Cheap Talk and Strategic Rounding in Libor Submissions

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Abstract

Interbanking rates were, until recently, based on judgmental estimates of borrowing costs. We interpret this as a cheap talk game that allowed banks to communicate non-verifiable information about their opportunity cost to potential counterparties. Under normal market conditions there is a welfare maximizing equilibrium where banks truthfully disclose their borrowing cost, but, in times of financial stress, only “coarse” equilibria survive. We take this prediction to the data and show that banks round more frequently if the risk of the bank increases. Rounding is also more frequent for the more liquid short term rates and certain benchmark maturities.

Key Words: Interbank market, Interest rate fixings, Libor, Cheap Talk, Search Frictions.

JEL-Classification: D43, D49, D82, D83, G21
1 Introduction

Interest rate benchmarks play a key role in financial markets. The most widely used, the London Interbank Offered Rate (Libor), served as a reference in contracts with notional values estimated to amount to up to $800 trillion (Wheatley, 2012). Surprisingly, Libor, before its reform in 2013, was determined with a mechanism resembling an informal daily opinion poll: The British Banker’s Association (BBA) asked a panel of banks to submit the rate at which they could potentially borrow in the interbank market and the Libor rate was calculated as the interquartile mean of these submissions. In this mechanism, the banks seem to have had a high level of discretion.¹

In this paper, we demonstrate that, even if misrepresentation is not penalized, banks will have incentives to honestly report their borrowing rates if certain conditions hold.² This not only explains why this surprisingly informal mechanism largely performed well and allowed Libor to become a widely followed benchmark, but also shows why it failed when these conditions did not hold any more in the unfolding of the financial crisis between 2008 and 2011. Our model also predicts a number of patterns in the precision of the banks’ submissions that can be identified in the data and seem difficult to explain otherwise.

We exploit a specific feature of the Libor mechanism: Until April 2013, the BBA not only published the benchmark rate, i.e. the interquartile mean, but also each bank’s individual submissions (see ICE Benchmark Administration, 2016). These individual submissions seem to

¹Concretely banks in the Libor panel were asked the question: “At what rate could you borrow funds, were you to do so by asking for and then accepting inter-bank offers in a reasonable market size just prior to 11am London time?” (Duffie and Stein, 2015). Note that there is no definition of “reasonable market size” and that banks can submit estimates even if they have not “asked for” or accepted interbank offers. The BBA further specifies in its provisions that: “Therefore, submissions are based upon the lowest perceived rate at which it could go into the London interbank market and obtain funding in reasonable market size, for a given maturity and currency. BBA Libor is not necessarily based on actual transactions.” (UK House of Commons, 2012) Not surprisingly, in the recent lawsuits, it has turned out to be very difficult to prove that banks were providing wrong answers to this question. Instead, courts based their convictions on seized internal communication that demonstrated that banks were conspiring to manipulate the outcome of the rate setting mechanism.

²Our findings can also be rationalized in a setting with verifiable reports if the cost of lying is sufficiently small. We explain why in Footnote 14.
have conveyed crucial information\textsuperscript{3} about a bank’s borrowing cost to potential counterparties in the opaque, phone-brokered interbank market.\textsuperscript{4}

Building on Kim and Kircher (2015), we model this setting as a search market where individual cheap talk reports emitted by banks guide counterparties seeking to lend to one of these banks. Under normal conditions, truthful reporting is an equilibrium. When a bank understates its true borrowing costs, potential counterparties expect lower gains from trade and thus submit fewer offers to this bank, which, not surprisingly, reduces its profits. On the contrary, overstating true borrowing costs attracts more offers, but in the model of Kim and Kircher (2015), these additional offers turn out to be at rates that are too high to be attractive for the bank. This truth revealing equilibrium also maximizes welfare, as the increased transparency leads to a better matching of borrowers and lenders.

Unfortunately, this equilibrium is upset when there is uncertainty about the bank’s financial health such as in the financial crisis between 2008 and 2011. In this case, high individual borrowing costs may not only reflect short term liquidity needs but also indicate a possible risk of failure.\textsuperscript{5} This creates incentives to understate the true borrowing cost.\textsuperscript{6} As in the canonical cheap talk model of Crawford and Sobel (1982), these incentives to misstate the truth limit the precision of information that can be transmitted in equilibrium. The reason for this result is straightforward: Banks can no longer credibly disclose their precise borrowing costs because in any truth revealing equilibrium they would have incentives to slightly understate

\textsuperscript{3}Ridley and Jones (2012) cite a banking official “recalling the intense scrutiny as soon as Libor rates were published at 11 a.m. ‘Trading rooms would be watching and dissecting what rates people had been putting in each day, looking for a major change in behavior.’ ”

\textsuperscript{4}See MacKenzie (2008) for a description of the functioning of the market.

\textsuperscript{5}For instance, the specialized press writes: “The Libor setting process is public and closely watched, so a bank that put in relatively high rate estimates could spark investor concern about its strength.” Financial Times, Feb. 9, 2012 (\textit{Probe Reveals Scale of Libor Abuse}). The Wheatley Review states that “While individual submissions reflect elements other than solely idiosyncratic counterparty credit risk, changes in a particular bank’s submission may be interpreted by some observers as an implicit signal as to the creditworthiness of that contributor.” (Wheatley, 2012)

\textsuperscript{6}Again, we have direct evidence for the existence of these incentives. For example, in a recorded electronic chat on September 26, 2008 an HBOS submitter wrote to an employee of another financial institution “youll like this ive been pressured by senior management to bring my rates down into line with everyone else.” (CFTC, 2014). For more evidence see also CFTC (2012), page 19 ff.
their true costs. However, if the signal space is partitioned into sufficiently large intervals, a small deviation from the truth is not possible. As large deviations remain unprofitable, with a coarse signal space there exists an equilibrium where banks truthfully indicate the interval in which their borrowing costs are situated.

In our model, the coarseness of the signal space, i.e. the size of these intervals, will depend on two factors. One is uncertainty about the bank’s default probability: The more a bank needs to demonstrate that it is healthy, the stronger are its incentives to understate its true borrowing cost and thus the less precise is the information that can be credibly transmitted in the cheap talk equilibrium. A second, less straightforward and more surprising factor is market liquidity: If there are many potential lenders, they compete more fiercely and thus submit, on average, lower quotes. Thus, a deviation that sheds away the high quotes is less costly for the bank. Finally, note that the two effects reinforce each other: In the absence of uncertainty, there is a fully revealing equilibrium independently of the liquidity of the interbank market and hence no liquidity effect, but with increasing risk, liquidity will have a stronger effect on coarseness.

We take these hypotheses to the data and show that our model can explain a number of so far undocumented patterns in the precision of the Libor submissions. Similar to Backus, Blake, and Tadelis (2019) we interpret rounding as a natural way to partition the signal space into intervals of a certain size and implement the coarse equilibria predicted by the cheap talk model. An increase in the frequency of rounded numbers during times of uncertainty then corresponds to a lower informational content of the reports. We use the banks’ 1-year Credit Default Swap (CDS) spreads\(^7\) as measure of the banks’ financial health and demonstrate that the banks’ use of rounding increases with this measure of risk. The data also confirm our second and third prediction: Rounding increases with liquidity and this effect becomes stronger with higher risk. In particular, rounding is more common for the shorter, more liquid maturities as

\(^7\)A CDS is an insurance contract against default risk. A company’s CDS spread corresponds to the annual per dollar price of insuring this company’s debt.
well as for certain frequently traded reference tenors. If rounding was simply the consequence of uncertainty about market conditions, we would expect to see less, not more, rounding for liquid maturities.

Note, that our paper entirely focuses on the banks’ “signaling” (Gandhi, Golez, Jackwerth, and Plazzi, 2019) or “reputational” (Youle, 2014) concerns, i.e. their incentives to underreport their borrowing costs to appear less risky. The alleged “Libor suppression” (Libor I, 2013) resulting from these incentives was the original source of concerns about the Libor (Mollenkamp and Whitehouse, 2008).

In contrast, almost all of the Libor related lawsuits as well as much of the academic literature focus on a second, different issue. They analyze the banks’ incentives to manipulate the final Libor benchmark in order to benefit the bank’s trading portfolio. The legal literature refers to this as “trader-based manipulations” (Libor VI, 2016), motivated by “cash flow” (Gandhi, Golez, Jackwerth, and Plazzi, 2019) or “portfolio” incentives (Youle, 2014).

We argue that “trader-base manipulations” are orthogonal to our findings and can be treated as noise in our empirical analysis: First, the communications between traders and submitters seized during the different investigations only mention longer maturities such as the 1 month, 3 month or 6 month tenors, which are used as reference rates for loan and derivative contracts, but not the short rates such as the overnight and one week rates for which rounding is particularly strong.\footnote{To the best of our knowledge, there are no financial contracts that are directly indexed to the very short term Libor rates. The only derivatives that depend on overnight rates are Overnight Index Swaps (OIS), but the standard reference rate used in these contracts is the fed funds rate not the overnight Libor. We therefore think that it is unlikely that banks’ trading portfolios have a strong exposure to the overnight and one week Libor rates.} Note also that when traders and submitters discuss numbers, these are almost never rounded.\footnote{For example in March 16, 2006, a Barclays submitter replies to a swaps trader’s request for a high one-month and low three-month US Dollar Libor as follows “For you ... anything. I am going to go 78 and{92.5}{basis points}. It is difficult to go lower than that in threes looking at where cash is trading. In fact, if you did not want a low one I would have gone 93 at least.” (CFTC, 2012)} In addition, while “trader-based manipulations” were relatively common, overall only a comparatively low fraction of the total submissions seems to have been affected. For example,
the Financial Services Authority (2012) reports that at Barclays, one of the most notorious manipulators, “between January 2005 and May 2009, at least 173 requests for US dollar Libor submissions were made to Barclays Submitters”\(^{10}\) This corresponds to only 1% of the total number of submissions made during this period.\(^{11}\) Finally, “reputational manipulation” and “trader-based manipulations” seem to have happened at different periods. Snider and Youle (2014) demonstrate that “trader-based manipulations” will result in the bunching of quotes around the first and third quartiles. They find evidence for “bunching” but show that this behavior disappears in times of high stress, exactly when the rounding behavior documented in this paper is strongest.

The shortcomings of Libor have given rise to a recent literature on benchmark setting mechanisms and their design: Abrantes-Metz and Evans (2012), Duffie, Skeie, and Vickery (2013), Chen (2013), Diehl (2013), Duffie and Dworczak (2014), Duffie and Dworczak (2014), Hou and Skeie (2014), Duffie and Stein (2015), Coulter, Shapiro, and Zimmerman (2018), Eisl, Jankowitsch, and Subrahmanyam (2017), Duffie, Dworczak, and Zhu (2017) and Gandhi, Golez, Jackwerth, and Plazzi (2019). This literature largely focuses on mechanisms that would prevent future “trader-based manipulation”. The gist of this literature is that incentives to manipulate a benchmark can only be countered by tying submissions closely to verifiable transactions and introducing explicit incentives to tell the truth.\(^{12}\)

In contrast to these papers, our paper demonstrates that, under certain circumstances, even a benchmark based on non-verifiable, judgmental submissions can generate a reliable indicator of market conditions. We also show that this type of benchmark will improve the functioning

\(^{10}\)In this count a request for example for a high 3 month and low 6 month rate would be counted as two requests and a request for high or low submissions which did not specify a particular maturity would be counted as three requests (for one month, three month and six month submissions) unless the context of the communication indicates otherwise.

\(^{11}\)Banks submit daily reports for 15 different maturities, which results in 17265 submissions for the 1150 trading days between January 2005 and May 2009.

\(^{12}\)These insights have been taken into account in the new design of the Libor mechanism which is largely based on verifiable market information, although ICE, the current administrator of Libor has maintained the possibility of submitting estimates that are not based on market transactions, if no other information is available.

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of illiquid and otherwise opaque “over the counter” (OTC) search markets. In this sense, our paper is complementary to Duffie, Dworczak, and Zhu (2017), who have first demonstrated how benchmarks can improve the efficiency of OTC markets. The difference though is that Duffie, Dworczak, and Zhu (2017) focus on the information conveyed by the final Libor rate, not the individual submissions, as in our model. More generally, our paper also contributes to a fast growing literature on the implications of search frictions in the interbank market: Ennis and Weinberg (2013), Afonso and Lagos (2015a), Afonso and Lagos (2015b), Bech and Monnet (2016).

Obviously, our results do not imply that the original design of the Libor mechanism is still adequate today. In a world with large and liquid markets for bank funding, it does not make sense to base such a benchmark on a cheap talk equilibrium, in particular if the benchmark is used as basis for other financial contracts. However, a judgmental, “cheap talk” mechanism might have been the only possibility to generate a benchmark in the late 1970s and early 1980s, when interbank markets were barely developed (MacKenzie, 2008; O’Malley, 2014). Thus, Libor’s setup was likely not flawed from the outset, but, although clearly inadequate now, may well have contributed to the spectacular evolution of the interbank market by improving transparency and liquidity in the early days. Similar mechanisms might be appropriate even today to support the development of other illiquid markets.

The evidence developed in this paper might also have consequences for the ongoing Libor lawsuits. Price stability is generally considered a sign of collusion and the stability of the Libor quotes together with bunching of the quotes around certain rates has been cited as evidence for cartelized behavior (Abrantes-Metz, Kraten, Metz, and Seow, 2012). Our paper indicates that at least part of this price stability might rather be the result of a strategic choice of signal precision. Note, however, that our paper does not allow us to draw conclusions about the suppression of Libor. In cheap talk models there is no specific anchoring of the signal

\[\text{\textsuperscript{13}}\text{See, e.g. Worstall (2017) and Dye (2017).}\]
and therefore, even in a truth telling equilibrium, the signal does not need to correspond to the underlying parameters. For example, a submission strategy that prescribes “submit your borrowing cost minus 20 basis points” qualifies as a fully revealing equilibrium, if the receiver of the signal is aware of this bias. In fact, this type of strategy might be a reasonably good description of what happened during 2008-2009, when almost all panel banks are alleged to have systematically understated their true financing costs (Binham and Thompson, 2017).14

Our empirical part is related to the growing literature analyzing the statistical properties of Libor submissions, but our paper differs in several aspects from much of this literature. Existing studies have largely focused on identifying specific patterns of co-movements or differences between the Libor submissions and other measures of borrowing costs (Abrantes-Metz, Kraten, Metz, and Seow, 2012; Kuo, Skeie, and Vickery, 2012; Monticini and Thornton, 2013; Fouquau and Spieser, 2015; Bariviera, Guercio, and Martinez, 2015). An exception is Abrantes-Metz, Villas-Boas, and Judge (2011) and Rauch, Goettsche, and Mouaouy (2013) who analyze the distribution of digits using Benford’s law and observe strong anomalies that are consistent with our findings. They have, however, neither explained the origin of these anomalies nor uncovered the pattern of rounding that we document in this paper.

Rounding has also been the focus of a number of papers in the accounting literature that provide an alternative motivation (Herrmann and Thomas, 2005; Dechow and You, 2012). These papers show that rounding can be related to a lack of information: For example financial analysts who spend less effort produce more rounded forecasts. We don’t think that this can explain our finding, though. In particular, this is not consistent with the fact that rounding is more common for the shorter, and thus more liquid, maturities, for which information is more

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14 Our model could be modified to incorporate such predictions by including a direct cost of misreporting as modelled by Chen (2013). In his model, as the cost of misreporting goes to zero the reports tend to minus infinity. Obviously, this is only possible because Chen does not put any restriction on the reported rates. In reality, however, one may expect that banks cannot report arbitrarily low rates. If one adds this as a constraint, we expect that the only equilibrium displays pooling at the lowest feasible rate when the net incentives to deviate downwards are sufficiently strong. Thus, a variation of our model that allows for small costs of misreporting would imply the same comparative static predictions as our model and would also predict low rates in the case of uninformative submissions.
readily available.

Finally, our model is related to the recent game theory literature that explores how cheap talk communication can improve the outcome of markets with search frictions (Menzio, 2007; Kim and Kircher, 2015; Backus, Blake, and Tadelis, 2019). The difference with our theory part is that these models do not deliver comparative statics about the informativeness of the cheap talk reports. Our empirical test of these comparative statics may also be of more general game theoretic interest. To the best of our knowledge this is the first paper that explicitly tests the negative relationship between the level of disagreement and signal precision predicted by cheap talk models. These results add to the increasing literature suggesting that cheap talk equilibria do exist outside experimental settings (Goel and Thakor, 2015; Backus, Blake, and Tadelis, 2019) and have the properties predicted by theory.

2 The Model

We start with an informal description of the institutional features of the interbank market and explain how we capture this setup with the three main elements of our theoretical model: The Libor panel banks, the lenders and the stock market.

The members of the Libor panel are large banks that operate at the core of the interbank market. They centralize most of the trades and borrow from a stable network in a frictionless market up to the point of exhausting all their gains from trade.\textsuperscript{15} This determines for each of these banks a (marginal) borrowing cost that we model, for simplicity, as exogenous and assume to be the bank’s private information. It is this interest rate that the Libor mechanism tries to elicit from each of the panel banks.

The panel banks can also borrow from non-relationship lenders that participate in a second

\textsuperscript{15}These banks, sometimes called money-center banks, carry out the majority of transactions in the interbank market with a limited number of counterparties with whom they have stable relationships (Cocco, Gomes, and Martins, 2009), often formalized by contingent credit lines, see Müller (2006).
tier of the interbank market.\footnote{See Craig and Von Peter (2014) for a detailed description of this two tiered structure of the interbank market.} Although most of these lenders are also banks, we refer to them as lenders and use the term banks only for the panel banks. Lenders only operate occasionally and their trades are affected by search costs. In our model, the information disclosed by the Libor reports can mitigate these search costs by helping the lenders to find the most profitable lending opportunity among the banks. The formal search protocol is described below.

Finally, bank management not only wants to make profits from the interbank operations, but is also concerned about the market’s perception of the bank’s financial health. The precise reasons for why banks wanted to project financial health are still being discussed in the ongoing lawsuits (Binham and Thompson, 2017), but it is intuitive that a large number of the bank’s activities, including the interbank market operations, will be negatively affected by a perception of high failure risk. In our model we summarize all of these effects by assuming that, in periods of financial stress, high refinancing costs negatively affect the bank’s fundamental value. Management is assumed to maximize the bank’s stock price, computed mechanically as the bank’s expected fundamental value conditional on the Libor reports, plus the profits from the interbank market.

**Model Setup**

To capture how search frictions interact with the Libor reports we adapt the model of Kim and Kircher (2015). Similar to their setup, we want to abstract from strategic interaction between banks. We therefore assume for simplicity a continuum of (panel) banks\footnote{The main complication of assuming a finite set of banks is that a unilateral deviation of a bank has a nonzero effect on the distribution of quotes received by the other banks and thus on the opportunity cost faced by lenders when making an offer to the deviating bank. Instead of assuming a continuum of banks it is also possible to focus on a single bank and assume that instead of competing with other banks to attract lenders it just competes with a fixed outside option. The results in this case are similar.} with a measure normalized to be one. We also assume a continuum of lenders with measure $\beta$. Lenders are homogeneous but banks differ in their borrowing costs in the first tier of the interbank market.
Efficient bargaining in the first tier of the interbank market means that each bank’s value of an (additional) unit of credit in the second tier is equal to its (marginal) borrowing cost in the first tier. For simplicity, we assume the borrowing cost in the first tier, and thus the value the bank puts on one (additional) unit of credit in the second tier, as exogenous with a cumulative distribution $F$ over the set $\mathcal{V} = [\underline{v}, \overline{v}]$, where $F(v)$ denotes the fraction of banks with a value (i.e. borrowing cost) less than $v$. For technical reasons we assume $F$ to have a density $F'$ bounded above by $\eta > 0$ and below by $\frac{1}{\eta}$.

In this setup, banks and lenders make the following decisions: Banks decide about the report $m$ they submit to the Libor panel after having privately learned their value $v$. Lenders observe these reports and use them to select with which banks to trade. For the sake of tractability, we assume a stylized search and bargaining protocol: Each lender submits a quote, if any, to only one bank and each bank chooses the lowest quote received, unless it is higher than its value. Other search and bargaining protocols should lead to similar albeit less clear-cut predictions.

**Strategies**

We follow the description of the strategies and payoffs of Kim and Kircher (2015) simplified for the sake of readability. A (pure) communication strategy of a Libor panel bank is described by a function $Q : \mathcal{V} \rightarrow \mathcal{M}$ that maps each bank’s value $v \in \mathcal{V}$ into a report $m \in \mathcal{M}$. The lenders’ (mixed) strategy is described by a cumulative distribution function $P$ on $\mathcal{M} \times [0, \overline{v}]$, where $P(m, b)$ is the fraction of lenders that submit a quote less than $b$ to banks that reported less than $m$.\(^{18}\) There is no need to allow for bids above $\overline{v}$ as they are always rejected by the banks. Public beliefs about a bank’s value conditional on a given submission can be characterized by a distribution function $\mu : \mathbb{R}_+ \times \mathcal{M}$, where $\mu(v, m)$ denotes the fraction of banks whose values are believed to be strictly below $v$ among those banks that reported $m$.

\(^{18}\)Note that we have dropped the dependence of the lender’s strategy on the observed distribution of reports submitted to the Libor panel. This is without loss since our assumption that there is a continuum of banks means that a unilateral deviation of a bank does not change the observed distribution of reports.
As in Kim and Kircher (2015), we are interested in interval partition equilibria. These are equilibria in which the banks’ communication strategy partitions \( \mathcal{V} \) into intervals. Each of these intervals is what we call a pooling interval as all banks with a value in the pooling interval submit the same report and banks with values in different pooling intervals submit different reports. Our main result is in terms of the size of these pooling intervals, which we refer to as the coarseness of the equilibrium. For notational convenience, we assume that the set of possible reports \( \mathcal{M} \) is identical to the set of values \( \mathcal{V} \) and assign higher reports \( m \in \mathcal{V} \) to higher pooling intervals. Thus, in an interval partition equilibrium, the communication strategy of banks is an increasing function \( Q \) and the induced distribution of reports is characterised by

\[
Q_{\mathcal{M}}(m) = F(\sup\{v : Q(v) = m\}),
\]

where \( Q_{\mathcal{M}}(m) \) is the fraction of banks that report \( m \) or less to the Libor panel.

**Payoff Functions**

To provide a more intuitive description of the payoff functions, we describe the lenders’ strategy using the concept of “queue length” borrowed from the literature on markets with a continuum of agents and search frictions. Formally, the queue length \( \lambda(\cdot, b) \) is the Radon-Nikodym derivative of \( \beta P(\cdot, b) \) with respect to the distribution of reports submitted to the Libor panel \( Q_{\mathcal{M}} \).\(^{19}\) Intuitively, \( \lambda(m, v) \) is the expected number of quotes weakly below a certain threshold \( b \) received by a bank that submits a report \( m \). Thus, integrating over the set of possible reports

\[
\int_{[v,m]} \lambda(\tilde{m}, b) dQ_{\mathcal{M}}(\tilde{m}) = \beta P(m, b), \text{ for any } m \in \mathcal{V}.
\]

It combines the likelihood of the lenders making an offer to a bank with report \( m \) with the likelihood of the bank having chosen \( m \) and the lenders’ conditional distribution of quotes after having chosen to make an offer to a bank with report \( m \).

\(^{19}\)This is a function whose integral with respect to \( Q_{\mathcal{M}} \) is equal to \( \beta P(\cdot, b) \):
\( \mathcal{M} \) one obtains the total mass of quotes:

\[
\int_{\mathcal{M}} \lambda(\tilde{m}, \mathcal{P}) \, dQ_{\mathcal{M}}(\tilde{m}) = \beta. \tag{1}
\]

The usefulness of \( \lambda \) comes from the following observation. In a version of our model with finitely many agents, the equilibrium distribution of quotes less than \( b \) received by each bank having reported \( m \) follows a binomial distribution. This distribution converges to a Poisson distribution with parameter \( \lambda(m, b) \) as the number of agents tends to infinity.\(^2\) Thus, \( e^{-\lambda(m,b)} \) is the limit probability that a bank reporting \( m \) does not receive any quote weakly below \( b \). We use this limit probability below, to characterize the bank’s payoffs.

The bank chooses its report to maximize the expected fundamental value plus its profits in the interbank market. For simplicity, we assume that the bank only borrows one additional unit of credit in the second tier of the interbank market and thus its profits are equal to the difference between the value \( v \) that the bank puts in this unit of credit and the lowest quote received, if less than \( v \). As explained above, the probability that the lowest quote is less than \( b \) is \( 1 - e^{-\lambda(m,b)} \). Besides, the bank’s fundamental value is computed from the public beliefs \( \mu(v, m) \) under the assumption that the bank’s fundamental value is a function \( w(v) \) differentiable and decreasing in the bank’s value \( v \). The bank’s expected payoff is thus:

\[
V(m, v; \lambda, \mu) \equiv \int_{b \leq v} (v - \tilde{b}) \, d\left(1 - e^{-\lambda(m, \tilde{b})}\right) + \int w(\tilde{v}) \, d\mu(\tilde{v}, m). \tag{2}
\]

A lender offering an interest rate \( b \) to a bank reporting \( m \) is accepted if no other lender submits a lower quote and the bank’s value is larger than \( b \). The probability of the former event is, again by a limit argument, \( e^{-\lambda(m, b)} \) in the case in which \( e^{-\lambda(m, r)} \) has no atom at \( b \), and the

\(^{20}\)Formally, this equation is a direct consequence of the definition of \( \lambda \) as a Radon-Nykodym derivative of \( P \), see Footnote 19.

\(^{21}\)For a formal derivation see Kim and Kircher (2015) Footnote 16 and Section 4.

\(^{22}\)To avoid confusion, we use a tilde on the variable of integration here and in what follows.
probability of the latter event is $1 - \mu(b, m)$. Since we assume that the lender’s opportunity cost of funds is normalized to zero and that the interbank loans are risk-free,\footnote{Assuming a probability of default increasing with the bank’s value plays a similar role as our assumption that the bank’s fundamental value is decreasing in its value for additional credit: it gives the bank incentives to submit a lower report. The analysis of this type of setup is, however, slightly more complex as an increasing probability of default affects the equilibrium search of lenders. This is the reason we only consider the stock market effect.} the lender’s expected payoff is:

$$U(m, b; \lambda, \mu) = e^{-\lambda(m,b)} (1 - \mu(b, m)) b,$$

(3)

if $e^{-\lambda(m,\cdot)}$ has no atom at $b$. The former expression has to be modified to account for ties if $e^{-\lambda(m,\cdot)}$ has an atom at $b$. This case is less relevant for our analysis because a Bertrand argument implies here that the distribution of quotes cannot have atoms. The formal details can be found in Kim and Kircher (2015).

Note that the lenders’ payoff function (3) is only well-defined for reports $m$ in the support of $Q_M$. This is sufficient for equilibrium existence because we can always define out-of-equilibrium beliefs that guarantee lack of incentives to deviate outside the support of $Q_M$. For instance, we could fix the interpretation of any out-of-equilibrium messages as identical to one of the equilibrium messages. This is a common property of games with cheap talk communication. Besides, standard equilibrium selection arguments are not used as messages are not directly payoff relevant, see the discussion in Banks and Sobel (1987), Section 5.

We are interested in the case in which the ratio of lenders to banks is sufficiently high to guarantee that all banks have a positive probability of receiving a quote in equilibrium. This is consistent with the fact the Libor panel only includes banks that are active in the interbank market. As we show in the proof of Lemma 1, a sufficient condition for this is,

$$e^{\beta} > \frac{\int_0 \bar{v} dF(\bar{v})}{\bar{v}}.$$  

(4)
Equilibria

**Definition:** An interval partition equilibrium is a communication strategy $Q$, a distribution of quotes $P$ (with associated queue length $\lambda$) and a belief system $\mu$ such that:

- **Bank’s Optimality:** For any $v \in V$,

$$V(Q(v), v; \lambda, \mu) \geq V(m', v; \lambda, \mu), \forall m' \in M.$$ 

- **Lender’s Optimality:** $(m, b) \in \text{supp } P(\cdot, \cdot)$ implies

$$U(m, b; \lambda, \mu) \geq U(m', b'; \lambda, \mu), \forall (m', b') \in M \times \mathbb{R}^+.$$ 

- **Belief Consistency:** For each $m \in M$, if $Q^{-1}(m) = \{v\}$ then $\mu(v, m)$ puts all its probability mass at $v$, and if $Q^{-1}(m) = [v, v']$, $v' > v$, then,

$$\mu(\hat{v}, m) = \frac{F(\hat{v}) - F(v)}{F(v') - F(v)} \text{ for } \hat{v} \in [v, v']. \quad (5)$$

In the next lemma, we pin down the value of $\lambda$ from the lender’s optimality condition that requires that the lender must get the same payoff, denoted by $u^*(Q)$, in the support of its strategy.

**Lemma 1.** For any given communication strategy $Q$ and report $m$ in the range of $Q$, the lender’s optimality and (1) are satisfied if and only if $\lambda(m, b) = 0$ for $b < u^*(Q)$ and:

$$\lambda(m, b) = \sup_{b \in [0, b]} \left\{ \ln \left( \frac{1 - \mu(\hat{b}, m) \hat{b}}{u^*(Q)} \right) \right\} \text{ for } b \geq u^*(Q), \quad (6)$$
where

\[ u^*(Q) \equiv e^{\int_{\hat{b} > b}^\infty \ln((1 - \mu(\hat{b}, \tilde{m}) \hat{b})) dQ_{\hat{b}}(\tilde{m})} e^{\beta}. \] (7)

The value of \( u^*(Q) \) is computed as the unique solution to the equation resulting from substituting (6) into (1).

For \( v \) equal to the infimum of the pooling interval associated to \( m \) and \( \delta \) its size, the right hand side of (6) has a particularly simple form in two cases that we use in our next discussion:

(C1) \[ \lambda(m, b) = \ln \left( \frac{b}{u^*(Q)} \right) \text{ if } b \in [u^*(Q), v]. \]

(C2) The support of \( \lambda(m, \cdot) \) is equal to \([u^*(Q), v]\) if \( \delta \leq \frac{v}{\eta^2}. \)

The former property holds true because \( \mu(\hat{b}, m) = 0 \) for \( \hat{b} < v \), and the latter because the sup-term in (6) is decreasing in \( \hat{b} \), and thus \( \lambda \) constant, for \( \hat{b} \geq v \) when \( \delta \) is sufficiently small.

Property (C1) is the key insight of the work of Kim and Kircher (2015) that simplifies our analysis. It means that our particular bargaining and search protocol has the property that the distribution of quotes below \( v \) does not vary with a larger report. Property (C2) simplifies the bank’s payoff description and it is consistent with our interest in equilibria with small pooling intervals.

In what follows, we take as given the lenders’ play in Lemma 1 and study the bank’s optimality condition. As is usually the case in cheap talk games, a necessary and sufficient condition is that the banks with values in the extreme points of the pooling intervals do not have incentives to deviate. In our case, this is a consequence of the supermodularity of (2) in \((m, v)\) when \( \lambda \) is as in (6). We study these incentives to deviate next.

\(^{24}\)The condition \( \delta \leq \frac{v}{\eta^2} \) implies that \(-bF'(\hat{v}) + F(\hat{v} + \delta) - F(\hat{v}) \leq -\frac{\delta}{\eta} + \delta \eta \leq 0 \) for any \( b \geq v \). This inequality and (5) implies that the term between curly brackets in (6) is increasing in \( \hat{b} \) up to \( \hat{b} = v \) and then decreasing. Hence, the supremum in (6) is achieved at \( \hat{b} = v \) when \( b \geq v \), which means that \( \lambda \) is constant in \([b, \overline{v}]\) and thus puts zero measure in \([b, \overline{v}]\).
Take the lower bounds \( v \) and \( v' \) corresponding to the pooling intervals associated to the reports \( m \) and \( m' \). (C1) means that two different reports \( m \) and \( m' \) induce the same queues at any price \( b \) less than both \( v \) and \( v' \). This implies that a bank with value \( v \) does not improve by submitting a higher report \( m' > m \): The deviation does not change the distribution of quotes below \( v \) but implies a lower fundamental value. (C1) also implies that submitting a lower report \( m' < m \) only changes the distribution of quotes above \( v' \). Besides, if (C2) applies, the bank does not get any quote in \([v', v]\) with the deviation \( m' \). Thus, the incentive to deviate from \( m \) to \( m' \) is equal to:

\[
\left( \int w(\tilde{v})d\mu(\tilde{v}, m') - \int w(\tilde{v})d\mu(\tilde{v}, m) \right) - \int_{v'}^v (v - \tilde{b})d \left( 1 - \frac{u^*(Q)}{b} \right).
\]

If (C2) does not apply, the loss of quotes in a downward deviation is smaller and thus (8) becomes a lower bound to the incentives to deviate.

The first term of (8) is the increase in the bank’s fundamental value when it deviates and submits a lower report \( m' \). The second term corresponds to the forgone profits in the interbank market due to the lost quotes after the deviation. The first term is absent when the bank’s value (i.e. cost of borrowing) is irrelevant for the fundamental value, formally when \( w(v) \) is constant in \( v \). In this case, there is a fully revealing equilibrium. However, there is no fully revealing equilibrium whenever the stock market is sensitive to the bank’s value. This can be deduced from (8): Submitting a report \( m' \) marginally lower than \( m \) gives first order gains from improving the fundamental value, whereas the losses in the interbank market are of second order as only quotes close to \( v \) are lost. A variation of this argument gives a lower bound to the size of the pooling intervals which is the basis for our empirical analysis.

**Proposition 1.** There is no interval partition equilibrium strategies \((Q, P, \mu)\) in which the size
of any of the pooling intervals is less than:

\[
\inf |w'(v)| \cdot e^\beta \cdot \frac{v^2}{\eta^2}. \tag{9}
\]

A variation of the above argument also implies that there exists an equilibrium with pooling intervals smaller than a similar bound to (9), if some additional conditions are met.

**Proposition 2.** If \( \sup |w'(v)| \leq \frac{v^2}{2\eta^4 \epsilon^4} \) and \( \int_{v'}^{v} w(\tilde{v}) dF(\tilde{v}) \) is concave in \( v \),\(^{25}\) then there exists some interval partition equilibrium strategies \((Q, P, \mu)\) in which the size of each of the pooling intervals is less than:

\[
\sup |w'(v)| \cdot e^\beta \cdot \frac{2\eta^2}{v^4}. \tag{10}
\]

The particular value of the bounds is derived in the proof from approximations to the elements of (8) using Lemma 1 and the implication of (7) that \( u^*(Q) \) must lie in \([\frac{v}{e^\beta}, \frac{v}{e^\beta}]\).

The size of the pooling intervals reflects the coarseness of the information disclosed. Thus, Propositions 1 and 2 show how the maximum equilibrium information disclosure varies with the slope of \( w \) and the ratio of lenders to banks \( \beta \). The former is intuitive. The more the stock market is sensitive to the report, the more the bank benefits from underreporting, which explains why the maximum equilibrium information disclosure decreases. The ratio of lenders to banks affects coarseness through the competitiveness of the interbank market. The greater this ratio, the greater the competition among lenders and thus the lower their expected quotes. Thus, a deviation that sheds away high quotes is less costly for the bank.

\(^{25}\)The first condition of Proposition 2 guarantees that the upper bound in (10) is sufficiently small for (C2) to apply. The second is a regularity condition that it is satisfied, for instance, when \( w \) is linear and \( F \) uniform. The role of this regularity condition is to guarantee that the indifference condition that usually defines the size of the pooling intervals in a cheap talk equilibrium has no more than one solution. An equivalent condition is satisfied in Crawford and Sobel (1982) as a consequence of their assumption that a sender’s utility is concave in the receiver’s action.
Note that the two effects pointed out in the paragraph above reinforce each other. The greater the stock market sensitivity, the greater is the effect of the ratio of lenders to banks on the minimum coarseness of the equilibrium. To see why, notice that whereas the minimum equilibrium coarseness varies with the ratio of lenders to banks if the stock market is sensitive to the submission, this is not the case otherwise. The reason is that there exists a fully revealing equilibrium when the fundamental value is not sensitive to the reports, and thus the ratio of lenders to banks has no effect on the minimum coarseness of the equilibrium.

Proposition 2 also sheds some light on the welfare implications of the design of the Libor panel. On the one hand, it shows that the feedback of the Libor reports on the functioning of the interbank market can be sufficient to induce honest Libor reporting in the cases in which the Libor reports have a negligible effect on the stock market. However, this deterrence is not sufficient once the stock market becomes more reactive to the Libor reports which seems plausible in periods of crisis. On the other hand, the analysis of Kim and Kircher (2015) shows that full information disclosure in the Libor panel implies that the equilibrium queues of Lemma 1 maximize the sum of the payoffs of banks and lenders. Together with Proposition 2, this implies that, as the effect on the fundamental value becomes negligible, the information disclosed by the Libor reports induce the maximum social surplus that can be created in the interbank market subject to the search costs of Lemma 1.

3 Data and Graphical Evidence

In this section, we describe the data and illustrate the pattern of rounding with a series of simple graphs. Our data set consists of daily reports submitted by the up to 18 panel banks\footnote{We use the Reuters Libor codes for the banks. The banks are Bank of America (BAFX), Bank of Tokyo-Mitsubishi UFJ Ltd (BTML), Barclays Bank plc (BARL), Citibank NA (CTGL), Credit Suisse (CSBL), Deutsche Bank AG (DBBL), HSBC (HSBL) and JP Morgan Chase (JPML) BNP Paribas (BPGL), Credit Agricole CIB (CALL), Lloyds Banking Group (LOYL), The Norinchukin Bank (NORL), Rabobank (RABO), Royal Bank of Canada (RBCL), The Royal Bank of Scotland Group (RBSL), Sociétè Générale (SGBL), Sumitomo Mitsui Banking Corporation (SUML) and UBS AG (UBSL). We have not included West LB and} for all
15 maturities obtained from Reuters. We complete these data with the banks’ credit default swap (CDS) spreads (see Figure S1 in the Online supplementary material). CDS contracts are standardized insurance contracts against credit risk, which are traded in comparatively liquid markets. They are usually written for longer maturities, ranging from 1 to 30 years. To maximize comparability with the short term Libor rates, we use spreads on the 1 year contract, obtained from Markit.

We use ten years of data from January 2005 to December 2014. The main analysis focuses on the period between 1.1.2005 and 30.4.2013, during which CDS data are available for almost all banks and during which the Libor setting process remained unchanged. Our panel is not fully balanced as a number of banks entered and left the Libor panel during and after the crisis.

After April 2013 the ICE Benchmark Administration (IBA) who had succeeded the BBA as the administrator of Libor implemented two key changes: It eliminated eight of the less liquid tenors out of the original 15 maturities to make sure that Libor submissions can be explicitly supported by transaction data and it delayed the publication of individual submissions by three months so that individual submissions cannot be “interpreted as signals (often erroneously) of a change in the creditworthiness of a submitter” (ICE, 2015, p.30). As an illustration for the submission process and the structure of the data, Table 1 presents all submissions from the US dollar Libor panel banks to the BBA on the 10th of October 2006.

[Table 1 about here.]

HBOS which failed during the financial crisis.

27 Until April 2013, Libor submissions were provided for the following maturities: Overnight (ON), one week (SW), two weeks (2W), one to 11 months (1M, 2M,..., 11M) and one year (1Y).

28 Different types of CDS contracts are used in different geographic regions. To maximize liquidity and data availability we have used the junior contract with the MR/Modified Restructuring clause for North American banks, the MM/Modified Modified Restructuring for European and the CR/Complete Restructuring clause for Japanese banks. For 2014, we replaced these contracts with the corresponding contracts following the new 2014 credit derivatives definitions.

29 See Snider and Youle (2014) for a precise description of changes in the panel.

30 The remaining tenors are the overnight (“ON”), single week (“SW”), one, two, three and six month (“1M”, “2M”, “3M”, “6M”) as well as the one year rate (“1Y”).
Figure 1 shows the overall evolution of the Libor benchmark rates (i.e. the interquartile means) for all 15 maturities. The two vertical lines indicate two key dates in the crisis. The first one indicates July 31st, 2007, when Bear Stearns’ High-Grade Structured Credit Fund collapsed. The second line indicates September 15th, 2008, when Lehman Brothers filed for bankruptcy. The spike in refinancing rates corresponding to the crisis entering its climax in the second half of 2008 is clearly visible. Figure 2 plots the daily standard deviation of the submissions across different banks for each maturity. This illustrates that the crisis period was not so much exceptional for the absolute size of the panel banks’ borrowing costs, but rather for the strong dispersion of refinancing rates between banks.

[Figure 1 about here.]

[Figure 2 about here.]

The crisis period also corresponds to a clearly visible change in the banks’ submission patterns. Table 2 again presents the full number of submissions by panel banks, but now two years later, for the 10th of October 2008, roughly one month after the Lehman failure. Compared to Table 1 the increase in rounding is obvious. Not only do most banks now submit only two instead of three decimals, the second decimal also is rarely different from “5”.

[Table 2 about here.]

That coarseness is not caused by a lack of information becomes particularly clear when one compares the evolution of bidding behavior before and after the 2013 reform that delayed the publication of individual submissions. As an example for the change in the bidding behavior we plot in Figure 3 the submissions by BNP Paribas for the seven tenors that remained after the reform. Clearly, more information is revealed in the bids submitted after April 2013. Interestingly, while BNP Paribas is not the only bank to change its behavior, this is not true
for all banks.\footnote{For example Bank of America’s rounding behavior remains unchanged by the reform (see Figure S2 in the online supplementary material).} Explaining the heterogeneity in submission strategies after the reform is beyond the scope of this paper as our model requires submissions to be visible, but the change in bidding strategies as well as the differences in bidding strategies across banks strongly suggest that the coarseness of submissions is the result of a strategic choice.\footnote{Figures S5 and S6 in the online supplementary material provide some additional graphical evidence for the change in bidding strategies. Until the 2013 reform, successive bids were strongly correlated, but this autocorrelation abruptly decreases after the reform. We think that this is related to the recommendation of the Wheatley report (Wheatley, 2012) and the new ICE submission guidelines implemented after April 2013, which encouraged banks to rely less on “expert judgment and instead justify their submissions by documenting underlying market transactions. It seems that, as a consequence, banks’ submissions became less stable, as they started to reflect not only overall market conditions, but also transaction-related idiosyncratic factors such as bid size, transaction costs and the relative bargaining power of the lender and borrower. Note, that the heterogeneity in bidding strategies has been identified as a problem by the IBA. They state in their first position paper that after the reform that “each benchmark submitter has developed its own methodology for establishing LIBOR submissions. A variety of approaches now exists” and encourage banks to work on a conversion of approaches (ICE Benchmark Administration Limited, 2014).} 

To understand the bidding behavior in more detail, we analyze the distribution of digits used for the second decimal of the individual submissions. A change in this decimal corresponds to one basis point, the usual unit of measure in fixed income markets.\footnote{As a basis point is 1/100th of a percentage point.}

Our theoretical model predicts that the coarseness of Libor submissions depends on the sensitivity of the bank’s expected value to the information from the interbanking market. Keeping other things equal, this sensitivity is expected to be high, when a bank’s CDS spread is high, as in this case, investors are wary of additional bad news. With a series of univariate comparisons we try to illustrate that, indeed, as predicted, CDS spreads and liquidity are related to the coarseness of submissions and that they reinforce each other.

Figures 4 plots the distribution of digits for banks that have, on a given day, a CDS spread in the top and bottom quartile of CDS spreads observed during 2005-2014. Note that we adapted the y-axis to better visualize the differences. The horizontal line at 0.1 corresponds to the frequency we would expect if the distribution was uniform across all digits.
As predicted, the distribution of digits strongly varies with CDS spreads. Evidence of rounding is clearly visible in Figure 4 (b) corresponding to the most risky periods and banks, with the digits “0” and “5” occurring roughly 50% more often than other digits. In contrast, the distribution for periods and banks with low CDS spreads in Figure 4 (a) is closer to the uniform distribution.

[Figure 4 about here.]

Our theoretical model also predicts that coarseness of the Libor submissions should vary with the liquidity of the underlying interbank market. Measuring liquidity is, however, more difficult as few detailed data about volumes in interbank markets are available. As time and cross-bank differences in liquidity are strongly correlated with the market’s perception of credit risk, we will exploit cross-maturity differences in transaction volumes. Afonso, Kovner, and Schoar (2011) and Kuo, Skeie, Vickery, and Youle (2014) demonstrate that liquidity is higher for short term rates, such as the overnight or one week rates, as well as for a number of commonly used reference tenors, such as the 1 month, 3 month and 1 year maturities. If we compare the distribution of digits as a function of liquidity/maturity a clear pattern emerges. As an example we reproduce in Figure 5 the distribution of digits for the (very liquid) overnight and the (very illiquid) eight month maturities.

[Figure 5 about here.]

To generate a simple measure of rounding across the 15 different maturities, we next construct a dummy variable “Rounding in the 2nd Decimal”, indicating that a bank submitted “0” or “5” as the second decimal and did not provide any smaller decimals. If banks submit only two decimals and do not round, we would expect the average value of this dummy to be 0.2. With three or more decimals the dummy should always be zero. This measure captures
coarseness in the most relevant range, but we lose some information about more extreme levels of coarseness, as rounding may also take place in the first, third and fourth digits.\textsuperscript{34}

Figures 6 (a) and (b) represent the frequency of rounding in the second decimal across maturities for banks with CDS spreads in the top and bottom quartiles. Again, a clear pattern can be recognized. Whereas the frequency of rounding is low and relatively stable across maturities for quiet markets, it is much higher and decreases overall with maturity in turbulent markets. Interestingly, certain more heavily traded reference rates such as the 3 month, 9 month and 1 year tenors exhibit more rounding than the neighboring less liquid tenors. These patterns fit perfectly with our predictions, but are difficult to reconcile with alternative explanations for rounding, as explained in more detail in Section 4.3.

[Figure 6 about here.]

The evolution of rounding corroborates these patterns. Figure 7 plots a 63 day\textsuperscript{35} moving average of rounding in the second decimal across all maturities for the different panel banks. The change in rounding behavior at the two key dates corresponding to the failures of the Bear Stearns fund and Lehman Brothers is clearly visible. There is also a visible difference in the extent of rounding between banks. A similar picture can be drawn for the evolution of rounding in the different maturities averaged out across banks (Figure 8). The high frequency of rounding for the very short term rates between the two key events of the unfolding crisis is particularly striking. Again this is perfectly in line with our theory: During this time, serious concerns about bank stability had emerged, but banks did not yet rely on central banks for liquidity provision and interbank markets still had high trading volumes. The highest frequency of rounding is visible in the overnight rates at the crisis dates, other key rates such as the three month or nine month rate also show a higher occurrence of rounding.

\textsuperscript{34}See the online supplementary material for graphical evidence using the number of submitted decimals as an alternative proxy for coarseness.

\textsuperscript{35}This corresponds to the approximate number of trading days in three months.
4 Multivariate Analysis

In this section, we present simple multivariate tests to complement the graphical evidence from the previous section and assess the statistical significance of the observations made above. Recall that Propositions 1 and 2 from our model predict that 1) the minimum coarseness of the Libor submissions increases when the bank’s expected market value becomes more sensitive to the information from the interbank market, 2) coarseness increases with the liquidity of the underlying interbank market and, 3) these effects interact positively, i.e. the coarseness of the submissions reacts stronger to the sensitivity to information from the interbank market in more liquid markets.

As in the previous section, we focus on the banks’ one year subordinate CDS spreads as a proxy for the sensitivity of the bank’s market value with respect to the interbank borrowing costs and measure the coarseness of the submissions with the dummy variable “Rounding in the 2\textsuperscript{nd} Decimal” defined above. To proxy for liquidity we introduce a new indicator variable “Liquid Maturities” that takes the value of 1 if the submission is for one of the tenors that survived the 2013 reform.

We analyze again 10 years of data from January 2005 to December 2014 with a focus on the period until April 2013, as from May 2013 on the Libor reform ended the immediate disclosure of individual submissions and removed more than half of the submitted tenors. This corresponds to a total of 530,952 individual submissions across different banks, maturities and trading dates. For 489,158 of these observations we have CDS spreads, as for the non-listed Norinchukin Bank and Rabobank 1 year CDS data are not available in the first years of our
sample. Summary statistics are simple: The average CDS spread is 1.04 percent with a median of 0.58 and a standard deviation of 1.34 percent. The highest CDS spread in our sample is 15.3% attained by Citigroup in April 2009 and the lowest 0.012% for Rabobank in May 2007. The average frequency of rounding in the second decimal is 20.3 percent.

4.1 Baseline Specification

To test how risk and liquidity affect the coarseness of the banks’ submissions, we regress the “Rounding” dummy on the banks’ CDS spreads together with the “Liquid Maturities” dummy and an interaction term for these two variables. Formally, we run the following regression:

\[
\text{Rounding}_{t,b,m} = \beta \cdot \text{CDS spread}_{t,b} + \gamma \cdot D(\text{Liquid Maturities})_m + \lambda \cdot D(\text{Liquid Maturities})_m \times \text{CDS spread}_{t,b} + \mu_b + \mu_t + \epsilon_{t,b,m},
\]

where \(\mu_b\) and \(\mu_t\) stand for bank fixed effects and time controls, respectively. Despite our dependent variables being binary, we choose to use an ordinary least square estimation (OLS) of a linear probability model rather than logit or probit. This will considerably simplify the analysis of autocorrelated errors with no clear structure in our panel setting. Given that for our preferred specification with annual time controls all predicted values in our sample lie in the interval \([0, 1]\), the potential bias of the linear probability model is likely to be small.\(^{36}\)

Simple OLS standard errors for this regression will be biased due to the presence of heteroskedasticity and serial as well as cross sectional correlation. To cope with these biases, we determine robust standard errors that are two-way clustered at the bank and time levels.

The results are presented in Table 3. In models (1) and (2) with annual time controls, all

\(^{36}\)See Horrace and Oaxaca (2006). In the specification with daily dummies, predicted values lie outside the interval \([0, 1]\).
coefficients have the predicted sign and are significant. Both, a higher CDS spread and more liquid maturities are associated with coarser submissions i.e. a higher frequency of rounding. The sign of the interaction term is positive, indicating that coarseness is more sensitive to CDS spreads for more liquid maturities. In models (4) and (5), with full time fixed effects at daily frequency, the effect of liquidity on rounding remains significant. The coefficient for the interaction term is again positive with similar size, but only significant at the 10% level, and the coefficient for the CDS spread still has the correct sign, but is not significant.

[Table 3 about here.]

These results imply that the relationship between CDS and rounding is mostly identified from the time variation. Indeed, the average risk of the banking sector seems to be more relevant than the individual CDS spreads. In model (3), we add the daily cross-bank average of CDS spreads to the baseline regression. The results are clear: This measure of the overall banking sector risk is highly significant, whereas individual CDS spreads become non-significant. This surprising result might be caused by poor data quality. We only have quotes, not transaction data for CDS spreads, and liquidity was likely low, especially at the height of the crisis. Possibly, some of this noise can be reduced by taking the cross sectional mean.

However, there might also exist a more fundamental explanation. The large banks contained in the Libor panel are affected by similar risk factors and strongly interconnected. If one or several banks in the sample are known to be in trouble, reflected by high CDS spreads, investors will be particularly attentive to signals that could indicate that any other bank might have a problem as well, either because it is exposed to the same risk factors or because it is connected to the risky bank. Clearly, in this situation, a bank that would disclose high borrowing costs (even if its CDS spread was initially low) could suddenly raise alarm about its prospects. Possibly, banks considered to be healthy might even be more concerned about releasing negative signals.

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37 These results do not change if we exclude the submitting bank’s CDS spread when calculating the average.
than banks that are known to have problems and can, in any case, be expected to face high borrowing costs in the interbanking market. In this context, the main driver for the sensitivity of a bank’s expected value to the information from the interbanking market might not be the banks own CDS spread, but rather the overall perception of the riskiness of the banking sector reflected by the average CDS spread.

In the following sections, we will for convenience mostly keep individual CDS spreads rather than the cross sectional average as independent variable and use annual time controls. Average CDS spreads and alternative time controls provide largely similar results, but if too many time controls are added, the coefficient for CDS spreads becomes insignificant.\(^{38}\) Further evidence confirming the robustness of our results is presented in the next section, where we analyze the evolution of rounding pattern and demonstrate that our results remain significant for several non-overlapping sub-periods.

We have also run the above regressions separately for each of the 18 panel banks.\(^ {39}\) These tests demonstrate that there is a fair level of heterogeneity. Some banks do not seem to exhibit the rounding pattern that we have identified in the regressions above. We think that this is not inconsistent with our theory. Our model can only provide a lower bound for the length of signal intervals, but we cannot exclude that some banks play equilibria with larger intervals or the “babbling equilibrium”, where submissions have no informational content.

As an additional robustness test we have replicated our baseline regressions with other measures for coarseness of the submissions. In the online supplementary material, we provide the results of using the number of submitted digits instead of “Rounding in the 2nd Decimal” as the dependent variable. With this measure of coarseness our results are verified for 2005-2010,

\(^{38}\)See Table S1 in the online supplementary material for a version of models (1) and (2) in Table 3, where individual CDS spreads are replaced by average CDS spreads. We cannot add average CDS spreads to models (4) and (5) since they are perfectly multicollinear with daily fixed effects. Still, the cross effect of the average CDS spread with liquidity is positive and significant at the 5% level. For alternative time controls, see Table S2 in the online supplementary material presenting our baseline regression with time controls according to the different phases of the crisis as defined by Kuo, Skeie, Vickery, and Yonle (2014).

\(^{39}\)The results are in Table S5 in the online supplementary material.
where very few banks submitted more than three decimals (See Figure S3 and S4 in the online supplementary material), however in the later period the high number of decimals submitted by certain banks render the coefficient on CDS spreads not significant for the full period.

4.2 Evolution of Rounding Patterns

More evidence comes from a finer analysis of how the banks’ rounding behavior changes over time. Figures 9 and 10 depict the evolution of regression coefficients when estimating equation (11) for moving two-year subperiods. To generate the quarterly results used for the graph, we have run separate regressions, each with 24 month of data, covering a time window stretching from 12 months before to 12 months after the beginning of the respective quarter.\footnote{The time windows at the beginning and the end of the period are truncated for dates below 2005 and above 2015.} Note, that in order to be able to include the years 2013 and 2014 we only use in these regressions the seven maturities remaining after the 2013 reform. As a consequence we have to use a more restrictive definition of liquid maturities. We know from Kuo, Skeie, Vickery, and Youle (2014) that there exist a number of reference rates with higher liquidity, such as the overnight ("ON"), three month ("3M"), six month ("6M") and one year ("1Y") tenor and we use these reference tenors to identify more liquid markets.

[Figure 9 about here.]

[Figure 10 about here.]

The relationships between rounding, maturity and liquidity predicted by our model are both clearly present over the crisis period from 2007 to 2009, but then fade out rapidly. This is probably because successful liquidity injections by central banks reduced the informativeness of lending conditions in the interbanking market as a sign of financial distress and therefore weakened the sensitivity of a banks expected value to the interbank rates. In addition, starting
in 2009, banks seem to have become increasingly aware of the legal risks related to suspicious Libor submissions pattern. The trigger likely was the British Financial Services Authority’s (FSA) inquiry into Barclays in late 2009, followed in Nov. 2009 by the BBA’s issuance of new guidelines on setting Libor rates. At this time attention shifted from potential “Libor suppression” (Mollenkamp and Whitehouse, 2008) to the suspicion of “trader-based manipulations” (Snider and Youle, 2014) and banks started to avoid patterns that could be interpreted as the result of these manipulations. In particular, several banks rapidly increased the number of reported decimals (see Figure S3 in the online supplementary material). It seems likely that they did this to minimize legal risk rather than for reasons related to our model.

In 2013, at the time of the fundamental Libor reform, the two effects predicted by our model had already become insignificant.\(^{41}\) This makes it likely that the explanation for the change in bidding strategies after the reform that we documented with Figure 3 in Section 3 lies outside our model.

### 4.3 Excluding Alternative Explanations

The results listed above are consistent with our model, but there exist alternative mechanisms that might be able to explain our observations. For example, it is conceivable that the relationship between rounding and liquidity is actually generated by a relationship between rounding and maturity. Shorter maturities may require less precision given that the monetary consequences of investing at a rounded rate are less important.\(^{42}\) This story, however, does not seem consistent with the fact that the liquidity effect that we identify here fades gradually after 2009, as shown in Figure 10, or the fact that coarseness seems to be a strategic choice, as we discuss

\(^{41}\)Using “Number of submitted decimals” as the dependent variable confirms these patterns, see Figure S7 and Figure S8 in the online supplementary material.

\(^{42}\)Maturity dependent rounding exists in other markets. For example, bids in government auctions for US Treasury bills, which have a maturity of less than one year, are made in discount rates quoted in three decimals with 0.005 increments. Bids in the longer term Treasury bond auctions are made in yields with 0.001 increments (Department of the Treasury, 2004). Note, that in this example rounding is in the third decimal of the yield. This implies a much finer grid than rounding in the second decimal.
in Section 3. More importantly, we can exploit the fact that the relationship between both variables is not monotonous to test whether rounding is driven by maturity or liquidity. As mentioned above, certain references rates have higher liquidity than the neighboring tenors. If rounding is driven by maturity, the more liquid reference rates should not exhibit more rounding than surrounding less liquid rates with, on average, the same maturity.

Table 4 presents the result of regressing rounding on liquidity and CDS spreads for different sub-samples, each including a liquid reference rate such as the ON, 3M, 6M and 1Y tenor together with the neighboring less frequently traded tenors. Only for the six month rate the level of rounding is not significantly different from the surrounding rates, likely because liquidity is low. In all other cases, submissions for reference rates exhibit significantly more rounding than the surrounding less liquid maturities. In particular, the one year rate is more frequently rounded than the submissions for the shorter, but less liquid 11 month tenor and the three month rate is more frequently rounded than the surrounding two month and four month rates. That the overnight rate exhibits more rounding than the less liquid and longer one week tenor is not surprising, but still demonstrates that similar maturities with different liquidity can exhibit different coarseness.

Another possibility is that the patterns of rounding we observe are caused by collusive behavior. This would, however, have to be a different type of collusion than the “Trader based manipulations”, which, as discussed in the introduction, are not likely to have caused rounding. Abrantes-Metz and Metz (2012) argue that banks have colluded to submit identical values. Rounding might have been a way to facilitate this coordination. To test this hypothesis we report in Table S6 in the online supplementary material a version of our baseline regression

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43 We have excluded the one month rate as (Kuo, Skeie, Vickery, and Youle, 2014) document that during and after the crisis period maturities between one week and less than a month far dominated the volume of transactions executed at the one month tenor.

44 A version of Table 4 with daily controls yields similar results for the coefficient of “Reference Tenors.” However, the coefficient of “CDS spread” becomes non significant.
were we have added a dummy variable “Duplicated”, indicating if another bank has made the same submission. Our results remain significant, which suggests that the relationships we have identified are not related to the potential coordination of submissions on the same numbers.\footnote{Our results also survive if duplicated submissions are removed from the sample.} Indeed, rounded numbers strongly vary across banks and rounding is particularly prevalent in times of market turmoil. This makes it unlikely that collusion is the source of rounding.

5 Conclusion

This paper constructs a model of directed search in the interbank market and tests its empirical implications with data from the Libor benchmark setting process. We demonstrate that in normal market conditions “cheap talk” announcements by banks about their financing costs can credibly convey non-verifiable information and improve the functioning of the “over the counter” interbank markets. We think that this is a reasonably realistic model of how Libor worked in the early days. In particular it explains why, until the advent of the crisis, the surprisingly informal Libor setting mechanism largely produced reliable numbers.

Benchmark setting mechanisms based on cheap talk are, however, fragile. If panel members have additional reasons to understate their borrowing costs, the truth revealing equilibrium collapses. As is common for cheap talk models, in this case banks can only credibly convey some information by using a coarse signalling space, i.e. a signal space that is divided into intervals of a certain size. Our model provides a lower bound for the length of these intervals and shows that the length increases with a bank’s default probability and the liquidity of the interbank market. We argue that submitting rounded numbers is a simple and intuitive way to implement cheap talk equilibria with coarse signal spaces and provide evidence for patterns of rounding that are consistent with this explication.

We hope that beyond the insight generated about the Libor process, our model will con-
tribute to a deeper understanding of other market benchmarks and benchmark setting mechanisms. Market benchmarks are used in many illiquid OTC markets and the calculation of benchmarks in these markets is based on a bewildering range of different mechanisms involving past transactions, binding or partially binding quotes and pure cheap talk signals. Following the Libor investigations a number of these other market benchmarks have come under the suspicion of manipulation.\textsuperscript{46} Our results should help reforming these benchmarks in a way that preserves their efficiency enhancing properties.

\textsuperscript{46}In addition to interest rate benchmarks such as ISDAfix, RONIA and SONIA foreign exchange benchmarks such as the WM/Reuters FX rates as well as commodity benchmarks such as the Gold/Silver Fixings and energy benchmarks such as the Platts, ICIS and Argus have recently come under investigation.
References


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Libor I (2013): “Naomi Reice Buchwald United States District Judge, UNITED STATES DISTRICT COURT SOUTHERN DISTRICT OF NEW YORK In re: Libor-Based Financial Instruments Antitrust Litigation..”


APPENDIX: Proofs

Proof of Lemma 1

Proof. Eq. (3) means that the lender’s optimality is equivalent to:

$$e^{-\lambda(m,b)} (1 - \mu(b,m)) b \leq U, \forall m \in M, b \geq 0,$$

for some $U \in \mathbb{R}_+$, with equality in the support of $P$. For any $m$ in the support of $Q_M$, $(m,b)$ belongs to the support of $P$ if and only if $\lambda(m,\cdot)$ is strictly increasing at $b$. Thus, a simple algebraic rearrangement of (12) implies that the lender’s optimality is equivalent to:

$$\lambda(m,b) \geq \ln \left( \frac{1 - \mu(b,m)}{U} \right) b, \forall m \in M, b \geq 0,$$

with equality at any point in which $\lambda(m,\cdot)$ is strictly increasing in $b$. The only function $\lambda$ satisfying this condition and $\lambda(m,0) = 0$, and that it is also non-negative, continuous and increasing in $b$ is:

$$\lambda(m,b) = \max \left\{ 0, \sup_{\tilde{b} \in [0,b]} \ln \left( \frac{1 - \mu(\tilde{b},m)}{U} \tilde{b} \right) \right\}, \forall m \in M, b \geq 0.$$

We require that $\lambda(m,0) = 0$, and that $\lambda(m,\cdot)$ is non-negative, continuous and increasing because it is a cumulative distribution function with support in $\mathbb{R}_+$ and no atoms.
Our next argument uses that $u^*(Q) < \underline{v}$, which is a consequence of:

\[
\begin{align*}
    u^*(Q) &= \frac{e^\int_M \sup_{b \in \mathbb{R}_+} \ln((1-\mu(b, \tilde{m}))b)dQ_M(\tilde{m})}{e^\beta} \\
    &\leq \frac{e^\int_M \ln(\int_V \tilde{v}d\mu(\tilde{v}, \tilde{m}))dQ_M(\tilde{m})}{e^\beta} \\
    &\leq \frac{\int_M \int_V \tilde{v}d\mu(\tilde{v}, \tilde{m})dQ_M(\tilde{m})}{e^\beta} \\
    &= \frac{\int_V \tilde{v}dF(\tilde{v})}{e^\beta} \\
    &< \underline{v},
\end{align*}
\]

where the first step uses (7), the second step uses that:

\[
    (1 - \mu(\tilde{b}, m))\tilde{b} \leq \int_{\tilde{b}}^\sigma \tilde{v}d\mu(\tilde{v}, m) \leq \int_V \tilde{v}d\mu(\tilde{v}, m),
\]

the third step uses Jensen’s inequality, the fourth step uses the law of iterated expectations, and the fifth step uses the assumption in (4).

Next, we argue that the value of $\lambda$ defined in the statement of the lemma is equal to the value in (14) for $U = u^*(Q)$. This is because $u^*(Q) < \underline{v}$ means that $\mu(u^*(Q), m) = 0$, and thus, the right hand side of (14) at $U = u^*(Q)$ is equal to zero for $b < u^*(Q)$ and to the right hand side of (6) for $b \geq u^*(Q)$.

Finally, our last step is to note that (7) means that $U = u^*(Q)$ solves the equation resulting from substituting (14) into (1). Besides, $u^*(Q) < \underline{v} \leq \sup_{b \in \mathbb{R}_+} \left(1 - \mu(\tilde{b}, m)\right)\tilde{b}$ means that there is no other solution to this equation because its left hand side is decreasing in $U$, strictly if $U \leq \sup_{b \in \mathbb{R}_+} \left(1 - \mu(\tilde{b}, m)\right)\tilde{b}$. 

\[\blacksquare\]
Proof of Proposition 1

Proof. We take a message $m$ and a lower message $m'$ whose respective pooling intervals have lower bounds $v$ and $v'$ and prove the first item by contradiction: A bank with type $v$ has strict incentives to deviate and report $m'$ instead of $m$ if the difference $v - v'$ lies between zero and the proposed bound. This is a direct consequence of the observation that (8) is a lower bound to the incentive to deviate and the following chain of inequalities and equalities:

$$
\int w(\bar{v}) d\mu(\bar{v}, m') - \int w(\bar{v}) d\mu(\bar{v}, m) \geq \frac{\int_{v'}^{v} (w(\bar{v}) - w(v)) F'(\bar{v}) d\bar{v}}{\int_{v'}^{v} F'(\bar{v}) d\bar{v}} - u^*(Q) \int_{v'}^{v} (v - \bar{b}) d\bar{b}
$$

In the first step, we use three arguments. First, $\int w(\bar{v}) d\mu(\bar{v}, m) \leq w(v)$ since $w$ is a decreasing function and $v$ is the lower bound of the support of $\mu(\cdot, m)$. Second:

$$
\int w(\bar{v}) d\mu(\bar{v}, m') \geq \frac{\int_{v'}^{v} w(\bar{v}) F'(\bar{v}) d\bar{v}}{\int_{v'}^{v} F'(\bar{v}) d\bar{v}},
$$

since $w(\bar{v})$ is decreasing in $\bar{v}$ and the distribution with density $\frac{F'(\bar{v})}{\int_{v'}^{v} F'(\bar{v}) d\bar{v}}$ in the support $[v', v]$ first order stochastically dominates the distribution $\mu(\cdot, m')$, see (5) and note that its support is $[v', \hat{v}]$ for some $\hat{v} \leq v$. And third, we use that the last integral can be simplified applying standard algebra. In the second step, we use that $F'(x) \in \left(\frac{1}{\eta}, \eta\right)$, the infimum of the derivative of $w$ and that $\bar{b} \geq v$. In the third step, we just compute the integrals and take a common factor.

In the fourth step, we use that $u^*(Q) \leq \frac{\eta}{\varepsilon}$ that can be deduced from (7).

\[\square\]
Proof of Proposition 2

Proof. We prove the proposition by showing that there exists an interval partition equilibrium in which the range of $Q$ has finitely many messages whose pooling intervals are of size smaller than (2). Since $\sup |w'(v)| \leq \frac{v^2}{2\pi^2 \eta^2}$, implies that pooling intervals of size less than (2) meet the condition in (C2) we are going to restrict attention to equilibria in which (C2) is satisfied.

The communication strategy $Q$ can be described by the set of (decreasing) bounds of the corresponding pooling intervals, that we denote by $\{v_i\}_{i=0}^{n}$ for $v_0 = \bar{v}$ and $v_n = \underline{v}$, and a labelling convention for the messages. We shall use that a necessary and sufficient condition for the banks’ optimality condition is that a bank with type $v_i$ is indifferent between reporting the message corresponding to the pooling interval $[v_{i+1}, v_i]$ and the message corresponding to the pooling interval $[v_i, v_{i-1}]$. This is a consequence of the fact that the derivative of the bank’s payoff function (2) with respect to $v$ weakly increases with larger messages (i.e. it is supermodular in $(m, v)$). Formally, our necessary and sufficient condition can be deduced from (5) and (8) to be:

$$\Delta(v_{i+1}, v_i, v_{i-1}) \equiv \left( \frac{\int_{v_i}^{v_{i+1}} w(\bar{v})F'(\bar{v})d\bar{v}}{F(v_i) - F(v_{i+1})} - \frac{\int_{v_{i-1}}^{v_{i}} w(\bar{v})F'(\bar{v})d\bar{v}}{F(v_{i-1}) - F(v_{i})} \right) - u^*(Q) \int_{v_{i+1}}^{v_i} \frac{v_i - \bar{b}}{\bar{b}^2} d\bar{b} = 0, \quad (16)$$

for any $i \in \{1, \ldots, n - 1\}$.

To show that our required solution exists, we show that $\Delta(v_{i+1}, v_i, v_{i-1})$ crosses zero as we vary $v_{i+1}$. First, we apply the bounds derived in (15) to conclude that:

$$\Delta(v_{i+1}, v_i, v_{i-1}) > 0, \text{ if } v_i - v_{i+1} < \delta \equiv \inf |w'(x)| \cdot e^\beta \cdot \frac{v^2}{\eta^2}. \quad (17)$$

Similarly, $F'(x) \in \left( \frac{1}{\eta}, \eta \right)$ and $u^*(Q) \geq \frac{\lambda}{e^\beta}$, by (7), means that the right hand side of (16) is
less than:

\[
\sup |w'(x)| \cdot \eta^2 \cdot \left( \int_{v_i}^{v_{i+1}} (v_i - \tilde{v}) \, d\tilde{v} + \int_{v_i}^{v_1} (\tilde{v} - v_i) \, d\tilde{v} \right) - \frac{v}{\tilde{v}^2} \beta \int_{v_i}^{v_{i+1}} (v_i - \tilde{b}) \, d\tilde{b} \\
= \sup |w'(x)| \cdot \eta^2 \cdot \left( \frac{v_i - v_{i+1}}{2} + \frac{v_{i-1} - v_i}{2} \right) - \frac{v}{2\tilde{v}^2} \beta (v_i - v_{i+1})^2
\]

To give a sign to this expression, we note that the last line has the same sign as:

\[\kappa(\delta_i + \delta_{i-1}) - \delta_i^2,\]

for \(\delta_i \equiv v_i - v_{i+1}, \delta_{i-1} \equiv v_{i-1} - v_i\) and \(\kappa \equiv \sup |w'(x)| \cdot e^\beta : \frac{\eta^2}{\tilde{v}}.\) The application of the formula for the solution of a quadratic equation means that the last expression is equal to:

\[
\left( \frac{\kappa + \sqrt{\kappa^2 + 4\kappa \delta_{i-1}}}{2} - \delta_i \right) \left( \delta_i + \sqrt{\kappa^2 + 4\kappa \delta_{i-1} - \kappa} \right).
\]

The second term is positive and the first term is strictly less than \((2\kappa - \delta_i)\) if \(\delta_{i-1} < 2\kappa.\) Hence:

\[\Delta(v_{i+1}, v_i, v_{i-1}) < 0, \text{ if } v_i - v_{i+1} > \delta \equiv \sup |w'(x)| \cdot e^\beta : \frac{2\beta \eta^2}{\tilde{v}} > v_{i-1} - v_i.\]  \hspace{1cm} (18)

The eqs. (17) and (18) imply that \(\Delta(v_{i+1}, v_i, v_{i-1})\) moves from a strictly negative value to a strictly positive value as \(v_{i+1}\) moves from \(v_i - \delta\) to \(v_i - \delta\) if \(v_{i-1} - v_i < \delta.\) Consequently, the continuity of \(\Delta\) means that for any value of \(v_1 \in (\overline{\tau} - \delta, \overline{\tau}],\) we can define recursively the functions \(\nu_{i+1}(v_i), i = 1, 2, \ldots,\) as a solution in \(v_{i+1} \in (\nu_i(v_i) - \delta, \nu_i(v_i) - \delta)\) to \(\Delta(v_{i+1}, v_i, v_{i-1}(v_i)) = 0,\) and where \(\nu_1(v_1) = v_1\) and \(\nu_0 = \overline{\tau}.\) The concavity of \(\int_{\overline{\tau}}^{\nu} \frac{w''(\nu)}{F(\nu) - F(\overline{\tau})} d\nu\) in \(v\) implies that \(\Delta(v_{i+1}, v_i, v_{i-1})\) is concave in \(v_{i+1}.\) This implies that the functions \(\nu_i\) are uniquely defined and, thus, continuous in \(v_1\) since \(\Delta\) is also continuous. Another useful property of the functions \(\nu_i,\) that we call (P1), is that \(\nu_{i+1}(\overline{\tau}) = \nu_i(\nu_2(\overline{\tau}))\) for \(i = 2, 3, \ldots.\) This property can be proved recursively using that \(\nu_{i+1}(v_1)\) is the unique value that solves \(\Delta(v_{i+1}, v_i, v_{i-1}(v_i)) = 0,\) and that \(\nu_1(v_1) = v_1\) and
\(\nu_0(v_1) = \overline{v}.\)

We let \(n^*\) be the maximum index \(i\) for which \(\nu_i(\overline{v})\) is defined. This means that:

\[
\Delta(\overline{v}, \nu_{n^* - 1}(\overline{v}), \nu_{n^* - 2}(\overline{v})) \leq 0, \tag{19}
\]

and,

\[
\Delta(\overline{v}, \nu_{n^*}(\overline{v}), \nu_{n^* - 1}(\overline{v})) > 0. \tag{20}
\]

(20) and (P1) means that \(\Delta(\overline{v}, \nu_{n^* - 1}(\nu_2(\overline{v})), \nu_{n^* - 2}(\nu_2(\overline{v}))) > 0\), which together with (19) and the continuity of \(\nu_i\) and \(\Delta\) means that there exists a \(v^* \in (\nu_2(\overline{v}), \overline{v}]\) such that

\[
\Delta(\overline{v}, \nu_{n^* - 1}(v^*), \nu_{n^* - 2}(v^*)) = 0. \tag{21}
\]

This means that \(\nu_{n^*}(v^*) = \overline{v}\). Hence, we have found, as desired, a decreasing sequence \(\{v_0, \ldots, v_{n^*}\} = \{\nu_0(v^*), \ldots, \nu_{n^*}(v^*)\}\) that goes from \(\overline{v}\) to \(\overline{v}\), that solves (16) and whose pooling intervals \(v_i - v_{i+1}, i = 0, 1, \ldots, n^* - 1\), are less than \(\delta\). \(\blacksquare\)
Figure 1: Evolution of the Libor benchmark rates for all maturities. The first vertical line indicates 31st of July 2007, when Bear Stearns’ High-Grade Structured Credit Fund collapsed. The second vertical line indicates September 15th 2008, when Lehman Brothers filed for bankruptcy.
Figure 2: Daily standard deviation of Libor quotes for each maturity. The first vertical line indicates 31st of July 2007, when Bear Stearns’ High-Grade Structured Credit Fund collapsed. The second vertical line indicates September 15th 2008, when Lehman Brothers filed for bankruptcy.
Figure 3: Daily submissions by BNP Paribas. The vertical line indicates April 2013, when the Libor reform delayed the publication of individual submissions.
Figure 4: Frequencies of digits in the 2$^{nd}$ decimal of submissions corresponding to: CDS spreads in the bottom quartile (a) and CDS spreads in the top quartile (b).
Figure 5: Frequency of digits in the 2\textsuperscript{nd} decimal of submissions for the overnight rate (a) and the eight month rate (b).
Figure 6: Rounding per maturity for banks with CDS spreads in the bottom quartile (a) and CDS spread in the top quartile (b). “1Y” stands for one year loans, “ON” stands for overnight loans, “SW” for one week loans, “2W” for two weeks loans, and “xM” for x month loans.
Figure 7: 3 months moving average of rounding in the 2nd decimal for different banks. The first vertical line indicates 31st of July 2007, when Bear Stearns’ High-Grade Structured Credit Fund collapsed. The second vertical line indicates September 15th 2008, when Lehman Brothers filed for bankruptcy.
Figure 8: 3 months moving average of rounding in the 2nd decimal for different maturities. The first vertical line indicates 31st of July 2007, when Bear Stearns’ High-Grade Structured Credit Fund collapsed. The second vertical line indicates September 15th 2008, when Lehman Brothers filed for bankruptcy.
Note: This figure shows the evolution of the regression coefficient for the CDS spread when estimating equation (11) for a 24 month interval surrounding the beginning of each quarter. The blue area indicates two-way clustered standard errors at bank and day and at the 10% significance level. The regression is estimated without interaction term and with bank fixed effects but without time controls and includes only data for maturities that survive the 2013 reform. CDS spreads are winsorized at the 1st and 99th percentiles. “Reference Tenor” is an indicator variable that takes the value of 1 if the submission is for the overnight (“ON”), three month (“3M”), six month (“6M”) or one year (“1Y”) tenors and 0, otherwise. *p<0.1; **p<0.05; ***p<0.01.

Figure 9: Evolution of the Relationship between Risk and Rounding.
Note: This figure shows the evolution of regressions coefficients for liquid maturities, when estimating equation (11) for a moving 24 month interval surrounding the beginning of each quarter. The blue area indicates two-way clustered standard errors at bank and day and at the 10% significance level. The regression is estimated without interaction term and with bank fixed effects but without time controls and includes only data for maturities that survive the 2013 reform. CDS spreads are winsorized at the 1st and 99th percentiles. Liquid maturities are measured with the indicator variable “Reference Tenor” that takes the value of 1 if the submission is for the overnight (“ON”), three month (“3M”), six month (“6M”) or one year (“1Y”) tenors and 0, otherwise. Data are winsorized at the 99% level to remove outliers. *p<0.1; **p<0.05; ***p<0.01.

Figure 10: Evolution of the Relationship between Liquidity and Rounding.
Table 1: Submissions from US-$ Libor panel banks on October 10th, 2006.

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Table 3: Risk, Liquidity and Rounding in Libor Submissions

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<td>(1)</td>
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<td>CDS spread</td>
<td>1.802**</td>
<td>1.521**</td>
<td>0.449</td>
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<td></td>
<td>(0.715)</td>
<td>(0.646)</td>
<td>(0.696)</td>
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<tr>
<td></td>
<td>p = 0.012</td>
<td>p = 0.019</td>
<td>p = 0.520</td>
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<tr>
<td>Liquid Maturities</td>
<td>0.035***</td>
<td>0.029***</td>
<td>0.035***</td>
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<tr>
<td></td>
<td>(0.006)</td>
<td>(0.006)</td>
<td>(0.006)</td>
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<tr>
<td></td>
<td>p = 0.000</td>
<td>p = 0.000</td>
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<tr>
<td>CDS spread x Liquid Maturities</td>
<td>0.601**</td>
<td>0.536*</td>
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</tr>
<tr>
<td></td>
<td>(0.289)</td>
<td>(0.301)</td>
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<tr>
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<tr>
<td>Daily mean CDS spread</td>
<td>3.183***</td>
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<tr>
<td></td>
<td>(0.733)</td>
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<td></td>
<td>p = 0.000</td>
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<table>
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<th>Time controls: Bank fixed effects:</th>
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<td>R²</td>
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<td>0.053</td>
<td>0.098</td>
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<td>Adjusted R²</td>
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<td>0.093</td>
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<tr>
<td>Residual Std. Error</td>
<td>0.395 (df = 489,128)</td>
<td>0.395 (df = 489,127)</td>
<td>0.395 (df = 489,126)</td>
<td>0.387 (df = 486,585)</td>
<td>0.387 (df = 486,584)</td>
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</table>

Note: This table provides the results of regressing coarseness in Libor submissions on proxies for risk and liquidity. Models (1), (2) and (3) include time controls at annual frequency. Models (4) and (5) are estimated with full time fixed effects with daily frequency. “Rounding in the 2nd Decimal” is a dummy variable taking the value 1 if the second and last decimal of the Libor submission is either “5” or “0”. “CDS Spread” is the spread on the bank’s one year senior Credit Default Swap. “Daily mean CDS Spread” is the daily average of “CDS Spread” and “Liquid Maturities” is an indicator variable that takes the value of 1 if the submission is for one of the tenors that remained after the 2013 reform. Standard errors are two-way clustered at bank and time level. *p<0.1; **p<0.05; ***p<0.01.
Table 4: Liquidity and Rounding for reference rates and surrounding tenors

<table>
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<tr>
<td>ON/SW 2M/3M/4M</td>
<td>5M/6M/7M</td>
<td>11M/1Y</td>
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<tr>
<td>CDS spread</td>
<td>3.120***</td>
<td>1.033*</td>
<td>1.280</td>
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<tr>
<td>(1.031)</td>
<td>(0.609)</td>
<td>(0.848)</td>
<td>(0.609)</td>
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<tr>
<td>p = 0.003</td>
<td>p = 0.091</td>
<td>p = 0.131</td>
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<tr>
<td>Reference Tenors</td>
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</tr>
<tr>
<td>0.045**</td>
<td>0.029***</td>
<td>−0.003</td>
<td>0.029***</td>
</tr>
<tr>
<td>(0.019)</td>
<td>(0.009)</td>
<td>(0.006)</td>
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<tr>
<td>R²</td>
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<td>Adjusted R²</td>
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<tr>
<td>Residual Std. Error</td>
<td>0.419 (df = 55419)</td>
<td>0.375 (df = 85418)</td>
<td>0.377 (df = 83333)</td>
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</table>

Note: This table provides the results of regressing rounding in Libor submissions on CDS spreads and liquidity for subsamples including each a reference rate and the surrounding less liquid tenors. In regression (1) we only include the overnight and single week tenor. The sample for (2) comprises the two, three and four month rates, (3) analyzes the five, six and seven month rates and (4) the eleven month and one year rates. “Rounding in the 2nd Decimal” is a dummy variable taking the value 1 if the second and last decimal of the Libor submission is either “5” or “0”. “CDS Spread” is the spread on the bank’s one year senior Credit Default Swap and “Reference Tenor” is an indicator variable that takes the value of 1 if the submission is for the overnight (“ON”), three month (“3M”), six month (“6M”) and one year (“1Y”) tenor and 0, otherwise. Standard errors are clustered at the bank and time level. *p<0.1; **p<0.05; ***p<0.01.