1, z, w) = \begin{cases} 
1, & \text{if (4) holds}, \\
0, & \text{otherwise}.
\end{cases}
\]

There are three assumed trade-offs between the college and no-college paths: (i) Human capital is not productive during college education but is when work is chosen. (ii) The technology for human capital accumulation in college is not the same as the technology for human capital accumulation on the job. (iii) The elasticity of human capital investment on the job differs between education levels. Each of these three trade-offs shape the effect of the price sequence on the college decision. The decision rule in eq. 4 can be rewritten as

\[
\left( \frac{1}{R} \right)^2 \left[ V_3(h_3,e^*); z, w \mid C \right] - V_3(h_3; z, w \mid H) \geq \left( 1 + \frac{1}{R} \right) e^* + V_1(h_1; z, w \mid H) - V_3(h_3; z, w \mid H),
\]

where \( e^* \) indicates the optimal level of college quality, as it results from eq. 3. In eq. 5, on the left-hand side are the gains of college — that is, the additional earnings received from age 3 onwards, and on the right-hand side are the costs of college — that is, total expenses on college quality and forgone earnings. By substituting the functional forms for the value function and earnings:

\[
\left( \frac{1}{R} \right)^2 w_3 \left\{ a_3 \left( h_3^C(h_1, e^*) - h_3^H(h_1, i_1, i_2) \right) + \left( b_3^C z^{1-\gamma_C} - b_3^H z^{1-\gamma_H} \right) \right\} \geq \left( 1 + \frac{1}{R} \right) e^* + w_1 h_1(1 - i_1) + \frac{w_2}{R} h_2^H (1 - i_2),
\]

where, for ease of notation, the time subscript on the price is replaced by the age subscript and I focus on the case of no full-time accumulation on the job — that is, \( j_0 = 1 \) for the no-college path and \( j = 3 \) for the college path. Starting from the latter of the three trade-offs, higher price growth over the lifecycle implies higher returns to human capital investment on the job. When the elasticity of human capital investment on the job for the college path is at least as big as that for the no-college path, the returns to college increase as price growth increases (notice the term \( b_3^C z^{1-\gamma_C} - b_3^H z^{1-\gamma_H} \)). On the second trade-off, a higher rental

\[\text{This mechanism is related to the reverse causality mechanism from anticipated TFP growth to educa-}\]
price per unit of human capital at high school graduation (henceforth “price level”) increases the net return to college quality, where else it leaves the net return to accumulating human capital on the job unchanged. The cost and benefit of human capital accumulation on the job both increase proportionally with the price level. However, only the benefit of college quality increases with the price level (compare the pair \(1/R^2 w_3 a_3 h_3^H\) and \(w_1 h_1 i_1 + 1/R w_2 h_2^H i_2\) to the pair \(1/R^2 w_3 a_3 h_3^C\) and \((1 + \frac{1}{R}) e^*\)). Lastly, on the first trade-off, the opportunity cost of accumulating human capital in college relative to accumulating human capital on the job decreases with higher price growth. The commitment to four-year full-time investment in human capital after high school graduation associated with college becomes relatively less burdensome as price growth increases.

Who goes to college? On average, individuals with high innate ability go to college. They are more productive learners, both in college and on the job, and therefore obtain higher returns from attending college. An individual’s initial human capital also influences the college choice. Because human capital and college quality are complements in the accumulation of human capital in college, individuals with higher initial human capital face bigger returns to college quality. However, because human capital is not productive during college, individuals with lower initial human capital have a lower opportunity cost of spending four years in college (lower forgone earnings). Notice that the importance of the margin associated with initial human capital in the college decision depends on the lifetime price sequence. As the price level increases and its growth over the lifecycle decreases, initial human capital becomes less and less important in the college decision; and the college decision is ruled more and more by an individual’s innate ability.

The fraction of cohort \(\tau\) acquiring a college education is determined from the cumulative distribution of initial endowments \(\Gamma_\tau(z,h_1)\):

\[
CA_\tau(w_\tau,\Gamma_\tau) = \int_{(z,h_1)\in R} 1(h_1, z, w_\tau) \ d\Gamma_\tau(z, h_1).
\]

3 Calibration

The quantitative strategy consists of setting the model in line with the path of unconditional earnings for the 1884-1970 cohorts and then exploring the model implications for educational attainment in Bils and Klenow (2000).
specific earnings and college attainment for those cohorts. The approach I follow to set the model’s parameters consists of three steps. First, I set a number of parameters a-priori. Second, I calibrate the parameters that are common to all cohorts (deep parameters) to a number of key moments in the NLSY79 dataset for the early 1960s cohorts. Third and last, I use IPUMS-USA data to infer the evolution of the rental price of human capital over time and that of the distribution of initial human capital across cohorts. The following subsections detail each of these steps.

I use two main data sources: the Integrated Public Use Micro Data Series for the United States (IPUMS-USA by Ruggles, Alexander, Genadek, Goeken, Schroede, and Sobek, 2010) and the 1979 National Longitudinal Survey of the Youth (NLSY79 by Bureau of Labor Statistics, 2002). IPUMS-USA provides quantitative information on long-time changes in earnings; the NLSY79 provides a constant panel that follows a limited number of individuals over time. I focus on a sample of white males in the labor force between the ages of 19 and 58 who have achieved either a high school diploma or a four-year college degree. Since earnings statistics in the data are computed for people in the labor force only, earnings statistics in the model ignore agents with full-time post-schooling investment. The IPUMS-USA data is not a fixed panel, and therefore I compute cohort data by constructing synthetic cohorts. See Appendix A for full descriptions of each data set and further details on sample selection.

### Table 1: Calibration. Deep parameters: parameters chosen without solving the model.

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>SYMBOL</th>
<th>VALUE</th>
<th>SOURCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model period</td>
<td></td>
<td>2 years</td>
<td></td>
</tr>
<tr>
<td>Gross interest rate</td>
<td>$R$</td>
<td>1.086</td>
<td></td>
</tr>
<tr>
<td>OTJ accumulation:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- college</td>
<td>$\beta_C$</td>
<td>0.871</td>
<td>Heckman, Lochner, and Taber (1998)</td>
</tr>
<tr>
<td>- high school</td>
<td>$\beta_H$</td>
<td>0.832</td>
<td>Heckman, Lochner, and Taber (1998)</td>
</tr>
<tr>
<td>- depreciation rate</td>
<td>$1 - \delta$</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

3.1 Deep parameters

I assume parameter values for which the literature provides evidence. The parameters that I calibrate without solving the model are reported in Table 1 together with the assigned values. I set the gross interest rate $R$ to 1.04 (annual rate). Estimates of the elasticity of human
Table 2: Calibration. Deep parameters: parameters computed by solving the model.

I calibrate the distribution of initial endowments, the in-college human capital accumulation function and the rental price of human capital in year 1980 to the age variation of unconditional earnings moments, college expenses, college attainment, and college premium for the 1961 to 1964 cohorts. I assume that the distribution of initial endowments, $\Gamma_\tau$, is jointly log-normal. This class of distributions is characterized by 5 parameters, $\{\mu_{\log(z)}, \mu_{\log(h_1)}, \sigma_{\log(z)}, \sigma_{\log(h_1)}, \rho\}$. Thus, the list of parameters that are calibrated within the model are:

$$\Lambda = \{\mu_{\log(z)}, \mu_{\log(h_1)}, \sigma_{\log(z)}, \sigma_{\log(h_1)}, \rho, \eta, w_{1980}\}.$$

I calibrate these parameters to the following statistics for the 1961-1964 cohorts:

1. Age variation of unconditional earnings moments: mean and coefficient of variation of the distribution of unconditional earnings at six points over the lifecycle, $j \in J = \{23 - 26, 27 - 30, 31 - 34, 35 - 38, 39 - 42, 43 - 45\}$ (Source: NLSY79.)

---

10 The human capital accumulation function in Heckman, Lochner, and Taber (1998) is $z_i^{\eta_S} h_i^{\beta_S}$ for $S \in \{C, H\}$. They work with four ability types and two education levels (high school and 4 years of college or more) and estimate the human capital accumulation function with NLSY79 data on white-male earnings for the period 1979-1993.

11 Huggett, Ventura, and Yaron (2006) show that, in this set-up, in terms of replicating life-cycle earnings dynamics, the gains of going from a parametric to a non-parametric approach for the distribution of initial endowments are not substantial.

12 At this stage only the distribution of initial endowments for the 1961 to 1964 cohorts is calibrated. The marginal distributions of initial human capital for the 1920 to 1970 cohorts are calibrated in the next section with the cohort-specific parameters.

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean earnings: growth btwn age 23-26 and 43-45</td>
<td>73.43%</td>
<td>73.36%</td>
</tr>
<tr>
<td>CV of earnings: age 23-26</td>
<td>0.499</td>
<td>0.474</td>
</tr>
<tr>
<td>CV of earnings: growth btwn age 23-26 and 43-45</td>
<td>83.85%</td>
<td>94.71%</td>
</tr>
<tr>
<td>College premium</td>
<td>1.40</td>
<td>1.58</td>
</tr>
<tr>
<td>College attainment</td>
<td>35.26%</td>
<td>34.02%</td>
</tr>
<tr>
<td>Average college expenditures</td>
<td>41.54%</td>
<td>39.88%</td>
</tr>
</tbody>
</table>

2. College premium for the 23- to 26-year-old: ratio of median earnings of four-year college graduates to median earnings of high school graduates (Source: IPUMS-USA.)

3. Education composition: fraction of high school graduates with a four-year college degree (Source: IPUMS-USA.)


There are a total of 14 targets. Formally, the calibration strategy consists of minimizing the following equation:

$$\min_{\Lambda} \sum_{u=1}^{14} \left( \frac{x_u(\Lambda)}{\bar{x}_u} - \tilde{x}_u \right)^2.$$  

For a given $\Lambda$, I compute the model moments, $x_u(\Lambda)$, that correspond to the targets described above, $\tilde{x}_u$.

Even though the parameter values are chosen simultaneously to match the data targets, each parameter has a first-order effect on some targets. The elasticity of substitution of the in-college human capital accumulation function is disciplined by data on college expenses. Data on college expenses use Trends in College Pricing (The College Board, 2007) data on average tuition and fees for private and public colleges in the United States. The rental price of human capital in year 1980 is important for matching the educational composition. The

---

13 Earnings in target 1 are normalized to mean earnings at ages 23-26. Because of the choice of units for the rental price of human capital, it is unreasonable to expect the model to match the level of earnings.
moments in targets 1 and 2 discipline the distribution of initial endowments. The argument
for identification behind this exercise follows Huggett, Ventura, and Yaron (2006).

The only source of earnings inequality in the model is due to initial endowments. An individual’s type \((z, h_1)\) implies a profile of human capital accumulation over the lifecycle and therefore a profile of earnings over the lifecycle. Within a cohort, a distribution of types maps into a distribution of earnings over the lifecycle. Thus, initial endowments can be identified through the evolution of the distribution of earnings over the lifecycle. The key assumption is that systematic differences in growth rates are the major driving force behind earnings dynamics over the lifecycle. This assumption is supported by empirical studies that estimate earnings processes from micro data sets (see, for example Guvenen, 2009). The NLSY79 dataset has a fixed-panel structure and allows me to infer initial endowments from life-cycle earnings. Lastly, I include the college premium as an additional source of discipline. The college premium contains information on the college-selection mechanism in terms of the \((z, h_1)\) types that choose college. Further details on the identification of each parameter of the distribution of initial endowments in the context of my model are in Appendix C.

The model is solved numerically. I simulate the earnings and schooling paths of 100,000 individuals in each of the 1961-1964 cohorts. The values of calibrated parameters are reported in Table 2, while the model’s performance on targets is reported in Table 3 and shown in
Table 4: Results. Life-cycle earnings dynamics for high school graduates (HS) and for college graduates (COL), 1961-1964 cohorts. Source: NLSY-79, IPUMS-USA, The College Board (2007), and the author.

<table>
<thead>
<tr>
<th>Earnings:</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean, growth btwn age 27-29 and 43-45</td>
<td>2.18% 42.12%</td>
<td>1.84% 37.58%</td>
</tr>
<tr>
<td>CV, growth btwn age 27-29 and 43-45</td>
<td>18.01% 40.18%</td>
<td>0.02% 35.70%</td>
</tr>
<tr>
<td>Median/mean, age 27-29</td>
<td>1.080 1.028</td>
<td>1.058 1.066</td>
</tr>
<tr>
<td>Median/mean, growth btwn age 27-29 and 43-45</td>
<td>-2.19% 12.76%</td>
<td>-0.50% 11.44%</td>
</tr>
</tbody>
</table>

Figure 4. Overall, the model is successful in matching the data. The mean age-earnings profile is reproduced in its growth and concavity. This results from human capital investment being disproportionately more convenient at younger ages. The life-cycle pattern of the coefficient of variation is also well reproduced. The coefficient of variation of earnings at ages 23-26 is 0.5 in the model as well as in the data. The college premium is slightly overestimated: it equals 1.58 in the model and 1.40 in the data. College premium could be delivered exactly at the price of a worse fit on the lifecycle pattern of the coefficient of variation of earnings. A smaller dispersion of initial human capital and a smaller correlation between innate ability and initial human capital are needed to match the college premium exactly. However, dispersion of initial human capital together with dispersion of innate ability are key to match the lifecycle pattern in the coefficient of variation of earnings.

Results: 1961-1964 cohorts I assess the merit of the model based on moments that are not targets of the calibration exercise. I pick those moments to my evaluation of the model as a model of life-cycle earnings in a context of a college choice. Figure 5 and table 4 display the model performance on the age variation of education-specific earnings moments for the 1961-1964 cohorts. The first panel shows the mean age-earnings profile for high school graduates and that for college graduates. First, notice that in the data the mean age-earnings profile for high school graduates is essentially flat, while that for college graduates has a considerably positive slope. Mean earnings of high school graduates grow 2 percent between the ages of 27 and 45 in the data, while those of college graduates grow of 42 percent. The model generates this fact as a result of positive association between college and innate ability. The average innate ability of college graduates is 0.2281, while that for high school graduates is 0.1621. The model predicts that, absent heterogeneity in human capital, agents with high innate ability have steeper profiles of human capital accumulation than agents with
Figure 5: Results. Life-cycle earnings dynamics for high school graduates and for college graduates, 1961-1964 cohorts. Earnings are normalized to mean earnings at ages 23-26. Data (solid lines) vs. Model (dashed lines). Source: NLSY79 and the author.

low innate ability. The difference between life-cycle earnings growth for college graduates and that for high school graduates is over-predicted by the model. Between the ages of 27 and 45, college graduate earnings grow 36 percentage points more than high school graduate earnings in the model and 40 percentage points more in the data.\footnote{One extension of the model carrying the necessary degrees of freedom to match earnings growth for both high school graduates and college graduates features education-specific prices of human capital, i.e., a price for high school human capital and a price for college human capital. I am not pursuing this extension because (i) of the parsimony of the single-price model, (ii) the single price model performs quite well on education-specific earnings moments, and (iii) in the single price model, education-specific earnings moments can be used as a metric of the merit of the model.} The model is consistent with a faster rise in the dispersion of earnings over the lifecycle for college graduates relative
to that for high school graduates. This is because college graduates have larger mean and larger dispersion of innate ability than high school graduates and the incentives to human capital accumulation increase more than proportionally with an individual’s innate ability. Finally, the model under-estimates the growth of the coefficient of variation of earnings over the lifecycle for both education groups and over-estimates its differential between college and high school graduates of 10 percentage points. To further check the model implications on life-cycle earnings inequality, I consider the performance of the model on an additional moment: the asymmetry of the earnings distribution measured by the ratio of median to mean earnings. The model generates life-cycle patterns of the asymmetry of the earnings distribution close to the data. Right skewness follows because incentives for human capital investment increase more than proportionally with an individual’s innate ability (see eq. 7 and recall that \( \beta < 1 \)). The positive association of innate ability with college is the reason for: (i) a higher average life-cycle skewness of college graduate earnings relative to that of high school graduate earnings and (ii) a higher rise in the skewness of college graduate earnings over the lifecycle relative to that of high school graduate earnings.

### 3.2 Cohort-specific parameters

Cohorts exogenously differ by two dimensions: they face different life-cycle price sequences (a time effect) and they face different distributions of initial human capital across individuals (a cohort effect).\(^\text{15}\) I calibrate time and cohort effects to replicate the evolution of unconditional earnings moments, first and second moments, for the 1884 to 1970 cohorts.

The price sequence is identified with data on life-cycle earnings growth for successive cohorts. Consider mean earnings of individuals in cohort \( \tau \) at age \( j \)

\[
E_{\tau j} = w_{\tau + 2(j-1)}h_{\tau j}
\]

where \( h_{\tau j} = \int_{(z,h_1)} h_{\tau j}(1 - l_{\tau j})d\Gamma_{\tau}(z,h_1) \). Lifecycle earnings growth for cohort \( \tau \) is measured

\(^{15}\)The distribution of innate ability is assumed to stay constant across cohorts. This is possibly a restrictive assumption for the cohorts born between 1920 and 1940 considering the substantial expansion in high school education that happened during this period. Within the framework of this paper, exogenous changes in the distribution of innate ability of high school graduates cannot be separately identified from those in the distribution of human capital of high school graduates due to data restrictions.
by the change in mean earnings between age \( j \) and \( j' > j \):

\[
\frac{E_{\tau j'}}{E_{\tau j}} = (1 + g_w)^j' - j - 1 \tilde{h}_{\tau j'} = (1 + g_w)^j' - j - 1 (1 + res_1(g_w, g_\mu, g_\sigma))
\]

where \( res_1(\cdot) \) is a residual arising from human capital accumulation, \( g_w \) is price growth, \( g_\mu \) is growth in mean initial human capital, and \( g_\sigma \) is growth in the standard deviation of initial human capital. I am assuming these three growth rates to be constant for ease of exposure; this assumption is dropped in the actual calibration exercise. If there is no human capital accumulation on the job, lifecycle earnings growth equals \( 1 + g_w \) and perfectly identifies price growth. As the age at which life-cycle earnings growth is measured increases, the benefits of human capital accumulation decrease, and life-cycle earnings growth more closely mimics price growth (see Appendix C for further details). Therefore, I use life-cycle earnings growth late in the lifecycle for the 1884 to 1958 cohorts to discipline price growth (Figure 6, panel (c), solid lines). This methodology for recovering price growth was originally proposed by Heckman, Lochner, and Taber (1998) and more recently used by Bowlus and Robinson (2012) to measure the prices of human capital across various education levels.

The evolution of the distribution of initial human capital is identified with data on mean and dispersion of cross-sectional earnings over successive years. Cross-sectional earnings growth at age \( j \) is measured by the change in average earnings between \( j \)-year olds in cohort \( \tau \) and \( j \)-year olds in cohort \( \tau' \):

\[
\frac{E_{\tau j'}}{E_{\tau j}} = (1 + g_w)^j' - j - 1 \tilde{h}_{\tau j'} = (1 + g_w)^j' - j - 1 (1 + g_\mu + res_2(g_w, g_\mu, g_\sigma))
\]

where \( res_2(\cdot) \) is a residual arising from human capital accumulation. If there is no human capital accumulation on the job and college augments an individual’s initial human capital of \( \Delta \), cross-sectional earnings growth equals \( (1 + g_w)(1 + g_\mu) \frac{\Delta'}{\Delta} \) for \( \Delta' = \Delta CA_{\tau} + 1(1 - CA_{\tau}) \), and identifies changes in average initial human capital given \( g_w \) and a path of college attainment. Earnings dispersion is measured by the coefficient of variation of earnings. Thus, growth of cross-sectional earnings dispersion at age \( j \) is measured by the change in earnings dispersion between \( j \)-year olds in cohort \( \tau \) and \( j \)-year olds in cohort \( \tau' \):

\[
\frac{\sigma_{h_{\tau j'}}}{\sigma_{h_{\tau j}}} = \frac{1 + g_\sigma + res_3(g_w, g_\mu, g_\sigma)}{1 + g_\mu + res_2(g_w, g_\mu, g_\sigma)}
\]
where \( \sigma_{\tilde{h}_{\tau j}} = \int_{(z, h_1)}(h_{\tau j}(1 - l_{\tau j}) - \tilde{h}_{\tau j})d\Gamma_\tau(z, h_1) \) and \( \text{res}_3(\cdot) \) is a residual arising from human capital accumulation. Given a price sequence and a sequence of means and standard deviations of initial human capital, the model produces a sequence of mean and standard deviation of cross-sectional earnings. Thus, I discipline the evolution of distributions of initial human capital with data on the growth in mean and coefficient of variation of cross-sectional earnings for a specific age group over the 1940-2008 period (Figure 6, panels (b) and (c), solid lines).

![Figure 6](image_url)

(a) Earnings growth, cross-section
(b) Growth of cross-sectional earnings dispersion
(c) Earnings growth, lifecycle

Figure 6: Model fit. Earnings: life-cycle growth, cross-sectional growth, and growth of cross-sectional dispersion. Growth rates are calculated as decennial changes and presented at annual rates. Data (solid lines) vs. Model (dashed lines). Source: IPUMS-USA and author.
The calibration targets are collected in Figure 6. Each of them can be summarized quite well by two distinct growth rates: (i) pre-1970s and post-1970s growth for the time-series data and (ii) pre-1939 and post-1939 growth for the cross-cohort data. The calibration targets are therefore condensed in the following six moments:

1. growth of median earnings of white males between the ages of 31-40 for the 1940-1970 period and the 1980-2008 period (source: IPUMS-USA, Figure 6a),

2. growth of the coefficient of variation of earnings of white males between the ages of 31-40 for the 1940-1970 period and the 1980-2008 period (source: IPUMS-USA, Figure 6b), and

3. growth of average earnings between the ages 39-45 and 49-55 for the 1884-1920 cohorts and the 1924-1958 cohorts (source: IPUMS-USA, Figure 6c).

The age group in targets 2 and 3 is chosen to match the age group at which the college premium is measured, since the evolution of the distribution of initial human capital is a key determinant of the path of the college premium.

I structure the paths of the time effect and the cohort effect in the model to match the structure of the target moments in the data. That is, I allow a trend break to accour (i) in price growth in year 1970, (ii) in the growth of the mean of the distribution of initial human capital for the 1939 cohort, and (iii) in the growth of the standard deviation of the distribution of initial human capital for the 1939 cohort. More formally, the paths are as follows:

• for price growth:

\[
x_t = \begin{cases} 
  x_{t-1}[1 + g_{w,1}] & t \leq 1969 \\
  x_{t-1}[1 + \omega g_{w,1} + (1 - \omega)g_{w,2}] & t \in [1970, 1979] \\
  x_{t-1}[1 + g_{w,2}] & t \geq 1980,
\end{cases}
\]

for the distribution of initial human capital:

\[
x_{\tau} = \begin{cases} 
  x_{\tau-1}[1 + g_{x,1}] & \tau \leq 1939 \\
  x_{\tau-1}[1 + g_{x,2}] & \tau \geq 1940,
\end{cases}
\]

where \( x = \{\mu, \sigma\} \) and \( \tau \) indicates the year of birth of the cohort.

There is a total of 6 targets for 6 unknowns. Formally, the calibration strategy consists of solving a system of 6 equations in 6 unknowns. For a given \( \Gamma = \{g_{1,x}, g_{2,x}\}_{x=\{w,\mu,\sigma\}} \), I compute the model moments, \( X(\Gamma) \), that correspond to the targets described above. I then solve for the zero of the function \( F(\Gamma) \) defined by

\[
F(\Gamma) = \tilde{X} - X(\Gamma),
\]

where \( \tilde{X} \) are the targets described above.

I simulate the earnings and schooling path for 100,000 individuals in the 1884-1970 cohorts. Figure 6 and table 5 shows the performance of the model on targeted moments. From Figure 6, the model is able to replicate the main patterns in the targets by allowing just one trend break for the cohort effect and one trend break for the time effect. The calibrated \( g_w \) pair is \( \{1.57\%, -0.15\%\} \), calculated as annual rates. The calibration implies a slowdown in price growth starting in the 1970s.\(^{16}\) The estimated pattern of price growth is in line with Bowlus and Robinson (2012)’s estimates in both direction and magnitude. Bowlus and Robinson (2012) find a generalized trend break in the growth rates of the rental prices of human capital for various education levels in years 1974-1975. In their estimates, price growth for college and high school graduates declines of 1.97 percentage points (from an average growth of 1.47% to -0.05%), compared to my 1.72 percentage points decline. In Section 4.1, I explore the implications of the model when price growth is set to Bowlus and Robinson (2012)’s estimates.

The calibrated changes in the mean and standard deviation of the distribution of initial human capital are \( g_{\mu} = \{0.33\%, -0.05\%\} \) and \( g_{\sigma} = \{-2.94\%, 1.80\%\} \), calculated as annual

---

\(^{16}\)Possible reasons for the slowdown in price growth include a slowdown in productivity growth, which influences the demand of human capital. Also, an increase in the supply of human capital following the increase in female labor force participation, the increase in average years of schooling of females and non-white males, and the increase in cohorts size (the baby boom) can also have contributed to the slowdown in price growth.
rates. What is the significance of a change in the distribution of initial human capital over cohorts? An individual’s human capital is the amount of knowledge he possesses. Hence, the distribution of initial human capital is a measure of the “quality” of the high school graduates. The calibration implies an increase in the average “quality” of successive cohorts of high school graduates followed by a decline.\textsuperscript{17} This pattern is consistent with anecdotal evidence presented by Taubman and Wales (1972) and Bishop (1989) on cognitive skills of high school graduates. Taubman and Wales (1972) observe that test scores of high school graduates decline starting with the late-1920s cohorts, after increasing from the beginning of the century. Bishop (1989) reports a decrease in the average scores of high school graduates on normed tests, such as the ITED and ITBS, starting with the late-1940s cohorts and following 50 years of uninterrupted improvement. Lastly, the calibration implies that the dispersion of initial human capital increases starting with the 1940s cohorts. Even though the calibration is set to replicate average changes in earnings dispersion for the pre-1970s and post-1970s periods, the model matches the pattern of earnings dispersion for the various years within each period quite well.

\textsuperscript{17}An evident reason for the decline in the quality of successive cohorts of high school graduates is the expansion in high school education that happened between the 1920 and the 1940 cohorts. Among those born in 1920, the fraction of white males with at least a high school diploma was 57 percent. This fraction was 82 percent for those born in 1940. A positive correlation between schooling and innate ability and/or human capital, as it transpires from evidence on tests scores, implies that large changes in high school attainment can potentially have a significant selection effect on the average innate ability and average human capital associated with high school education.

<table>
<thead>
<tr>
<th>Earnings:</th>
<th>Data Grw 1</th>
<th>Grw 2</th>
<th>Model Grw 1</th>
<th>Grw 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross-sectional growth</td>
<td>2.71%</td>
<td>-0.25%</td>
<td>2.74%</td>
<td>-0.23%</td>
</tr>
<tr>
<td>Cross-sectional growth of dispersion</td>
<td>-2.00%</td>
<td>0.86%</td>
<td>-2.03%</td>
<td>0.91%</td>
</tr>
<tr>
<td>Life-cycle growth</td>
<td>1.79%</td>
<td>-0.07%</td>
<td>1.77%</td>
<td>-0.07%</td>
</tr>
</tbody>
</table>

Table 5: Model fit. Earnings: life-cycle growth, cross-sectional growth, and growth of cross-sectional dispersion. The column Grw 1 indicates the average growth rate for the pre-1970s period for cross-sectional data and for the pre-1939 cohorts for life-cycle data. The column Grw 2 indicates the average growth rate for the post-1970s period for cross-sectional data and for the for the post-1939 cohorts for life-cycle data.
4 Results

The main results of the paper are in terms of college attainment and college premium. In this section, I present the model implications for the patterns of college attainment and college premium for the 1920-1970 cohorts and I investigate the quantitative contribution of changes in the rental price per unit of human capital along with changes in the distribution of initial human capital to those patterns.

College attainment of the 1920-1970 cohorts is shown in Figure 13a and summarized in Table 6, column “Model/Baseline”. The model generates an increase and subsequent flattening of college attainment. After calibration, the model indicates that 36 percent of the 1961-1964 cohorts earned a college degree, which is close to the data. The fraction of high school graduates in the 1920-1950 cohorts that earned a college degree increases from 15.0 percent to 37.0 percent in the model and from 17.8 percent to 37.7 percent in the data. The positive trend contracts starting with the early-1950s cohorts. From the 1950 cohort to the 1970 cohort, college attainment decreases from 37.0 percent to 33.4 percent in the model, but increases from 37.7 percent to 39.7 percent in the data. Overall, for the 1920 to 1950 cohorts, the fraction of college graduates increases each cohort of 2.1 percentage points on average in the model and of 1.2 percentage points in the data. In contrast, that for the 1950-1970 cohorts decreases of 0.4 percentage points in the model and of 0.1 points in the data. The flat college attainment the model generates for the post-1950 cohorts is a critical result of the paper. However, the model implies a flat college attainment for the 1940s cohorts also. In Section 4.1, I discuss the role of price expectations on the college decision and conclude that if the slowdown in price growth was not foreseen by individuals, the slowdown in college attainment starts with the late 1940s cohorts both in the model and in the data.

College attainment is mostly driven by the path of the rental price per unit of human capital (the time effect). Figure 8 shows a decomposition exercise of the time and cohort effects on the pattern of college attainment. In a first experiment (“Time effect only”), I keep the initial human capital distribution of each cohort the same, so that the only difference between cohorts is the life-cycle price sequence. The resulting pattern of college attainment is almost the same as that in the baseline exercise. Individuals born in the 1920s face a steeper price

\[\text{The distribution of initial endowments across individuals is most important in determining the response of college attainment to changes in the rental price per unit of human capital. See, among others, Athreya and Eberly (2010), whose paper deals with asymmetric returns to college while investigating the magnitude of the response of college attainment to changes in the college premium.}\]
sequence than those born in the 1950s and therefore have a higher return to human capital investment on the job. However, those born in the 1950s face also a higher price at high school graduation than that faced by those born in the 1920s. This makes college a better deal for the later cohorts because of the higher return to college quality. Individuals born after 1950 face both diminished returns to human capital investment on the job and a flat profile of returns to college quality, as the rental price of human capital grows very slowly after 1970. Thus, college attainment flattens. Castro and Coen-Pirani (2013) contemplate the role of a decline in price growth on the slowdown in college attainment and find the magnitude of this effect to be relatively small. In their framework, price growth influences the college decision via the return to time investment in schooling. My framework allows price growth to also influence the returns to college quality and human capital accumulation on the job by endogenizing human capital accumulation in college and on the job.

The evolution of the distribution of initial human capital across cohorts (the cohort effect) plays only a minor role in the slowdown in college attainment. Figure 8, “Cohort effect only”, shows college attainment when price growth is kept constant to its average between pre-1970s growth and post-1970s growth, so that the only differences between cohorts are the initial human capital distribution and the price at age 19. College attainment does not slow down in this second experiment.

Figure 7: Results. College attainment and the college premium in the United States. Data (solid lines) vs. Model (dashed lines). Source: IPUMS-USA and author.
Table 6: Results. Average change in college attainment. The table reports the slope coefficient resulting from regressing the fraction of high school graduates with a college degree in a cohort on a constant and a cohort trend. Source: IPUMS-USA and author.

<table>
<thead>
<tr>
<th>Birth cohort</th>
<th>Data Baseline</th>
<th>Model Time effect only</th>
<th>Model Cohort effect only</th>
</tr>
</thead>
<tbody>
<tr>
<td>1920 to 1950</td>
<td>1.15%</td>
<td>2.10%</td>
<td>1.91%</td>
</tr>
<tr>
<td>1950 to 1970</td>
<td>-0.09%</td>
<td>-0.38%</td>
<td>-0.38%</td>
</tr>
</tbody>
</table>

Figure 13b plots the college premium for the 1920 to 1970 cohorts. The model generates an increase in the college premium for the cohorts 1920 to 1970: the college premium increases 13 percent both in the model and in the data. The cohort-over-cohort pattern of the college premium in the model follows the data quite nicely. Both in the data and in the model, the college premium persistently grows across successive cohorts born between 1920 and 1970, with the exception of the 1930s cohorts. During the 1930s cohorts, the college premium decreases by 7.43 percent in the model and by 4.28 percent in the data. Lastly, notice that the model generates an increasing college premium for the cohorts of the slowdown in college attainment, as shown in the data. Between the 1940s and the 1960s cohorts, the college premium increases by 19.2 percent in the model and by 12.1 percent in the data.

The increase in the college premium is generated by a combination of both exogenous forces in the model: the time effect and the cohort effect. Figure 9 shows a decomposition exercise of these two effects on the pattern of the college premium. In a first experiment ("Time effect only"), I keep the initial human capital distribution of each cohort to be the same, so that the only difference between cohorts is the life-cycle price sequence. The resulting college premium increases almost exclusively during the 1920s cohorts. In particular, the college premium increases of only 2.4 percent between the 1940s and the 1960s cohorts. The cohort effect is a composite of changes in mean and standard deviation of initial human capital (Figure 9, panels (c) and (d)). The standard deviation effect is predominant. The standard deviation of initial human capital decreases from the 1920 cohort to the 1930s.

The college premium is defined as median college graduate earnings relative to high school graduate earnings over ages 31 to 40. Several authors have documented that the fall and rise in the college premium in the United States in the twentieth century was largely due to changes among young workers, whereas the college premium among old workers wasn’t very much muted (Murphy and Welch, 1992). In the model, starting in the 1990s, the college premium among older workers rises far less than it does among young workers. Because younger workers have a larger planning horizon, they respond to a change in prices much more strongly than older workers in the model.
cohorts, lessening the increase in the college premium that would have otherwise resulted from the time effect. The contemporaneous increase in the mean of initial human capital strengthens this effect only slightly. Symmetrically, the dispersion of initial human capital increases commencing with the 1940s cohorts, fueling the rise in the college premium.

The time effect influences the pattern of the college premium along with the cohort effect. Figure 9, “Cohort effect only”, presents the implied pattern of the college premium when price growth is constant and set to its average between pre-1970s growth and post-1970s growth, so that the only differences between cohorts are the initial human capital distribution and the price at age 19. The resulting college premium decreases between the 1920 cohort and the 1940s cohorts and increases thereafter, even if at a slower pace than in the baseline. A composition effect is at play. Price growth determines college attainment and so the average innate ability and average initial human capital of college graduates and high school graduates. Innate ability and initial human capital determine human capital investment and so the college premium, which is the ratio of the median human capital supplied to market work by college graduates relative to that supplied by high school graduates. Pre-1970s price growth fuels the increase in college attainment, which has a significant selection effect on the average innate ability and average initial human capital associated with a schooling level and exert a predominant role in the increase in the college premium during the cohorts 1920s to 1940s. Previous studies, such as Hendricks and Schoellman (2014) and Laitner (2000)
highlight the importance of selection on the college premium during times of substantial changes in college attainment. The slowdown in price growth after the 1970s, instead, encourages an increase in the college premium for the 1940s cohorts by strengthening the association between college and innate ability.

Figure 10 shows the distribution of initial endowments, innate ability, and initial human capital, conditional on the education level for two groups of cohorts, 1931-1934 cohorts and 1961-1964 cohorts. While the less-recent group features college graduates with lower innate ability than that of high school graduates, the more-recent group includes almost perfect
positive sorting by innate ability across schooling levels. The initial human capital margin does not matter for the college decision of the more recent cohorts of high school graduates, while it matters for the less-recent cohorts along with the innate ability margin. The reason is that the two sets of cohorts face differently shaped life-cycle sequences of the price. The 1930s cohorts face low price of human capital at high school graduation and high price growth over the life-cycle. On the other hand, the 1960s cohorts face high price of human capital at high school graduation and low price growth over the lifecycle. There is no direct measure of such a change in college selection over time in the data. However, Taubman and Wales (1972), Bowen and Turner (1999), and Gemici and Wiswall (2014) offer some anecdotal evidence in support of such a change. Overall, the path of the rental price per

\footnote{First, Taubman and Wales (1972) report for cohorts born between 1907 and 1950 the average percentile score on IQ tests for those who continue on to college and for those who do not. The trend for the former is positive, while the trend for the latter is negative. Second, Bowen and Turner (1999) document sorting across majors by SAT math and verbal score, and Gemici and Wiswall (2014) document an increase in the}
Table 7: Discussion. Average change in college attainment. The table reports the slope coefficient resulting from regressing the fraction of high school graduates with a college degree in a cohort on a constant and a cohort trend. Source: IPUMS-USA and author.

unit of human capital has both an intensive- and an extensive-margin effect on the average innate ability associated with college and so on the college premium.

4.1 Discussion

I explore the robustness of the model results on college attainment by considering alternative estimates of the price sequence. I also discuss the estimated elasticity of college attainment to price growth in light of relaxing model assumptions on college expenses and price expectations.

Expectations on price growth The timing of the slowdown in college attainment is not well reproduced by the model. College attainment slows down commencing with the early-1940s cohorts in the model and with the late-1940s cohorts in the data. A similar result appears for the rise in college attainment. The greatest increase in college attainment occurs for the 1930s cohorts in the model but for the 1940s cohorts in the data. As pointed out by Cunha and Heckman (2007) expectations of future price growth may play a major role in choosing to attend college or not. I explore the possible role of individuals’ expectations on the timing of the slowdown in college attainment by relaxing the assumption of perfect foresight. I consider the simple alternative scenario of individuals expecting the price growth observed at high school graduation to persist during their lifetime. Figure 11 and Table 7, column “Model/Alt 1”, show the implied pattern of college attainment. The timing of the rise and of the flattening of college attainment in the model aligns with the data under this alternative scenario. Castro and Coen-Pirani (2013) study the path of college attainment when the assumption of perfect foresight is relaxed and individuals expect education-specific fraction of students pursuing majors associated with higher SAT math and verbal scores starting with the late-1940s cohorts.
prices to remain constant at the level observed at age 17. They find expectations are not a key driver of the slowdown in college attainment but help improve the model fit.

**College expenses** Figure 12, panel (a), plots model implied average expenses on college quality and average tuition and fees for private and public colleges in the United States, by cohort.\(^{21}\) Between the 1920 cohort and the 1970 cohort, average expenses for college quality increase about five times in the model and average college tuitions and fees increases about 3 times in the data. The two series are comparable when expenses on college quality are presumed to reflect only pecuniary costs. I now work under this presumption and relax two model assumptions. In the preceding formulation, I have assumed that college goers choose the magnitude of expenses on college quality and that the relative price of college education is constant. First, I consider an alternative formulation where some college expenses might be beyond the control of individuals. I divert from the baseline model by assuming that individuals must pay a fixed cost \(K_{\tau}\) in order to complete college. Thus, the problem of an individual born in cohort \(\tau\) on the college path writes:

\[
\max_{\{i_j\}_{j=3}^{20}, e} \sum_{j=3}^{20} \left( \frac{1}{R} \right)^{j-1} E_j - \left( 1 + \frac{1}{R} \right) (e + K_{\tau})
\]

\(^{21}\)The College Board (2007)'s data on average tuition and fees for private and public colleges as a group are available from academic year 1967-1977 onwards. For academic year preceding 1967-1977, I measure college expenses as a weighted average between average tuitions and fees in private colleges and average tuitions and fees in public colleges. The weight is chosen so that college expenses for academic year 1967-1977 match the data on average tuition and fees for private and public colleges as a group.
subject to the usual constraints in eq. 2.2. I normalize $K_{1961-1964}$ to zero since 1961-1964 cohorts’ expenses on college quality already match the data on average college tuitions and fees from the calibration of the deep parameters. Then, I discipline the path of the fixed costs so that the model implied growth in total average expenses on college quality – that is, $(1 + \frac{1}{R})(e + K_x)$, is as close as possible to replicating the growth in observed average college tuition and fees in the data. The quality of the calibration is summarized in Figure 12, panel (b). Figure 12, panel (c), and Table 7, column “Model/Alt 2”, show the model implied pattern of college attainment. Pre-1950 cohorts, in particular the 1940s cohorts, are most affected. For these cohorts, a decrease in college expenses compared to the baseline
fuels an increase in college attainment, aligning the model implied timing of the slowdown in college attainment closer to the data. Donovan and Herrington (2013) find college costs a key driver of college attainment for the 1930 to 1950 cohorts.

Second, I extend the model by allowing the relative price of college education $p_t$ to change over time. The problem of an individual born in cohort $\tau$ on the college path now writes:

$$\max_{\{i_j\}_{j=3}^{20},e} \sum_{j=3}^{20} \left( \frac{1}{R} \right)^{j-1} E_j - \left( p_\tau + p_{\tau+1} \frac{1}{R} \right) \left( e + K_\tau \right)$$

subject to the usual constraints in eq. 2.2. I normalize $p_{1980-1986}$ to 1 and restrict the other relative prices with data on the growth of the cost of higher education net of the growth of the GDP implicit price deflator. Calculations are based on the Higher Education Price Index and the Consumer Price Index adjusted to a school-year basis by Snyder and Dillow (2011), table 34. Lastly, I re-calibrate the fixed costs $K_\tau$ such that the model implied growth in total average expenses in college quality – that is, $(p_\tau + p_{\tau+1} \frac{1}{R}) (e + K_\tau)$, is as close as possible to replicating the growth in observed college tuitions and fees in the data. Figure 12, panel (d), and Table 7, column “Model/Alt 3”, show the model implied pattern of college attainment. I conclude that abstraction from explicit costs of college education does not significantly alter the measured elasticity of college attainment to price growth.
Alternative measures of price growth. Bowlus and Robinson (2012) estimate price growth across four education groups for the period 1963-2008. I average their estimates for high school graduate and college graduate prices to construct an alternative price sequence. I then compute average price growth for two periods: 1) 1963-1970 and 2) 1980-2008. I use the resulting average growth rates, $g_{w,t=1963-1970}$ and $g_{w,t=1980-2008}$, to check the robustness of my results. First, I set $g_{w,1} = g_{w,t=1963-1970} = 1.58\%$ and $g_{w,2} = g_{w,t=1980-2008} = -0.39\%$. Second, I recalibrate the model parameters to deliver the targets of the baseline calibration with the exception of cross-cohort life-cycle earnings growth, which was previously used to discipline price growth. Figure 13 and Table 7, column “Model/Alt 4”, show the implied pattern of college attainment and college premium. Consistent with the baseline exercise, college attainment in the alternative exercise slows down starting with the 1940s cohort. The college premium in the alternative exercise follows the baseline very closely, with the exception of the 1920-1930s cohorts for which it better aligns with the data.

5 Conclusion

In this paper I assess the quantitative importance of the growth rate of the rental price per unit of human capital in generating patterns of US college attainment for white males born between 1920 and 1970. I argue that price growth is a key factor in the pattern of college attainment. In particular, a decrease in price growth in the 1970s causes college attainment to remain flat for the cohorts born after 1950 in the US.

Since earnings reflect both the quantity and the price of human capital, the rental price per unit of human capital is not observable. I write a model of human capital accumulation in college and on the job to identify the rental price per unit of human capital and to quantify its importance for the path of college attainment. I calibrate the model to major patterns of earnings growth and earnings inequality, both across time and over the lifecycle, for the 1920-1970 cohorts. The calibration implies a decrease in price growth starting in the 1970s. As price growth decreases, the returns to human capital investment decrease and the opportunity cost of human capital accumulation in college increases relative to that on the job. Hence, college attainment flattens.

22Because of the structure imposed on price growth, in particular the choice of $\omega$, price growth between 1970 and 1980 also matches Bowlus and Robinson (2012) estimates for the period. See section 3.2.
One short coming of the model is that it generates a slowdown in college attainment that starts earlier than in the data. In an alternative exercise, I show that individual expectations influence the timing of the slowdown in college attainment. When I assume individuals expect price growth observed at high school graduation to persist during their lifetime, the model replicates the timing of the slowdown in college attainment as shown in the data. However, I only scratch the surface of the potential role of individuals’ expectations on the timing of the slowdown in college attainment.

The slowdown in college attainment is part of a wider phenomenon that involves all levels of education (see Appendix A). For example, Heckman and LaFontaine (2010) report a flattening of high school graduation rates. As Castro and Coen-Pirani (2013) point out, the observation that the slowdown in attainment spreads across all levels of education simultaneously, supports the idea of a common factor behind such slowdown. The mechanism I consider produces symmetric implications across schooling groups and, therefore, is qualitatively consistent with a general flattening of educational attainment. It would be interesting to extend the quantitative analysis in this paper to include levels of education beyond a four-year college degree.

References


40


A Data

IPUMS-USA. I use 1 percent samples for 1940-1970, and 5 percent samples for 1980-2008. I restrict the sample to employed white males. Observations are weighted. My measure of educational attainment is the IPUMS variable EDUC, which distinguishes among nine levels of education, of which I use two: (i) 12 years of schooling (high school, H), (ii) 16 years of schooling (four-year college, C). My measure of earnings is the IPUMS variable INCWAGE. It reports total pre-tax wage and salary income, i.e., money an employee received in the previous calendar year, as midpoints of intervals (instead of exact dollar amounts). I compute real earnings by applying Consumer Price Index (CPI) weights.

NLSY79. I restrict the sample to white males with no missing observations on earnings for ages 23 to 45. Among the available cohorts, I focus on cohorts born between 1961 and 1964, to maximize sample size. The final sample contains 403 individuals, 283 high school graduates (highest grade completed is 12th) and 120 4-year college graduates (highest grade completed is 16th). Observations are weighed. My measure of earnings includes wages, salaries, bonuses, and two-thirds of business income. I compute real earnings by applying CPI weights.

College attainment

![Graph](image)

Figure 14: College attainment in the United States (employed white individuals): fraction of individuals with a high school diploma that went on to complete a four-year college degree. Source: IPUMS-USA.
Educational attainment

Figure 15: Educational attainment in the United States (employed white individuals): fraction of individuals in a cohort by highest degree attained. Source: IPUMS-USA.

B Model Derivations

On-the-job human capital accumulation. If an agent of type \((z, h_1)\) never returns to full-time investment once he stops full-time investment, the on-the-job accumulation problem has a closed form solution. This condition is satisfied if (i) \(\delta \in (0, 1]\), and (ii) price growth does not increase “too much” over the lifecycle. The analytical solution of the on-the-job accumulation problem is as follows:

\[
V_j(h_j; z, w | S) = \begin{cases} 
  w_j \left[ h_j a_j + b_j^S z \frac{1}{1 - \beta S} \right] & h_j \geq h_j^*(z, w | S) \\
  \frac{1}{R} V_{j+1}(zh_j^S + \delta h_j; z, w | S) & h_j < h_j^*(z, w | S), 
\end{cases}
\]

where \(h_j^*\) is the cutoff level of human capital at age \(j\) under which the individual spends all his time on human capital accumulation. The recursive formulation of the two constants is:

\[
a_j = \begin{cases} 
  \frac{1}{R} & j = T \\
  1 + \frac{1 + g_j}{R} a_{j+1} & j < T
\end{cases}
\]

\[
b_j^S = \begin{cases} 
  0 & j = T \\
  \gamma \left( \frac{1 + g_j}{R} a_{j+1} \right) \frac{1}{1 - \beta S} + b_{j+1} \frac{1 + g_j}{R} & j < T,
\end{cases}
\]
for $T = 20$ and $\gamma = \beta S^{1-\gamma} - \beta S^{1-\gamma}$. This can be written in non-recursive form as:

$$a_j = \sum_{u=0}^{T-j} \delta^u \prod_{k=1}^{u} \frac{1 + g_{j-k+1}}{R},$$

$$b_j^* = \begin{cases} 0 & j = T, \\ \gamma \left( \frac{1+g_j}{R} \right)^{\frac{1}{1-\gamma_S}} + \sum_{u=1}^{T-j-1} \left( a_{j+u+1} \prod_{k=1}^{u} \frac{1+g_{j+k}}{R} \right)^{\frac{1}{1-\gamma_S}} \times \left( \prod_{k=1}^{u} \frac{1+g_{j+k-1}}{R} \right)^{\frac{\delta_S}{1-\gamma_S}} & j < T. \end{cases}$$

**College quality.** For an agent of cohort $\tau$ and type $(z, h_1)$, the first order conditions for college quality for the case of no full-time accumulation on the job are:

$$u_1 e^{-\eta^2} \left[ \left( zh_1^{\eta-1}(e w_\tau)^{1-\eta^2} + (e w_\tau)^{\eta-1} \right) \eta^{-1} \left[ zh_1^{\eta-1} u_2 + e^{\eta-1} u_3 \right] + e^{-\eta} u_4 = u_5, \quad (6) \right.$$  

where

$$\eta w_\tau^{-\eta^2} (1 + g_\tau)^{1-\eta} = u_1, \quad w_\tau (1 + \eta) = u_2, \quad w_\tau^\eta = u_3, \quad w_\tau^{1-\eta} = u_4.$$  

$$\frac{R^2}{(1 + g_\tau)(1 + g_\tau + 2)} \left( 1 + \frac{1 + g_\tau}{R} \right) \frac{1}{(1-\eta) a_3 z h_1^{\eta}} = u_5.$$  

The LHS of eq. 6 is decreasing in $e$ and the RHS of eq. 6 is a constant greater than zero for $\eta \in (0, 1)$. Moreover, it is true that:

$$\lim_{e \to \infty} LHS = 0, \lim_{e \to 0} LHS = \infty.$$  

This assures that the solution for $e$ exists and is unique for each type $(z, h_1)$. 

45
C Calibration Details

C.1 Deep parameters

**Identification: initial endowments.** The distribution of initial endowments is identified with the age variation of unconditional earnings moments and the college premium. Earnings of a \( j \)-year-old individual of type \((z, h_1)\), born in \( \tau \), with education \( S \), are

\[
E_j(h_1, z, w \mid S) = w_{\tau+2(j-1)}h_j - w_{\tau+2(j-1)}i_jh_j, \quad \text{that is:}
\]

\[
= w_{\tau+2(j-1)}h_1\delta^{j-1} - \]

\[
w_{\tau+2(j-1)}z^{\frac{1}{1-\beta_S}} \left( \left( \beta_S \alpha_{j+1} \frac{1 + g_{\tau+2(j-1)}}{R} \right)^{\frac{1}{1-\beta_S}} - \sum_{u=1}^{j-1} \left( \beta_S \alpha_{u+1} \frac{1 + g_{\tau+2(u-1)}}{R} \right)^{\frac{1}{1-\beta_S}} \delta^{j-1-u} \right),
\]

for \( a \) as defined in Appendix B. Average innate ability influences the slope of the earnings profile. Agents with higher innate ability allocate more time to human capital accumulation and so have low initial earnings. Later in life, their earnings are higher following higher human capital investment (in college and on the job). The level of initial human capital influences the intercept of an individual’s earnings profile and its concavity. The coefficients of variation of innate ability and initial human capital influence the life-cycle dynamics of earnings dispersion. A lower dispersion in innate ability implies a lower increase in the coefficient of variation of earnings over the lifecycle. When all agents are born with equal innate ability but different initial human capital levels, the model generates a pattern of decreasing earnings dispersion over the lifecycle through human capital accumulation. Dispersion in initial human capital determines the concavity of the life-cycle profile of earnings dispersion. The correlation of innate ability and initial human capital disciplines how the two dimensions of heterogeneity come together to shape life-cycle earnings dynamics. The college premium helps in the identification of the dispersion of initial human capital and the correlation between innate ability and initial human capital.

C.2 Cohort-specific parameters

**Identification: price growth.** When investment in human capital is negligible:

\[
E_j = w_{\tau+2(j-1)}h_j - w_{\tau+2(j-1)}i_jh_j \simeq w_{\tau+2(j-1)}h_j,
\]

\[
E_{j+1} = w_{\tau+2(j)}h_j - w_{\tau+2(j)}i_{j+1}h_j \simeq w_{\tau+2(j)}h_j,
\]

and therefore \( g_{w,\tau+2(j-1)} \simeq g_{E_j} \).