Variation in suspended sediment yield across the UK – a failure of the concept and
interpretation of the sediment delivery ratio

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Abstract

The sediment delivery ratio (SDR) has been a common approach developed to understand
change in sediment yield and flux through a catchment. In this study we propose that the
underlying concept of the sediment delivery ratio is flawed for a number of reasons: its linear
extrapolation is physically meaningless; there is no evidence of the magnitude of storage
required by the SDR approach on annual to decadal timescales; and the SDR approach
assumes suspended sediment transport is conservative yet it is known to undergo both loss
and production in-channel.

This study considers the sediment yield from 192 UK catchments from 1974 to 2010
for catchment areas between 4 and 9948 km² and shows that linear extrapolation of the SDR
approach overpredicts source terms and underpredicts fluxes for large catchments. The SDR
approach hides a range of behaviours of suspended sediment flux within catchments with
patterns of net deposition, net increase or no change all apparent in UK catchments. The

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approach proved to be self-correlated which meant that it can result in spurious correlations when compared to catchment area. The change in yield with catchment area can be just as well understood as a change in sediment supply from channels rather than as a change in delivery from hillslope sources. We propose that suspended sediment flux change with catchment area be modelled as a more physically-meaningful Gompertz function (step function) rather than using the traditional SDR approach.

**Keywords:** suspended sediment flux; SDR; particulate organic matter; rivers

1. **Introduction**

The sediment delivery ratio (SDR) is a common approach used to explain changes in sediment flux through a catchment (Roehl, 1962; Burt and Allison, 2009). The concept was developed because of the need to link observed erosion at field or hillslope scale to sediment yields in larger catchments and to then predict sediment yield and flux at ungauged sites (e.g. Bouraoui and Dillaha, 1996). Here, flux is defined as the mass of suspended sediment per year (tonnes/yr) and the yield is the flux per unit area per year (tonnes/km$^2$/yr; yield is often referred to as export). White (2005) described the sediment delivery ratio as a conceptualisation of the erosion-transport-deposition cycle. The black-box nature of the interpretation of SDR has led to criticism (e.g. Trimble and Cosson, 2000) and its subsequent replacement by sophisticated deterministic models (e.g. Borah and Bera, 2004) which nonetheless remain opaque due to the complexity of their linkages and difficulties of parameterisation.

1.1 **Questioning the geomorphological basis for a Sediment Delivery Ratio**
Here we are interested in the SDR approach as it pertains to suspended sediment flux. We argue that, for a number of reasons, the use of the SDR approach could have misled the interpretation of suspended sediment flux. Firstly, the term SDR implies that the major reason for sediment yield to decrease with increasing catchment area is that delivery of sediment gets less and less, i.e. sediment is not delivered to lower reaches of the basin but rather is lost to storage (Fryirs, 2013). This can be seen in the review of SDR given by Walling (1983) where the spatial lumping of the SDR concept has been recognised but has been interpreted as difference in delivery across the catchment, with each unit of the catchment having its own SDR. Several authors have then been forced to explain a decline in sediment yield with increasing catchment area as due to a decline in connectivity as catchment area increases (Delmas et al., 2012; Fryirs, 2013) and, although they talk of changing erosion rates through a catchment, this is not accounted for. Relying on loss to storage to explain changes in sediment yield with catchment area implies large sinks, but such large sink sizes are difficult to explain, identify or justify given the known, measured rates of deposition.

Worrall et al. (2013) calculated the suspended sediment flux for the UK, for which the SDR would be 10%, although if an empirical rule were applied (Roehl, 1962), then the SDR would be approximately 4%. Based on the average suspended sediment flux from the UK from 1974 to 2010, at an SDR of 10%, then 84 Mtonnes/yr would have to be stored somewhere in the UK each year, this is equivalent to an average 344 tonnes/km²/yr stored for each km² of the UK. This equates to an annual net deposition across the UK of 0.23 mm/ha/yr based upon a bulk density of 1.5 tonnes/m³.

Sediment budgets are often estimated for small headwater catchments; for example, Walling et al. (2002) studied lowland agricultural catchments in the UK of between 0.96 and 2.67 km² and found fluxes at the catchment outlets were between 40 and 209 tonnes/km²/yr with SDR between 14 and 27%. The headwater catchments used by Walling et al. (2002)
were described as typical of lowland Britain yet the in-field storage, and storage between field and channel (an SDR of between 14 and 27% equates to between 72 and 86% sediment storage), were still not sufficient to achieve the amount of deposition required at the national scale.

If sediment storage in “typical” headwater catchments is not sufficient, are there additional processes operating at larger scales that could remove an even greater percentage of suspended sediment flux? Overbank sedimentation occurs in the lower courses of typical UK rivers, but estimates of overbank sedimentation give very low rates compared to those required. Walling et al. (1999) estimated overbank sedimentation for the Yorkshire Ouse (3313 km² - flux at outlet: 49 ktonnes/yr) as 30% of the outlet flux (23% of influent suspended sediment flux) and as 40% of the outlet flux (29% of influent flux) for the River Tweed (4390 km² – flux at outlet: 66 ktonnes/yr). At its maximum, the rate of overbank sedimentation was 6 tonnes/km²/yr. Therefore, compared to sediment storage and SDR required for the UK, the available studies do not show sufficient loss to overbank storage.

Overbank sedimentation is not the only storage process operating at large scales in river catchments and there could be in-channel storage. Collins and Walling (2007) give values of in-channel storage as between 18% and 57% of the outlet flux of two UK lowland streams but they note that most of this storage was transient. Indeed, Walling et al. (2002) noted that permanent in-channel storage was only between 2 and 5% of the catchment outlet flux. Therefore, there is no evidence for sufficient in-catchment storage if the sediment delivery problem is only interpreted in such terms as delivery, conveyance and connectivity – there must have been a change in supply. This can be observed in the study of Delmas et al. (2012) where there is a recognition that erosion rate changes with scale from hillslope scale up to a catchment area of 10,000 km² but there was no recognition that the rate of sediment delivery would change with scale, and therefore did not consider there would be any change upon the
estimated value of SDR with varying catchment size. In other words a headwater source area will have very different characteristics to one in the lower reaches of a 10,000km² catchment. The fact that slope angles decline towards the lower reaches of most catchments is well known (i.e. the hypsometric curve – Cohen et al., 2008). However, the SDR approach implies that each unit area of the catchment supplies material at the same rate no matter where in the catchment it is and that some areas are disconnected and fail to deliver. Even theoretical approaches to understanding SDR have chosen to take a storage rather than supply approach (Lu et al., 2005). Therefore, we argue that the use of the SDR term has emphasised change in storage rather than change in supply.

In general, the SDR is plotted in log-log space which disguises some important but implicit assumptions. When not plotted on the log-scale it becomes apparent that sediment yield actually decreases fastest in the upper parts of a catchment where flood plains are typically small and slopes highest; correspondingly, sediment yield hardly changes in the lower parts of the catchment where flood plains are large and slopes low. The classical picture of the river system is one of erosion in the headwaters, transport in the middle course and deposition in its lower course (Schumm, 1977), but that is not consistent with a sediment delivery that declines fastest in the headwater tributaries of a catchment but varies little further down the channel network. One explanation of this distribution of sediment yield is a common confusion of interpreting sediment yield as sediment flux. Parsons et al. (2006) have indeed argued that sediment delivery is a fallacy when considered via the medium of sediment yield (flux/area) rather than as flux itself.

1.2 Conservative and non-conservative sediment

The concept of SDR can be unhelpful if it is interpreted as if suspended sediment were chemically and biologically conservative. The average organic matter content of UK
suspended sediment at Harmonised Monitoring Sites (HMS) since 1974 is 33.5% with a 5\textsuperscript{th} percentile of 6% and a 95\textsuperscript{th} percentile of 75% (Worrall et al., 2014). The importance of the non-mineral content of suspended sediment fluxes has been underestimated because it is often quoted as particulate organic carbon content but it is forgotten that naturally occurring organic matter is typically only 45 to 50% carbon and so, if organic carbon concentration is compared to dry mass weight of suspended sediment, its importance is approximately halved.

This organic matter content means several problems for how a SDR has been interpreted. Firstly, approximately one third of the suspended sediment may change by independent physical processes such as turnover to dissolved or gaseous phases. For example, Walling and Collins (2008) give eight sediment budgets for agricultural catchments across UK and USA but in none of these sediment budgets is turnover considered or estimated. Secondly, although it is possible under some circumstances for mineral matter to be precipitated in a stream, such effects are rare, whilst the river itself can be a source of particulate organic matter from in-stream flora and fauna independent of erosion processes. Ni et al. (2012) use the SDR approach to predict sediment deposition of particulate carbon in small watersheds without reference to supply varying with scale or the non-conservative nature of the carbon. Similarly, Gaiser et al. (2008) when modelling the effect of soil erosion on carbon sequestration assumed that carbon in eroded sediment no longer influenced CO\textsubscript{2} emissions but rather particulate carbon was either deposited in river channels or lost to the sea. Furthermore, there are sources of suspended sediment that have nothing to do with soils, river banks or erosion, for example, sewage outfalls in the UK typically discharge water with a suspended solid concentrations of between 20 and 30 mg/l which will predominantly be particulate organic matter. Moody et al. (2013) found between 38 and 87\% in-stream removal of peat-derived POC over a 10 day period, but even at this rate of loss, or turnover of the
organic matter content of suspended sediment would still not be enough to fulfil the storage
requirement identified above.

1.3 The sediment delivery ratio as a mathematical function

The SDR approach may be more of an empirical then a physical law, however, the
“classical” variation in sediment delivery ratio is implicitly assumed to be a power law
function of area but this causes some consequences that are rarely interpreted. Firstly,
extrapolation to small values of area should tend towards erosion values at the field scale but
often do not (e.g. Vanmaercke et al., 2011) which implies that a single power law relationship
may not hold true at the lowest values of catchment area even though this was one of the
reasons that the SDR approach was developed, i.e. linking universal soil loss equations to
catchment management (Wischmeier and Smith, 1965, USDA SCS, 1972, Lim et al., 2005).
Equally, the correlation between sediment delivery ratio and catchment area is generally
negative and extrapolation of the trend to very large catchments implies that catchments exist
with no sediment yield and so, as with the relationship at low values of catchment area, the
relationships may not hold true at large values of catchment area. Secondly, the gradients of
the relationship between sediment delivery ratio and catchment area vary to the extent that, in
some cases, the relationship predicts a decline in actual suspended sediment flux through a
catchment. This has been neglected partly because many studies have confused yield and flux
(eg. Fryirs, 2012). We therefore suggest the SDR approach has worked poorly in linking
field or hillslope observations to both larger catchment scales and to the discharge of the
largest catchments.

Finally, a plot of sediment delivery ratio (sediment yield vs. catchment area) will be
subject to self-correlation (Kenney, 1982) and therefore presents a spurious relationship that
cannot - and should not - be interpreted as meaningful let alone used for prediction. Self-
correlation occurs whenever there is a variable common to both the predictor and response
variable, in this case the SDR is derived from the sediment yield (the ratio of suspended
sediment flux to catchment area) and then compared to catchment area. A variable common
to both response and predictor is likely to result in a linear correlation, which is even more
likely if the common variable is the dominant source of variance in the response variable. In
the case of SDR in the UK, catchment areas vary up to 10000 km$^2$ while sediment yield
varies up to 1000 tonnes/km$^2$/yr. Self-correlation in the SDR approach has not been tested,
yet correlations based upon sediment yield variation with catchment area are commonly used,
interpreted and discussed (e.g. Tetzlaff et al., 2013). It should be noted that self-correlation in
this context predicts a negative relationship between SDR and catchment area and so positive
relationships might be thought to be free of such issues. Such relationships between SDR and
catchment area have been observed in a number of studies (e.g. Church and Slaymaker, 1989)
and have been associated with, for example, areas of high channel incision. Here, we propose
that SDR is self-correlated and should therefore be replaced by measures not prone to
spurious correlation.

The purpose of this study is to consider the change in sediment yield across the UK as a
means of testing the sediment delivery ratio approach and whether the issues raised above can
be solved.

2 Methodology

This study used the suspended sediment flux data as calculated, corrected and analysed by
Worrall et al. (2013), which was based on data from the Harmonised Monitoring Scheme
(HMS - Bellamy and Wilkinson, 2001). There are 56 HMS sites in Scotland and 214 sites in
England and Wales (Figure 1). Rivers for monitoring were selected as the tidal limit of rivers
with an average annual discharge over 2 m$^3$/s; in addition, any tributaries that have an
average annual discharge above 2 m$^3$/s are also sampled. These criteria lead to a good spatial coverage of the coast of England and Wales, but in Scotland many of the west coast rivers are too small to warrant inclusion in the HMS. No data were available from Northern Ireland. This study only considered sites where sediment flux monitoring was coincident with flow monitoring, otherwise a flux calculation would be impossible.

Among the monitoring agencies, sampling frequencies vary, ranging from sub-weekly to monthly or even less frequently. Annual data were rejected at any site where there were fewer than 12 samples (sampling frequency (f) = 1 month) in that year with the samples in separate months. Flux was estimated using the method of Littlewood and Marsh (2005) who proposed an interpolation technique that accounts for differing sampling frequencies. This approach assumes that each sample taken at a site is equally likely to be representative of an equal proportion of the year as any other sample. The data from the HMS dataset have an inconsistent, and often only monthly, sampling frequency. So as to correct the flux data for sampling bias and to correct the flux estimates to the highest frequency flux estimates analysis of variance (ANOVA) was used to derive all the correction factors after the method and Worrall et al. (2013) and bring all site-year combinations to be equivalent to those with the highest sampling frequency. With respect to the suspended sediment concentration, there were 103162 measurements with 91604 measurements for which there were also measured flow from 270 catchments for all years from 1974 to 2010. An annual suspended sediment flux could be calculated for at least one year in all 270 catchments in the HMS scheme but across the 37 years of the HMS records considered and across the 270 catchment (9472 possible catchment-year combinations) a flux calculation was only possible for 6026 catchment-year combinations (66%).

The estimated suspended sediment fluxes were compared to a range of catchment properties. All suspended sediment fluxes were expressed as a yield (or an export) based
upon catchment areas derived for each monitoring point from the CEH Wallingford digital terrain model which has a 50 m grid interval and a 0.1 m altitude interval. The soils of each catchment were classified into mineral, organo-mineral and organic soils based upon the classification system of Hodgson (1997) and the land use was classified into: arable, grass and urban based upon the June Agricultural Census for 2004 (Defra, 2005a). In addition, the number of cattle and sheep in each 1 km² were counted within this census and converted to the “equivalent sheep per hectare” based upon a ratio of 3.1 sheep per cow. A range of hydrological characteristics for each catchment were calculated, these were: the BFI (Baseflow Index - Gustard et al., 1992), the average actual evaporation, the standard average annual rainfall; and the maximum altitude within the catchment. The comparison between suspended sediment fluxes and catchment characteristics was analysed firstly by multiple regression and secondly by logistic regression. For multiple regression, variables were assessed for normality (Anderson and Darling, 1952) and if necessary log-transformed – it did prove necessary for further transformation. The best-fit multiple regression was fitted using both step-up and step-down procedures with variables only retained if they were significantly different from zero at a 95% probability. Quality of fit was assessed using the coefficient of determination (R²). Binary logistic regression to understand the differences between groups of data found in the analysis. The best-fit binary logistic regression was fitted using maximum likelihood and again variables were only retained if they were significantly different from zero at the 95% probability. The quality of fit was assessed by considering concordance, i.e. the correct classification of the data. For the binary logistic regression the odds ratio was used to assess importance of individual variables. For both multiple regression and binary logistic regression the best-fit line was also chosen based on physical interpretability of the composition of the identified equations.
The data can be tested for self-correlation using a method adapted from Vickers et al. (2009). The principle is that in a self-correlated dataset, correlation will be observed even when the values are calculated from values drawn at random. To apply the method, a value of catchment suspended sediment yield is calculated from the catchment’s average suspended sediment flux and a catchment area drawn at random from the values within the whole dataset. The new value of sediment yield is paired with the value of catchment area drawn that was used to calculate the sediment yield. This process is repeated a sufficient number of times to create the same number of data pairs as in the original dataset. This random generation of a dataset of equivalent size to the original can be repeated to generate as many random datasets as required - in this case 100 random datasets were generated.

In these randomised datasets the correlation between the sediment yield and catchment area can be calculated and the distribution of gradients and intercepts compared to that for the correlation in the original dataset. If there is no significant difference between the best-fit line for the observed and that from the randomised data, then any line fitted to the observed data can be dismissed as spurious due to self-correlation. This approach differs from that of Vickers et al. (2009) because in this case no distribution of the data is assumed while Vickers et al. (2009) assumed a normal distribution and let that govern the randomisation process. In this study it is only linear relationships in log-log space that were considered and tested.

Given the potential issues of self-correlation in the traditional approaches to understanding sediment delivery ratio this study considers two alternative approaches. Firstly, an alternative approach to understanding the change in sediment yield with catchment area is not to consider the change in yield with catchment area but instead to consider the incremental change in yield with catchment area, that is the sediment yield of the i\textsuperscript{th} km\textsuperscript{2} in the catchment that is required to meet the change in sediment flux observed as catchment area increases. Secondly, the change in suspended sediment flux with catchment rather than the
change in sediment yield with catchment area was considered. The function of changing suspended sediment flux with catchment area was assessed using a range of sigmoidal functions (including logistic, hyperbolic, Weibull and Gompertz functions).

3 Results

For catchment-year combinations that met the criteria of sampling frequency \( f \) ≤ 1 month, the bias-corrected suspended sediment yields have a median of 22.2 tonnes/km\(^2\)/yr with a 5\(^{\text{th}}\) percentile of 5.4 tonnes/km\(^2\)/yr and 95\(^{\text{th}}\) percentile of 107.7 tonnes/km\(^2\)/yr. Comparing the sediment yield to the catchment area shows the expected decline and, perhaps not surprisingly, the sediment yield declines with increasing catchment area (Figure 2). Replotting Figure 2 without log-transformation shows that sediment yield declines dramatically across the range 0 to 100 km\(^2\) compared to the change in catchment size from 100 to 10000 km\(^2\) (Figure 3). This implies that most sediment deposition happens in the headwaters and not in the lower courses of a river.

3.1 Sediment trends

Figure 2 shows that there is not one trend of declining yield with area, but rather two trends that bound the study catchments. The Trends (A and B – Figure 2) were derived from a visual selection of the lowest bounding data points across all catchment areas in the dataset and the highest bounding values in the dataset, and then plotting the best fit line through them: note that point O was included in both the highest and lowest bounding datasets. Trends A and B are distinguished by their difference in the rate of decline of yield with increasing catchment area but both have a common point at point O (area 32 km\(^2\), yield 374 ± 130 tonnes /km\(^2\)/yr). Note that if Trends A and B were calculated without inclusion of point...
O then the intersection of Trend A and B would be predicted at area = 29 km$^2$ and yield = 441.6 tonnes/km$^2$/yr. Trend B was taken as only applying for catchment areas > 30 km$^2$ while Trend A applies across the entire range. The equations of the bounding trends are:

**Trend A**

$$\log_{10}(SS_{yield}) = 3.29 - 0.44\log_{10}(Area) \quad (i)$$

**Trend B**

$$\log_{10}(SS_{yield}) = 6.5 - 2.63\log_{10}(Area) \quad (ii)$$

where: $SS_{yield}$ = suspended sediment yield (tonnes/km$^2$/yr); and Area = the catchment area (km$^2$).

The projection of Trend B beyond point O would give incredibly large values of erosion for small catchments, but Trend A can be extrapolated to the y-axis. Trend A implies a suspended sediment yield of 1950 tonnes/km$^2$/yr at source, where source is taken as the y-intercept value of the trend at 1 km$^2$. It is, therefore, possible to estimate an SDR for Trend A (Figure 2). Given Equation (i), the SDR for the largest catchment (River Thames – 9948 km$^2$) is estimated to be 1.8%, but for some catchments represented by Equation (ii) the SDR would be as low as 0.1%. At a 1.8% delivery ratio and a source suspended sediment export of 1950 tonnes/km$^2$/yr, then 466 Mtonnes/yr of sediment has be retained in the catchment or turned over to the atmosphere – we would suggest that this is improbably large amount of sediment to be stored, processed or disposed each year in these UK catchments.

The value of 1950 tonnes/km$^2$/yr as source term would appear large in comparison to published values for the UK. Walling et al. (2002) working in 1.5 km$^2$ and 3.6 km$^2$ agricultural catchments on mineral soils in England showed soil erosion rates up to 466
tonnes/km²/yr with 80% removal, or deposition, by the outlet of the catchment – a SDR of
20%. Defra (2005b) gave median values of net soil loss from arable fields in England as 410
tonnes/km²/yr and from English grasslands as 60 tonnes/km²/yr. Worrall et al. (2011) gave a
value of 406 tonnes/km²/yr for a bare peat plot in the South Pennines. Worrall et al. (2013)
suggested source suspended sediment values between 241 and 825 tonnes/km²/yr depending
upon the nature of the land use and soil types within the catchment with higher values for
organic soils under grass. Even though the value of 1950 tonnes/km²/yr would be the lower
value predicted from Equations (i) and (ii). There is little evidence, therefore, to support such
a high source value.

The unreasonably high value of the suspended sediment flux at source predicted by
Trend A compared to field observations from the literature does suggest that at small
catchment sizes the SDR is not linear with catchment area in log-log space and is only linear
over a mid-range of catchment sizes. Nevertheless, the data from the smallest catchment
available from the HMS dataset was for a 4 km² subcatchment of the River Dearne and its
yield is consistent with Trend A, i.e. even at a scale of 4 km² the sediment yield is 1229
tonnes/km²/yr (12.29 tonnes/ha/yr). Non-linear relationships in log-log space for sediment
yield and catchment area have been noted by, for example, de Vente and Poessen (2005).

Because trends identified in Equations (i) and (ii) are bounding trends, it is then
possible to express all results as a mixture of the bounding trends. Using the available
catchment characteristics, and stepwise regression, it was then possible to explore what
controlled a catchment’s position between the two bounding trends for which the best-fit line
for the proportion of Trend A for a given catchment ($\phi_{trendA}$) was:

$$\phi_{trendA} = 0.51 + 0.29\log_{10}Area - 0.12\log_{10}Arable + 0.09\log_{10}Grass + 0.08\log_{10}Orgmin - 0.27\log_{10}AAR$$  (iii)

$$r^2 = 0.49, n = 192$$
Where: Arable = the area of arable land use in the catchment (km²); Grass = the area of grass land use in the catchment (km²); Orgmin = the area of organo-mineral soils in the catchment (km²); and SAAR = the standard average annual rainfall (mm). Only terms found to be significantly different from zero, at P < 0.05, were included in Equation (iii). Equation (iii) is not proposed as a predictive equation, rather that there is a physical explanation for the trends shown in Figure 2. Equation (iii) implies that larger catchments dominated by grass and organo-mineral soils are more like Trend A while those catchments with a greater arable land use and higher standard annual average rainfall are more akin to Trend B.

If the bounding trends to the sediment yield vs. catchment area curve are defined by change with catchment area (Equations (i) and (ii)) but the position between two bounding trends is controlled by nature of each catchment (Equation (iii)), then the position within the defined space is defined by an interaction between catchment area and its land use and soil types with a form of equation as:

\[
\log_{10} SS_{yield} = k_1 + k_2 f(\text{land use, soil type, hydroclimate}) \text{Area} + k_3 \log_{10} \text{Area} \quad \text{(iv)}
\]

Fitting this function to the available data gives a fit that can represent the bounding trends and extreme values although the RMSE is 0.46 which suggests the fit for individual catchments would be poor (Figure 4). Equation (iv) shows that land use, soils or hydrology becomes more or less important as catchment area increases or decreases. Although many studies have noted a variation in the relationship between sediment yield and catchment, very few have explained this variation as shown above. Milliman and Syvitski (1992) did use maximum altitude as well as catchment area to improve relationships but described land use, climate
and geology as of secondary importance – a result not supported here, although the study of Milliman and Syvitski (1992) was for larger catchments than any considered in this study.

The comparison of the nature of Trend A and Trend B suggests that, while catchments following along Trend A have increasing suspended sediment flux with increasing catchment area, for those catchments plotting along Trend B, the suspended sediment flux actually decreases with increasing catchment area beyond point O (Figure 2). There are several implications of this observation. Firstly, those catchments following Trend B are net depositing catchments where the maximum sediment flux is that defined by point O (11955 tonnes/yr) and that this value would decrease for larger catchments. This implies that there would be a catchment where almost no sediment was actually exported - in this case it would be a catchment with area 616 km$^2$ (it should be noted that such a catchment along this trend does not exist in this dataset). Therefore, just as for the small catchments where a linear response gave unreasonable high values of yield, then the implication of a catchment which has zero yield may also be unreasonable and an asymptote to the x-axis may be physically preferable.

Secondly, an implication of the rate of decline of yield bounded by the two identified trends, one of which implies increasing suspended sediment flux and the other implying a decrease in suspended sediment flux with area catchment, suggests there is a boundary between these two trends for which, although the yield declines, the actual flux does not. That is, there is a boundary discernible below which suspended sediment flux decreases with increasing catchment area, and above which it increases with increasing catchment area. Given the point O is common to both trends then it is easy to define the line from which the flux remains constant with increasing catchment area, ie. where the sediment flux remains the same as that estimated for the catchment representing point O (River Dee at Mary Culter Bridge 11955 tonnes/yr) across the range of catchment areas in the study dataset (Trend C -
Figure 2). All catchments can be defined relative to this line as being a net decreasing or a net increasing catchment. Logistic regression can be used to assess what controls the membership of these two groups. Here the best-fit logistic regression was:

\[
\ln \left( \frac{\theta}{1-\theta} \right) = 8.8 - 0.012 Evap_{act} - 0.0035 Org - 0.006 Grass \quad (v)
\]

Where: \( \theta = \) the probability of being a net increasing catchment; \( Evap_{act} = \) the average actual evaporation of the catchment (mm/yr); and \( Org = \) area of organic soils in the catchment (km\(^2\)). The equation was 85\% concordant with the data. The odds ratio suggests that none of the variables included are more important than any other. The equation implies that a catchment is more likely to be a net decreasing catchment if it is dominated by grassland and organic soils and high actual evaporation. It should be noted that high actual evaporation often correlates with higher rainfall rather than being indicative of drier catchments.

Since Trend C identifies no change in flux with increasing catchment area, this implies that such catchments are experiencing a form of equilibrium in that, relative to point O, they deliver to the outlet the same amount of sediment as enters the channel system. Therefore, it would be reasonable to describe this situation as having a delivery ratio of 1 because, even if the sediment yield decreased, all the flux that entered the catchment was delivered to the outlet. Often studies have used a decline in sediment yield with catchment area to describe catchments as inefficient (Hinderer, 2012, Fryirs, 2013) whereas it could be reasonable to describe catchments on these trends as very efficient as they export what comes in to them.

We use our words carefully as it is important to stress that it is often the interpretation of the SDR that we believe has been unhelpful and not the comparison of exports through a
catchment. When we describe catchments close to Trend C as efficient because they export all the sediment that enters them, this is not to say that they convey the same sediment right along their channel system but rather the same amount of sediment. These channels could therefore be in a steady-state equilibrium that makes them appear as a perfect conduit of sediment. Note that the back projection of Trend C to catchments sizes smaller than that represented by point O would again suggest unrealistically high sediment yields and so again there must be a curving to lower values so that the assumption of linearity is only true within a constrained range of catchment sizes.

3.2 Effects of self-correlation

The test for self-correlation shows that the distribution of self-correlated regression lines covers a considerable proportion of the results of the sediment yield against catchment area (Figure 5). However, both Trend A and Trend B both lie outside the significant range of the self-correlated random datasets and would therefore be considered as significantly different from the self-correlated result.

The identification of self-correlation leads to some important results. Firstly, there is a reasonable probability that any regression line on a graph of sediment yield against catchment area will be spurious, i.e. not significantly different from results that would be expected from the self-correlated nature of the comparison. Secondly, the self-correlation test does confirm that there are bounding trends to the sediment yield for UK catchments and given that one of these represents net deposition with increasing catchment area (Trend B) and the other represents increasing sediment flux with increasing catchment area (Trend A), there must be a point between these two trends where there is no change with catchment area and so the result illustrated by Trend C does have a physical basis.
Considering not the yield with catchment area but the change in yield required from the $i^{th}$ km$^2$ of the catchment for the catchments along Trends A and B. The graph of the yield of the $i^{th}$ km$^2$ with increasing catchment area shows that for Trend A the yield declines following a power law while that for Trend B shows a far more complex pattern that cannot readily be described by a power law (Figure 6). A power law description can be equated with a hyposometric curve for a catchment, i.e. slope change through a catchment is commonly described by a power law and it is reasonable to suggest that sediment yield is a function of slope among other things. If the change in sediment yield can be described by a power law akin to a slope descriptor, we may propose that sediment yield change through a catchment is not a matter of delivery but rather a matter of changing sediment supply through the catchment, i.e. if slope through a catchment declines then each new km$^2$ of the catchment has a lower capacity to produce sediment into the stream network.

For the River Thames catchment (9948 km$^2$) Trend A would now be interpreted as having a sediment yield of 1950 tonnes/km$^2$/yr in the first 1km$^2$ of the catchment and a sediment yield of 18.5 tonnes/km$^2$/yr in the very last km$^2$ of the catchment before the monitoring point while the average sediment yield for the Thames catchment to 9948 km$^2$ point was 36.5 tonnes/km$^2$/yr. Alternatively, for Trend B what is observed is a prediction that the sediment yield from the $i^{th}$ km$^2$ would rapidly become negative and there would have to be net deposition with this first occurring for the $47^{th}$ km$^2$ of the catchment and peaking in the $65^{th}$ km$^2$ of the catchment after which the rate of deposition declines becoming asymptotic to zero with further increases in catchment area. If the change of sediment yield for Trend A followed a curve which was readily interpretable as declining sediment source as the catchment grew larger, then a change which has net deposition is a catchment which is delivering less and less sediment into the next part of the catchment. The implications are reduced inputs from lower-angle hillslopes further down the catchment and reduced capacity.
for conveyance in lower-gradient channels notwithstanding the higher stream velocities and more efficient channel cross-sections. It may indeed be, for the Thames, that Schumm’s “transport zone” encompasses most of the channel network and that there is now very little additional net storage of sediment in the main floodplain system.

Given the self-correlated nature of the relationship between catchment area and sediment yield, it is reasonable to consider alternative approaches. A graph of suspended sediment flux (as opposed to sediment yield) against catchment area is be compromised by self-correlation. Figure 7 shows that, as expected, the sediment flux increases with increasing catchment size in the majority of cases and that a single linear relationship, although significant, does not capture the nature of the data; as before, the data may be better described as bounded by two trends (Figure 7). Note that in this case the two trends bounding these data do not quite correspond to those identified above (Figure 2 – Equations (i) and (ii)). The point O from Figure 2 is given on Figure 7 and the sites that constitute Trends A and B show that the upper bound in Figure 2 is that identified as Trend A above but Trend B curves down and away from Trend A but does not constitute the lower trend observed in Figure 2, i.e. trends of declining suspended sediment flux are possible within the bounds of the data projected on Figure 7.

3.3 Suspended sediment – a new interpretation

If Figure 7 is replotted on a double log plot, it is observed that the plot of suspended sediment flux against catchment area could be bounded by two straight lines. A straight line response on a log-log plot implies that a sigmoidal function would describe the data. Therefore, we propose the use of the Gompertz function which is one of a family of sigmoidal functions, but unlike others based upon hyperbolic functions, the parameters of a Gompertz function are physically interpretable. However, sigmoidal functions have a low
response for small values of x, in this case small catchment areas: this is not observed for UK
data and so a short range effect at low values of catchment area was included:

\[ SS_{flux} = \alpha e^{\beta e^{\gamma Area}} - \delta e^{-\epsilon Area} \]  
(vi)

Where: \( \alpha, \beta, \gamma, \delta \) and \( \epsilon \) = constants with \( \alpha \) = maximum asymptote (tonnes/yr); \( \beta \) = the y
displacement with respect to catchment area (km\(^2\)); \( \gamma \) = the growth rate (tonnes/km\(^2\)/yr); \( \delta \) =
The maximum small area correction (tonnes/yr); and \( \epsilon \) = the decay constant for the small area
correction (/km\(^2\)). Equation (v) was fitted to the data for the bounding trends (as for Figure 2
the highest and lowest bound datapoints were selected and then the best-fit of Equation (vi)
estimated), for Figure 7 and the best-fit equations are:

Trend D

\[ SS_{flux} = 350000e^{-3.16e^{-0.00089Area}} - 14219e^{-0.061Area} \]  
(vii)

Trend E

\[ SS_{flux} = 87652e^{-5.53e^{0.0035Area}} - 109e^{-1.132Area} \]  
(viii)

The fit of these equations shows that a maximum asymptote is apparent in the data that
represent a maximum sediment flux, in one case 350 ktonnes/yr and the other 87.7
ktonnes/yr.

The approach taken here can be considered for individual catchments. Within the
dataset considered here, it was possible to identify two catchments where multiple
measurements across scales were available – the rivers Trent and Ouse (Figure 8). Results for
individual catchments were included into the analysis illustrated in Figure 2 because plotting
in log space can act to cramp data points together. When plotted as sediment yield vs. catchment area then it can be observed that each catchment’s sediment yield evolves along a line away from Trend B towards Trend A but at larger catchment areas the trend in sediment yield for each specific catchment changes to trend parallel to Trend A. So, in individual catchments in this UK dataset we do see catchments that show negative and positive relationships between sediment yield and catchment area depending upon catchment location. This has been observed for a number of catchments (e.g. Yellow River – Jiongxin and Yunxia, 2005). When plotted as suspended sediment flux against catchment area, it is apparent that both catchments could be described by an equation of the form of Equation (v) where the best-fit equations were;

Trent

\[ SS_{flux} = 57980e^{-4.896e^{-0.00088Area} - 188e^{-0.102Area}} \quad (ix) \]

Ouse

\[ SS_{flux} = 62000e^{-5.0e^{0.016Area} - 188e^{-0.102Area}} \quad (x) \]

These observations suggest that catchments will evolve and trend in complex ways across the space bounded by Trends A and B.

4. Discussion

Does the approach here solve the problems that have been discussed for the SDR method of analysing suspended sediment flux? Partly. Describing the variation in sediment yield across a catchment using a source-decay approach does describe some of the data and in particular data close to Trend A. However, it would predict no net storage in the catchment which is not possible but clearly it is also absurd to have no change in source with catchment
Furthermore, the data along Trend B clearly show that net deposition would have to occur in some catchments. Therefore, the reality is some mix of varying supply and delivery and future research could be used to understand and compare the distribution of the magnitude of sediment sources with increasing catchment size compared to the observed change in sediment yield through the catchment.

Studies in Australia and Canada (e.g. Church et al., 1999) have shown positive relationships between sediment yield and catchment area, but for the UK sediment yield is always observed to decrease with increasing catchment area. Negative correlations between sediment yield and catchment area have been interpreted in terms of changing land use, hydrology and climatic factors, just as was possible here for the UK but the magnitude of the gradients does suggest that there are catchments in the UK where sediment flux actually declines through the catchment. It should be noted that the catchments that lie along Trend B only show this behaviour above a certain catchment size; for a catchment area of less than 30 km\(^2\) the sediment flux would increase with increasing catchment area. After all, suspended sediment flux cannot decline from zero but must decline from some initial value. In this case there appears to be a point at approximately 30 km\(^2\) at which point net deposition begins. Why would this occur? It is easy to envisage catchments with distinct breaks in slope distribution with high slopes at small scales and very low gradients at larger scales, e.g. transition from mountains to a “piedmont” zone (cf Lawler et al., 1999). Such changes in the sediment cascade with increasing catchment area are worthy of further research beyond the scope of this study.

Equally, the difference between Trend A and Trend B implies that there are catchments that have no net change in sediment flux across the catchment. This only occurs once a certain catchment area and flux have been achieved and therefore this does not mean that these catchments are at an equilibrium, rather that net change is not occurring in their
lower reaches. However, it should be noted that, whenever tested with the observations of sediment yield across two actual catchments, both showed complex changes with increasing catchment area not necessarily following any single trend. Either, catchments can switch between regimes with increasing scale, or the plot of sediment yield against catchment area is misleading given the uncertainty in the data and the self-correlated nature of the plot. This therefore may be the big question – how, where and why do catchments shift from transport to storage and over what length and time scales?

However, there should be a prior question of whether we would start from here? The SDR approach is used because it matches the information available from soil erosion models, originally the USLE approach (Wischmeier and Smith, 1965). Given that a plot of sediment yield vs catchment area, however drawn, is difficult to meaningfully interpret and suffers from self-correlation, then the focus of attention and interpretation should be the actual graph of suspended sediment flux vs. catchment area. This study has proposed that the curve of the suspended sediment flux can be best described by a sigmoidal function, specifically, a modified Gompertz function. The graph of the suspended sediment flux shows that, as catchments get larger, the growth in suspended flux declines and reaches a maximum described by the asymptote. It should be noted that no observed catchment had a sediment flux that actually equalled this maximum value, perhaps because UK catchments have relatively small areas compared to large river basins globally. The y-displacement in the Gompertz equation represents the position of the step and, taking this as the position of maximum gradient, then for equation (vii) the step position is at 1500 km² while for equation (viii) it is 5100 km²; for the River Trent it is 1300 km² and for the Ouse the 1100 km². As the y-displacement is here interpreted as the inflexion point in the step function, it represents the catchment area at which the rate of suspended sediment flux is no longer increasing. This could readily have a physical interpretation but, relative to the sediment delivery ratio
approach this shows the greatest rate of change is not at small catchment areas but well into
the catchment and so in this sense studying sediment yield from hillslope source areas alone
is misleading.

The growth rate in equation (vi) or the maximum rate of change observed for equation
(vii) is 114 tonnes/km$^2$ while for equation (viii) it is 11.14 tonnes/km$^2$. For the Trent the
maximum rate of change is 18.8 tonnes/km$^2$ and for the Ouse the maximum rate of change is
36.4 tonnes/km$^2$. However, given the small-area correction in equation (vi), then the
maximum rate change at the very smallest catchment areas, of the order of 1,280 tonnes/km$^2$
for equation (vii) and 110 tonnes/km$^2$ for equation (viii). The gradient of the equation (v)
represents the incremental sediment yield.

The approach cannot say anything about the turnover of the organic proportion of the
suspended sediment. Several studies have considered the fate of organic carbon through
rivers, and therefore the impact of rivers on concentration of atmospheric greenhouses gases
(Battin et al., 2009). These studies have progressively developed methods for understanding
each component of this problem for the UK to understand the fate of fluvial organic matter
and the release of greenhouses gases including CO$_2$, CH$_4$ and N$_2$O (Worrall et al., 2007,
2012a,b). Future research will focus upon reproducing this study for the organic matter flux
and then compare these results to those already produced for suspended sediment and so
assess the change in proportion of organic matter with catchment area relative to changes in
suspended sediment flux. We must always be mindful that change in POM flux and its
proportion across a catchment may be due to fractionation (i.e. not related to turnover) caused
by differential deposition, flocculation or transport of suspended sediment dependent upon its
organic matter content. Indeed, as source strength wanes across a catchment, slope, channel
or both, so does the proportion of organic matter, for example in the UK it would be common
to have peat soils in a headwater but not at the tidal limit of a catchment.
5. **Conclusion**

This study shows that the use of the sediment delivery ratio causes a number of misinterpretations. The SDR approach can be shown to suffer from a number of problems:

i) The graphical representation of sediment yield contains trends that encapsulate both increasing, decreasing and constant sediment flux with increasing catchment area. Some catchments showed perfect “delivery” of sediment whilst in others sediment yield declined with increasing catchment area.

ii) The relationship of SDR against catchment area cannot be linear over the range of catchment sizes.

iii) The relationship of SDR against catchment area suffers from considerable self-correlation and any relationship derived from the graph should tested before use or interpretation.

iv) Alternatively, a plot of suspended sediment flux against catchment area does not suffer from self-correlation and can be readily described by a simple empirical function based upon the combination of a Gompertz function and a short-range, small-area correction based upon an exponential decay function.

This study can find no reason why the SDR approach should continue to be employed in the form that it has been traditionally been used.

**Acknowledgements**

The authors are grateful to Abby Lane and Sarah Wheater of the Environment Agency of England and Wales for supplying the HMS data.
References


Figure 1. The location of the catchments used in this study for which suspended sediment flux could be calculated.

Figure 2. The log-log plot of the average suspended sediment yield against catchment area for all study catchments.

Figure 3. The average suspended sediment yield against catchment area for all study catchments showing that the suspended sediment yield changes most rapidly for the smallest catchments.

Figure 4. Fit of Equation (iv) to the suspended sediment yield against catchment area.

Figure 5. The best-fit self-correlated line and its 95% confidence interval in comparison to the annual average suspended sediment yield against catchment area.

Figure 6. The changes in incremental suspended sediment yield with catchment area for a) Trend A; and b) Trend B (Figure 2).

Figure 7. The average suspended sediment flux with catchment area for all study catchments.

Figure 8. The change in suspended sediment for sites through the catchments of the Rivers Trent and Ouse against their catchment area.