# Nonsupersymmetric smooth geometries and D1-D5-P bound states

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We construct smooth nonsupersymmetric soliton solutions with D1-brane, D5-brane, and momentum charges in type IIB supergravity compactified on $T^4 \times S^1$, with the charges along the compact directions. This generalizes previous studies of smooth supersymmetric solutions. The solutions are obtained by considering a known family of $U(1) \times U(1)$ invariant metrics, and studying the conditions imposed by requiring smoothness. We discuss the relation of our solutions to states in the CFT describing the D1-D5 system and describe various interesting features of the geometry.

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## I. INTRODUCTION AND SUMMARY

String theory has made tremendous advances in understanding the microscopic origins of black hole entropy [1,2]. In the original calculations, two different dual descriptions of a supersymmetric object were considered: a weakly coupled description in terms of perturbative strings and D-branes and a strongly coupled description as a classical black hole solution. The picture of this black hole, as a background for the perturbative string, is essentially the same as in semiclassical general relativity. We have a singularity in spacetime that is shielded (censored) by a horizon. The horizon area determines the Bekenstein-Hawking entropy $S_{\rm BH} = \frac{A}{4G_N}$. This entropy was successfully reproduced by counting the degenerate supersymmetric vacua in the dual perturbative D-brane description. This picture did not provide any understanding of where the microstates were in the strong-coupling black hole picture: smooth black hole solutions "have no hair," so the geometry is entirely determined by the charges [3]. There was, however, a suggestion that pure states would be dual to geometries which were not smooth at the event horizon [4].

The anti–de Sitter/conformal field theory (AdS/CFT) correspondence [5–7] provided a deeper understanding of the counting of black hole entropy in string theory. The black holes in AdS are identified with the thermal ensemble in the dual CFT. The CFT was conjectured to provide a fundamental, nonperturbative description of string theory with asymptotically AdS boundary conditions, so the microstates were fundamentally thought of as states in the CFT, and it did not appear that they could be thought of as living somewhere in the black hole geometry. The evolution of the states in the CFT is unitary. Certain states can be identified with classical geometries, but as has been emphasized in e.g. [7,8], the CFT provides a fully quantized description, and reproducing the behavior of the CFT from a spacetime point of view will in general involve a sum over bulk geometries.

In a series of papers, Mathur and his collaborators have challenged the conventional picture of a black hole in string theory (see [9] for a review). They argue that the black hole geometry is merely a coarse grained description of the spacetime, and that each of the $e^{S_{\rm BH}}$ microstates can be identified with a perfectly regular geometry with neither horizon nor singularity [10,11]. The black hole entropy is a result of averaging over these different geometries, which produces an "effective horizon," which describes the scale at which the $e^{S_{\rm BH}}$ individual geometries start to differ from each other. They further argue that if a system in an initial pure state undergoes gravitational collapse, it will produce one of these smooth geometries, and the real spacetime does not have a global event horizon, thus avoiding the information loss paradox associated with outgoing Hawking radiation [12]. Thus, the idea is that stringy effects modify the geometry of spacetime at the event horizon, rather than, as would be expected from the classical point of view, at Planck or string distances from the singularity. This is a radical modification of the expected geometry. There are similarities with the correspondence principle ideas [13], but unlike that picture, there is no obvious sense in which the spacetime as seen by an infalling observer will be different. It is difficult to see how the singularity behind the black hole's event horizon can arise from a coarse graining over nonsingular geometries.[1]

The evidence for this proposal comes from the construction of smooth asymptotically flat geometries in the D1-D5 system that can be identified with individual microstates in the CFT on the world volume of the branes. The theory considered is type IIB supergravity compactified on $S^1 \times T^4$ with $n_5$ D5-branes wrapping $S^1 \times T^4$, and $n_1$ D-strings wrapping the $S^1$. The near-horizon geometry is $\mathrm{AdS}_3 \times S^3 \times T^4$ and has a dual $1 + 1$ dimensional CFT description

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[1] Although it may be that most measurements in the dual CFT find it difficult to distinguish between regular geometries and the conventional semiclassical picture of a black hole [14].

 

with $c = 6n_1n_5$. The first such geometries were constructed in [15,16], and correspond to the Ramond-Ramond (RR) ground state obtained by spectral flow from the Neveu-Schwarz–Neveu-Schwarz (NSNS) vacuum state. This was subsequently extended [10,17,18] to find a family of smooth geometries corresponding to the whole family of RR ground states in the CFT. The D1-D5 system via string dualities is the same as a system with $n_5$ units of fundamental string winding and $n_1$ units of fundamental string momentum on a circle. The D1-D5 bound state corresponds to a multiwound F-string carrying momentum, and the geometries are characterized as functions of the displacement of the string in its transverse directions. As a test of whether the two-charge system indeed describes the correct physics, the collision time for left- and right-moving excitations on the component string was computed in field theory and compared to the time for graviton absorption and reemission in supergravity; the two are found to match [10,19]. The degeneracy of RR ground states in this theory gives a microscopic entropy which scales as $\sqrt{n_1n_5}$; this was found to match a suitable counting in a supertube description in [20,21]. However, this entropy is not large enough to correspond to a black hole with a macroscopic horizon. It is therefore important to extend the identification to states that carry a third charge $n_p$, momentum along the string. These states have a microscopic degeneracy $\sqrt{n_1n_5n_p}$, and were used in [2] in the calculation of the black hole entropy. Recently, Giusto, Mathur, and Saxena have identified smooth geometries corresponding to some of these states [22,23], although the geometries constructed so far correspond to very special states, the spectral flows of the RR ground states studied earlier.[2] The overall evidence for the picture of black holes advanced by these authors is, in our judgement, interesting but not yet compelling.

We will extend these investigations to find more general solitonic solutions in supergravity and to identify corresponding CFT states. We believe that whether or not the picture of black holes advanced by Mathur and collaborators proves to be correct, these solitonic solutions will remain of interest in their own right. It is particularly interesting that we can find completely smooth nonsupersymmetric solitons. These are, as far as we are aware, the first explicit examples of this type.

We find these solutions by generalizing an analysis previously carried out for special cases in [15,16,22,23]. We consider the nonextremal rotating three-charge black holes given in [31] and systematically search for values of the parameters for which the solution is smooth and free of singularities. We find that if we allow nonsupersymmetric solutions, there are two integers $m, n$ labelling the soliton solutions. The previously studied supersymmetric solutions correspond to $m = n + 1$. Thus, we find new nonsupersymmetric solitons. Further solutions, some of which are smooth, can be constructed by orbifolds of this basic family. This provides another integer degree of freedom $k$. Some of the supersymmetric orbifolds have not been previously studied.

We identify the basic family of smooth solutions labeled by $m, n$ with states in the CFT constructed by spectral flow from the NSNS vacuum, with $m + n$ units of spectral flow applied on the left and $m - n$ units of spectral flow applied on the right. We find a nontrivial agreement between the spacetime charges in these geometries and the expectations from the CFT point of view. This agreement extends to the geometries constructed as orbifolds of the basic smooth solutions. We have studied the wave equation on these geometries, and we find that as in [23], there is a mismatch between the spacetime result, $\Delta t_{\rm sugra} = \pi R \varrho$ and the expectation from the CFT point of view $\Delta t_{\rm CFT} = \pi R$. We believe that understanding this mismatch is a particularly interesting issue for further development. Finally, we discuss the appearance of an ergoregion in the nonsupersymmetric solutions. We find that the ergoregion does not lead to any superradiant scattering for free fields.

The existence of these nonsupersymmetric solitons, and the fact that they can be identified with states in the dual CFT, might be regarded as further evidence for the proposed description of black holes. However, we would advocate caution. We still find it questionable whether we can really describe a black hole in this way. First of all, the three-charge states described so far are very special. The orbifolds we consider provide examples where the CFT state is not the spectral flow of a RR ground state, but the geometries we consider all have a $U(1) \times U(1)$ invariance. It is unclear whether the techniques used to date can be extended to obtain even the geometries corresponding to spectral flows of the more general RR ground states of [10,18], let alone to reproduce the full $e^{\sqrt{n_1n_5n_p}}$ states required to explain the black hole entropy. The much more difficult dynamical questions—how the appearance of a global event horizon in gravitational collapse can always be avoided, for example—have not yet been tackled. Nonetheless, the study of these smooth geometries offers a new perspective on the relation between CFT and spacetime, and it is interesting to see that their existence does not depend on supersymmetry.

The remainder of the paper is organized as follows: In the next section, we recall the metric and matter fields for the general family of solutions we consider and discuss the near-horizon limit which relates asymptotically flat solutions to asymptotically $\mathrm{AdS}_3 \times S^3$ ones. In Sec. III, we discuss the constraints required to obtain a smooth soliton solution. We find that there is a basic family of smooth solutions labeled by the radius $R$ of the $S^1$, the D1 and D5

[2]Three-charge states were previously studied in the supertube description [24,25] in [26,27]. Other supersymmetric three-charge solutions have been found in [28–30], but the regular solutions have not yet been identified or related to CFT states.

brane charges $Q_1$, $Q_5$, and two integers $m$, $n$. Further solutions can be constructed as $\mathbb{Z}_k$ orbifolds of these basic ones; they will be smooth if $m$ and $n$ are both relatively prime to $k$. We also discuss the asymptotically $\mathrm{AdS}_3 \times S^3$ solutions obtained by considering the near-horizon limit. The asymptotically $\mathrm{AdS}_3 \times S^3$ solutions corresponding to the basic family of smooth solutions are always global $\mathrm{AdS}_3 \times S^3$ up to some coordinate transformation. In Sec. IV, we verify that the solutions are indeed smooth and free of closed timelike curves. In Sec. V, we identify the corresponding states in the CFT, identifying the coordinate shift in the global $\mathrm{AdS}_3 \times S^3$ solutions with spectral flow. In Sec. VI, we discuss the massless scalar wave equation on these solutions, and show that the nonsupersymmetric solutions always have an ergoregion. Finally, in Sec. VII, we discuss some directions for future research.

## II. GENERAL NONEXTREMAL SOLUTION

We will look for smooth solutions as special cases of the nonextremal rotating three-charge black holes given in [32] (uplifted to ten-dimensional supergravity following [33]). The original two-charge supersymmetric solutions of [15,16] were found in this way, and the same approach was applied more recently in [22,23] to find supersymmetric three-charge solutions. In the present work, we aim to find all the smooth solutions within this family.

In this section, we discuss this family of solutions in general, writing the metric in forms that will be useful for finding and discussing the smooth solutions. We will also discuss the relation between asymptotically flat and asymptotically $\mathrm{AdS}_3 \times S^3$ solutions. We write the metric as

$$\begin{aligned}
\mathrm{d}s^2 = {} & -\frac{f}{\sqrt{\tilde{H}_1\tilde{H}_5}}(\mathrm{d}t^2 - \mathrm{d}y^2) + \frac{M}{\sqrt{\tilde{H}_1\tilde{H}_5}}(s_p\mathrm{d}y - c_p\mathrm{d}t)^2 + \sqrt{\tilde{H}_1\tilde{H}_5}\left(\frac{r^2\mathrm{d}r^2}{(r^2+a_1^2)(r^2+a_2^2) - Mr^2} + \mathrm{d}\theta^2\right) \\
& + \left(\sqrt{\tilde{H}_1\tilde{H}_5} + (a_1^2 - a_2^2)\frac{(\tilde{H}_1 + \tilde{H}_5 - f)\cos^2\theta}{\sqrt{\tilde{H}_1\tilde{H}_5}}\right)\cos^2\theta\mathrm{d}\psi^2 + \left(\sqrt{\tilde{H}_1\tilde{H}_5} - (a_1^2 - a_2^2)\frac{(\tilde{H}_1 + \tilde{H}_5 - f)\sin^2\theta}{\sqrt{\tilde{H}_1\tilde{H}_5}}\right) \\
& \times \sin^2\theta\mathrm{d}\phi^2 + \frac{M}{\sqrt{\tilde{H}_1\tilde{H}_5}}(a_1\cos^2\theta\mathrm{d}\psi + a_2\sin^2\theta\mathrm{d}\phi)^2 + \frac{2M\cos^2\theta}{\sqrt{\tilde{H}_1\tilde{H}_5}}[(a_1c_1c_5c_p - a_2s_1s_5s_p)\mathrm{d}t \\
& + (a_2s_1s_5c_p - a_1c_1c_5s_p)\mathrm{d}y]\mathrm{d}\psi + \frac{2M\sin^2\theta}{\sqrt{\tilde{H}_1\tilde{H}_5}}[(a_2c_1c_5c_p - a_1s_1s_5s_p)\mathrm{d}t + (a_1s_1s_5c_p - a_2c_1c_5s_p)\mathrm{d}y]\mathrm{d}\phi \\
& + \sqrt{\frac{\tilde{H}_1}{\tilde{H}_5}}\sum_{i=1}^{4}\mathrm{d}z_i^2, \qquad (2.1)
\end{aligned}$$

where

$$\tilde{H}_i = f + M\sinh^2\delta_i, \qquad f = r^2 + a_1^2\sin^2\theta + a_2^2\cos^2\theta, \qquad (2.2)$$

and $c_i = \cosh\delta_i$, $s_i = \sinh\delta_i$. This metric is more usually written in terms of functions $H_i = \tilde{H}_i/f$. Writing it in this way instead makes it clear that there is no singularity at $f = 0$. As the determinant of the metric is

$$g = -r^2\frac{\tilde{H}_1^3}{\tilde{H}_5}\cos^2\theta\sin^2\theta, \qquad (2.3)$$

it is clear that the inverse metric is also regular when $f = 0$. The above metric is in the string frame, and the dilaton is

$$e^{2\Phi} = \frac{\tilde{H}_1}{\tilde{H}_5}. \qquad (2.4)$$

From [22], the two-form gauge potential which supports this configuration is

$$\begin{aligned}
C_2 = {} & \frac{M\cos^2\theta}{\tilde{H}_1}[(a_2c_1s_5c_p - a_1s_1c_5s_p)dt + (a_1s_1c_5c_p \\
& - a_2c_1s_5s_p)dy] \wedge d\psi + \frac{M\sin^2\theta}{\tilde{H}_1}[(a_1c_1s_5c_p \\
& - a_2s_1c_5s_p)dt + (a_2s_1c_5c_p - a_1c_1s_5s_p)dy] \\
& \wedge d\phi - \frac{Ms_1c_1}{\tilde{H}_1}dt \wedge dy - \frac{Ms_5c_5}{\tilde{H}_1}(r^2 + a_2^2 \\
& + Ms_1^2)\cos^2\theta d\psi \wedge d\phi. \qquad (2.5)
\end{aligned}$$

We take the $T^4$ in the $z_i$ directions to have volume $V$, and the $y$ circle to have radius $R$, that is $y \sim y + 2\pi R$.

Compactifying on $T^4 \times S^1$ yields an asymptotically flat five-dimensional configuration. The gauge charges are determined by

$$Q_1 = M\sinh\delta_1\cosh\delta_1, \qquad (2.6)$$

$$Q_5 = M\sinh\delta_5\cosh\delta_5, \qquad (2.7)$$

$$Q_p = M\sinh\delta_p\cosh\delta_p, \qquad (2.8)$$

where the last is the charge under the Kaluza-Klein gauge field associated with the reduction along $y$. The five-dimensional Newton's constant is $G^{(5)} = G^{(10)}/(2\pi RV)$; if we work in units where $4G^{(5)}/\pi = 1$, the Einstein frame Arnowitt-Deser-Misner (ADM) mass and angular momenta are

$$M_{\rm ADM} = \frac{M}{2}(\cosh 2\delta_1 + \cosh 2\delta_5 + \cosh 2\delta_p), \qquad (2.9)$$

$$J_\psi = -M(a_1 \cosh\delta_1 \cosh\delta_5 \cosh\delta_p - a_2 \sinh\delta_1 \sinh\delta_5 \sinh\delta_p), \qquad (2.10)$$

$$J_\phi = -M(a_2 \cosh\delta_1 \cosh\delta_5 \cosh\delta_p - a_1 \sinh\delta_1 \sinh\delta_5 \sinh\delta_p) \qquad (2.11)$$

(which are invariant under interchange of the $\delta_i$). We see that the physical range of $M$ is $M \geq 0$. We will assume without loss of generality $\delta_1 \geq 0$, $\delta_5 \geq 0$, $\delta_p \geq 0$, and $a_1 \geq a_2 \geq 0$.

We also want to rewrite this metric as a fibration over a four-dimensional base space. It has been shown in [34] that the general supersymmetric solution in minimal six-dimensional supergravity could be written as a fibration over a four-dimensional hyper-Kähler base, and writing the supersymmetric two-charge solutions in this form played an important role in understanding the relation between these solutions and supertubes in [18] and in an attempt to generate new asymptotically flat three-charge solutions by spectral flow [35]. The supersymmetric three-charge solutions were also written in this form in [36]. Of course, in the nonsupersymmetric case, we do not expect the base to have any particularly special character, but we can still use the Killing symmetries $\partial_t$ and $\partial_y$ to rewrite the metric (2.1) as a fibration of these two directions over a four-dimensional base space. This gives

$$\begin{aligned} ds^2 = {} & \frac{1}{\sqrt{\tilde{H}_1 \tilde{H}_5}}\{-(f-M)[d\tilde{t} - (f-M)^{-1} M \cosh\delta_1 \cosh\delta_5 (a_1 \cos^2\theta d\psi + a_2 \sin^2\theta d\phi)]^2 \\ & + f[d\tilde{y} + f^{-1} M \sinh\delta_1 \sinh\delta_5 (a_2 \cos^2\theta d\psi + a_1 \sin^2\theta d\phi)]^2\} + \sqrt{\tilde{H}_1 \tilde{H}_5}\Big\{\frac{r^2 dr^2}{(r^2+a_1^2)(r^2+a_2^2) - Mr^2} + d\theta^2 \\ & + (f(f-M))^{-1}[(f(f-M) + f a_2^2 \sin^2\theta - (f-M) a_1^2 \sin^2\theta)\sin^2\theta d\phi^2 + 2M a_1 a_2 \sin^2\theta \cos^2\theta d\psi d\phi + (f(f-M) \\ & + f a_1^2 \cos^2\theta - (f-M) a_2^2 \cos^2\theta)\cos^2\theta d\psi^2]\Big\}, \end{aligned} \qquad (2.12)$$

where $\tilde{t} = t\cosh\delta_p - y\sinh\delta_p$, $\tilde{y} = y\cosh\delta_p - t\sinh\delta_p$.

We can see that this is still a "natural" form of the metric, even in the nonsupersymmetric case, inasmuch as the base metric in the second {} is independent of the charges. This form of the metric is as a consequence convenient for studying the "near-horizon" limit, as we will now see.

In addition to the asymptotically flat metrics written above, we will be interested in solutions which are asymptotically $AdS_3 \times S^3$. These asymptotically $AdS_3 \times S^3$ geometries can be thought of as describing a "core" region in our asymptotically flat soliton solutions, but they can also be considered as geometries in their own right. It is relatively easy to identify the appropriate CFT duals when we consider the asymptotically $AdS_3 \times S^3$ geometries. To prepare the ground for this discussion, we should consider the near-horizon limit in the general family of metrics.

The near-horizon limit is usually obtained by assuming that $Q_1, Q_5 \gg M, a_1^2, a_2^2$ and focusing on the region $r^2 \ll Q_1, Q_5$. This limiting procedure is easily described if we consider the metric in the form (2.12): it just amounts to "dropping the 1" in the harmonic functions $H_1, H_5$, that is, replacing $\tilde{H}_1 \approx Q_1$, $\tilde{H}_5 \approx Q_5$, and also approximating $M\sinh\delta_1 \sinh\delta_5 \approx M\cosh\delta_1 \cosh\delta_5 \approx \sqrt{Q_1 Q_5}$ in the cross terms in the fibration. This gives us the asymptotically $AdS_3 \times S^3$ geometry

$$\begin{aligned} ds^2 = {} & \frac{1}{\sqrt{Q_1 Q_5}}\Big\{-(f-M)[d\tilde{t} - (f-M)^{-1}\sqrt{Q_1 Q_5}(a_1 \cos^2\theta d\psi + a_2 \sin^2\theta d\phi)]^2 + f[d\tilde{y} + f^{-1}\sqrt{Q_1 Q_5}(a_2 \cos^2\theta d\psi \\ & + a_1 \sin^2\theta d\phi)]^2\Big\} + \sqrt{Q_1 Q_5}\Big\{\frac{r^2 dr^2}{(r^2+a_1^2)(r^2+a_2^2) - Mr^2} + d\theta^2 + (f(f-M))^{-1}[(f(f-M) + f a_2^2 \sin^2\theta \\ & - (f-M) a_1^2 \sin^2\theta)\sin^2\theta d\phi^2 + 2M a_1 a_2 \sin^2\theta \cos^2\theta d\psi d\phi + (f(f-M) + f a_1^2 \cos^2\theta \\ & - (f-M) a_2^2 \cos^2\theta)\cos^2\theta d\psi^2]\Big\}. \end{aligned} \qquad (2.13)$$

This can be rewritten as

$$ds^2 = -\Big(\frac{\rho^2}{\ell^2} - M_3 + \frac{J_3^2}{4\rho^2}\Big)d\tau^2 + \Big(\frac{\rho^2}{\ell^2} - M_3 + \frac{J_3^2}{4\rho^2}\Big)^{-1}d\rho^2 + \rho^2\Big(d\varphi + \frac{J_3}{2\rho^2}d\tau\Big)^2 + \ell^2 d\theta^2 + \ell^2\sin^2\theta\Big[d\phi + \frac{R}{\ell^2}(a_1c_p - a_2s_p)d\varphi + \frac{R}{\ell^3}(a_2c_p - a_1s_p)d\tau\Big]^2 + \ell^2\cos^2\theta\Big[d\psi + \frac{R}{\ell^2}(a_2c_p - a_1s_p)d\varphi + \frac{R}{\ell^3}(a_1c_p - a_2s_p)d\tau\Big]^2, \quad (2.14)$$

where we have defined the new coordinates

$$\varphi = \frac{y}{R}, \qquad \tau = \frac{t\ell}{R}, \quad (2.15)$$

$$\rho^2 = \frac{R^2}{\ell^2}[r^2 + (M - a_1^2 - a_2^2)\sinh^2\delta_p + a_1a_2\sinh 2\delta_p] \quad (2.16)$$

and parameters

$$\ell^2 = \sqrt{Q_1Q_5}, \quad (2.17)$$

$$M_3 = \frac{R^2}{\ell^4}[(M - a_1^2 - a_2^2)\cosh 2\delta_p + 2a_1a_2\sinh 2\delta_p], \quad (2.18)$$

$$J_3 = \frac{R^2}{\ell^3}[(M - a_1^2 - a_2^2)\sinh 2\delta_p + 2a_1a_2\cosh 2\delta_p]. \quad (2.19)$$

Thus, we see that we recover the familiar observation that the near-horizon limit of the six-dimensional charged rotating black string is a twisted fibration of $S^3$ over the Banados-Teitelboim-Zanelli black hole solution [37].

## III. FINDING SOLITONIC SOLUTIONS

In general, these solutions will have singularities, horizons, and possibly also closed timelike curves. Let us now consider the conditions for the spacetime to be free of these features, giving a smooth solitonic solution.

Written in the form (2.1), the metric has coordinate singularities when $\tilde{H}_1 = 0$, $\tilde{H}_5 = 0$, or $g(r) \equiv (r^2 + a_1^2) \times (r^2 + a_2^2) - Mr^2 = 0$. In addition, the determinant of the metric vanishes if $\cos^2\theta = 0$, $\sin^2\theta = 0$, or $r^2 = 0$, which will therefore be singular loci for the inverse metric. The singularities at $\tilde{H}_1 = 0$ or $\tilde{H}_5 = 0$ are real curvature singularities, so we want to find solutions where $\tilde{H}_1 > 0$ and $\tilde{H}_5 > 0$ everywhere. The vanishing of the determinant at $\theta = 0$ and $\theta = \frac{\pi}{2}$ merely signals the degeneration of the polar coordinates at the north and south poles of $S^3$; these are known to be just coordinate singularities for arbitrary values of the parameters, so we will not consider them further.

The remaining coordinate singularities depend only on $r$. We can construct a smooth solution if the outermost one is the result of the degeneration of coordinates at a regular origin in some $\mathbb{R}^2$ factor; that is, of the smooth shrinking of an $S^1$. If this origin has a large enough value of $r$, we will have $\tilde{H}_1 > 0$ and $\tilde{H}_5 > 0$ there, and we will get a smooth solution. The coordinate singularity at $r^2 = 0$ cannot play this role, as we can shift it to an arbitrary position by defining a new radial coordinate by $\rho^2 = r^2 - r_0^2$. The determinant of the metric in the new coordinate system will vanish at $\rho^2 = 0$.

The interesting coordinate singularities are thus those at the roots of $g(r)$, and the first requirement for a smooth solution is that this function *have* roots. If we write

$$g(r) = (r^2 - r_+^2)(r^2 - r_-^2), \quad (3.1)$$

with $r_+^2 > r_-^2$, then

$$r_\pm^2 = \frac{1}{2}(M - a_1^2 - a_2^2) \pm \frac{1}{2}\sqrt{(M - a_1^2 - a_2^2)^2 - 4a_1^2a_2^2}. \quad (3.2)$$

We see that this function only has real roots for

$$|M - a_1^2 - a_2^2| > 2a_1a_2. \quad (3.3)$$

There are two cases: $M > (a_1 + a_2)^2$, or $M < (a_1 - a_2)^2$. Note that in the former case, $r_+^2 > 0$, whereas in the latter, $r_+^2 < 0$ (which is perfectly physical, since as noted above, we are free to define a new radial coordinate by shifting $r^2$ by an arbitrary constant).

Assuming one of these two cases hold, we can define a new radial coordinate by $\rho^2 = r^2 - r_+^2$. Since $r^2dr^2 = \rho^2 d\rho^2$, in this new coordinate system

$$g_{\rho\rho} = \sqrt{\tilde{H}_1\tilde{H}_5}\frac{d\rho^2}{\rho^2 + (r_+^2 - r_-^2)}, \quad (3.4)$$

and the determinant of the metric is $g = -\rho^2\frac{\tilde{H}_1^3}{\tilde{H}_5}\cos^2\theta\sin^2\theta$. Thus, in this coordinate system, the only potential problems are at $\rho^2 = 0$ and $\rho^2 = r_-^2 - r_+^2$, that is, at the two roots of the function $g(r)$.

To see what happens at $r^2 = r_+^2$, consider the geometry of the surfaces of constant $r$. The determinant of the induced metric is

$$g^{(ty\theta\phi\psi)} = -\cos^2\theta\sin^2\theta\tilde{H}_1^{1/2}\tilde{H}_5^{1/2}g(r). \quad (3.5)$$

Thus, at $r^2 = r_+^2$, the metric in this subspace degenerates. This can signal either an event horizon, where the surface $r^2 = r_+^2$ is null, or an origin, where $r^2 = r_+^2$ is of higher codimension. We can distinguish between the two possibilities by considering the determinant of the metric at fixed $r$ and $t$; that is, in the $(y, \theta, \phi, \psi)$ subspace. This is

$$\begin{aligned} g^{(y\theta\phi\psi)} = {}& \cos^2\theta\sin^2\theta\{g(r)(r^2 + a_1^2\sin^2\theta + a_2^2\cos^2\theta \\ & + M(1 + s_1^2 + s_5^2 + s_p^2)) + r^2M^2(c_1^2c_5^2c_p^2 \\ & - s_1^2s_5^2s_p^2) + M^2(M - a_1^2 - a_2^2)s_1^2s_5^2s_p^2 \\ & + 2M^2a_1a_2s_1c_1s_5c_5s_pc_p\}. \end{aligned} \tag{3.6}$$

This will be positive at $r^2 = r_+^2$ if and only if $M > (a_1 + a_2)^2$. If it is, the constant $t$ cross section of $r^2 = r_+^2$ will be spacelike, and $r^2 = r_+^2$ is an event horizon. Thus, we can have smooth solitonic solutions without horizons only in the other case $M < (a_1 - a_2)^2$.

To have a smooth solution, we need a circle direction to be shrinking to zero at $r^2 = r_+^2$. That is, we need some Killing vector with closed orbits to be approaching zero. Then by a suitable choice of period we could identify $\rho^2 = 0$ with the origin in polar coordinates of the space spanned by $\rho$ and the angular coordinate corresponding to this Killing vector. The Killing vectors with closed orbits are linear combinations

$$\xi = \partial_y - \alpha\partial_\psi - \beta\partial_\phi, \tag{3.7}$$

so a necessary condition for a circle degeneration is that (3.6) vanish at $r^2 = r_+^2$, so that some linear combination of this form has zero norm there. We can satisfy this condition in two different ways.

### A. Two-charge solutions: $a_1a_2 = 0$

The first possibility is to set $a_2 = 0$, so $a_1a_2 = 0$. Then for $M < a_1^2$, $r_+^2 = 0$, and we set (3.6) to zero at $r^2 = 0$ by taking one of the charges to vanish. We will focus on setting $\delta_p = 0$, since these solutions will have a natural interpretation in CFT terms. Recall that in string theory, we can interchange the different charges in this solution by U-dualities.

For this choice of parameters, the metric simplifies to

$$\begin{aligned} ds^2 = {}& \frac{1}{\sqrt{\tilde{H}_1\tilde{H}_5}}[-(f - M)(dt - (f - M)^{-1} \\ & \times Mc_1c_5a_1\cos^2\theta d\psi)^2 \\ & + f(dy + f^{-1}Ms_1s_5a_1\sin^2\theta d\phi)^2] \\ & + \sqrt{\tilde{H}_1\tilde{H}_5}\left(\frac{dr^2}{r^2 + a_1^2 - M} + d\theta^2 + \frac{r^2\sin^2\theta}{f}d\phi^2 \right. \\ & \left. + \frac{(r^2 + a_1^2 - M)\cos^2\theta}{f - M}d\psi^2\right). \end{aligned} \tag{3.8}$$

Since (3.6) vanishes at $r^2 = 0$, the orbits of a Killing vector of the form (3.7) must degenerate there. It is easy to use the simplified metric (3.8) to evaluate

$$\alpha = 0, \qquad \beta = \frac{a_1}{Ms_1s_5}. \tag{3.9}$$

That is, if we define a new coordinate

$$\tilde{\phi} = \phi + \frac{a_1}{Ms_1s_5}y, \tag{3.10}$$

the direction which goes to zero is $y$ at fixed $\tilde{\phi}$, $\psi$. To make $y \to y + 2\pi R$ at fixed $\tilde{\phi}$, $\psi$ a closed orbit, we require

$$\frac{a_1}{Ms_1s_5}R = m \in \mathbb{Z}. \tag{3.11}$$

Around $r = 0$, we then have

$$ds^2 = \ldots + \sqrt{\tilde{H}_1\tilde{H}_5}\left(\frac{dr^2}{a_1^2 - M} + \frac{r^2dy^2}{M^2s_1^2s_5^2}\right) + \ldots \tag{3.12}$$

This will be regular if we choose the radius of the $y$ circle to be

$$R = \frac{Ms_1s_5}{\sqrt{a_1^2 - M}}. \tag{3.13}$$

Thus, the integer quantization condition fixes

$$m = \frac{a_1}{\sqrt{a_1^2 - M}}. \tag{3.14}$$

With this choice of parameters, the solution is completely smooth, and $\theta$, $\tilde{\phi}$, $\psi$ are the coordinates on a smooth $S^3$ at the origin $r = 0$. We recover the smooth supersymmetric solutions of [15,16] for $m = 1$.

From the CFT point of view, it is natural to regard the charges $Q_1$, $Q_5$ and the asymptotic radius of the circle $R$ as fixed quantities. We can then solve (3.13) and (3.14) to find the other parameters, giving us a one integer parameter family of smooth solutions for fixed $Q_1$, $Q_5$, $R$. The integer (3.14) determines a dimensionless ratio $a_1^2/M$, while the other condition (3.13) fixes the overall scale ($a_1$, say) in terms of $Q_1$, $Q_5$, $R$.

### B. Three-charge solutions

Solutions with all three charges nonzero can be found by considering $a_1a_2 \neq 0$. Setting (3.6) to zero at $r^2 = r_+^2$ implies that

$$M = a_1^2 + a_2^2 - a_1a_2\frac{(c_1^2c_5^2c_p^2 + s_1^2s_5^2s_p^2)}{s_1c_1s_5c_5s_pc_p}, \tag{3.15}$$

and hence that

$$r_+^2 = -a_1a_2\frac{s_1s_5s_p}{c_1c_5c_p}. \tag{3.16}$$

The Killing vector which degenerates is (3.7) with[3]

$$\alpha = -\frac{s_pc_p}{(a_1c_1c_5c_p - a_2s_1s_5s_p)}, \qquad \beta = -\frac{s_pc_p}{(a_2c_1c_5c_p - a_1s_1s_5s_p)}. \tag{3.17}$$

The associated shifts in the $\phi$, $\psi$ coordinates are hence

[3]This choice of parameters is most easily derived by requiring $g_{ty} \to 0$ at $\rho^2 = 0$; having derived it, one can then check that it also gives $g_{yy} \to 0$ at $\rho^2 = 0$ as required.

$$\tilde{\psi} = \psi - \frac{s_p c_p}{(a_1 c_1 c_5 c_p - a_2 s_1 s_5 s_p)} y, \quad \tilde{\phi} = \phi - \frac{s_p c_p}{(a_2 c_1 c_5 c_p - a_1 s_1 s_5 s_p)} y, \tag{3.18}$$

and $y \to y + 2\pi R$ at fixed $\tilde{\phi}, \tilde{\psi}$ will be a closed orbit if

$$\frac{s_p c_p}{(a_1 c_1 c_5 c_p - a_2 s_1 s_5 s_p)} R = n, \quad \frac{s_p c_p}{(a_2 c_1 c_5 c_p - a_1 s_1 s_5 s_p)} R = -m \tag{3.19}$$

for some integers $n, m$. As in the two-charge case, requiring regularity of the metric at the origin fixes the radius of the $y$ circle. We do not give details of the calculation, but simply quote the result,

$$R = \frac{M s_1 c_1 s_5 c_5 (s_1 c_1 s_5 c_5 s_p c_p)^{1/2}}{\sqrt{a_1 a_2}(c_1^2 c_5^2 c_p^2 - s_1^2 s_5^2 s_p^2)}. \tag{3.20}$$

If we introduce dimensionless parameters

$$j = \left(\frac{a_2}{a_1}\right)^{1/2} \le 1, \qquad s = \left(\frac{s_1 s_5 s_p}{c_1 c_5 c_p}\right)^{1/2} \le 1, \tag{3.21}$$

then the integer quantization conditions determine these via

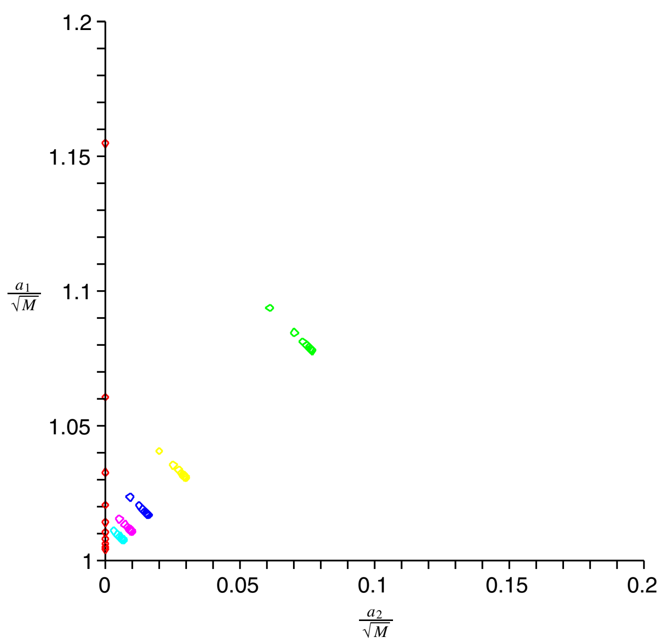


FIG. 1 (color online). The values of the dimensionless quantities $a_2/\sqrt{M}, a_1/\sqrt{M}$ for which smooth solitons are obtained are indicated by points. The highest point on the figure corresponds to $m = 2$, $n = 0$. Increasing $n$ moves diagonally downwards towards the diagonal, and increasing $m - n$ moves down towards (0,1). For each point, there is a set of orbifolds labeled by $k$. Solutions with event horizons exist in the region $a_1/\sqrt{M} + a_2/\sqrt{M} < 1$ (off the bottom of the plotted region).

$$\frac{j + j^{-1}}{s + s^{-1}} = m - n, \qquad \frac{j - j^{-1}}{s - s^{-1}} = m + n. \tag{3.22}$$

Note that this constraint is invariant under the permutation of the three charges. We note that we can rewrite the mass (3.15) as

$$M = a_1 a_2 (s^2 - j^2)(j^{-2} s^{-2} - 1) = a_1 a_2 nm (s^{-2} - s^2)^2, \tag{3.23}$$

so $M \ge 0$ implies $s^2 \ge j^2$ and $nm \ge 0$. Our assumption that $a_1 > a_2$ implies $n \ge 0$, so $m \ge 0$, and (3.22) implies $m > n$.

Thus, in this case, for given $Q_1, Q_5, R$, we have a two integer parameter family of smooth solutions. It is a little more difficult to make direct contact with the supersymmetric solutions of [22] in this case, since one needs to take a limit $a_1, a_2 \to \infty$, but these would correspond to $m = n + 1$, as it turns out that $s = 1$ and $M = 0$ if and only if $m = n + 1$. We can also think of the two-charge solutions in the previous subsection as corresponding to the case $n = 0$. To gain some insight into the values of the parameters for other choices of $m, n$, we have plotted the dimensionless quantities $a_1/\sqrt{M}, a_2/\sqrt{M}$ for some representative values in Fig. 1.

### C. Orbifolds and more general smooth three-charge solutions

So far, we have insisted that the solution be smooth. However, in the context of string theory, we may also consider solutions with orbifold singularities, since the corresponding worldsheet conformal field theory is completely well defined. In the context of the above smooth solutions, a particularly interesting class of orbifolds is the $\mathbb{Z}_k$ quotient by the discrete isometry $(y, \psi, \phi) \sim (y + 2\pi R/k, \psi, \phi)$.

In the two-charge case, the quotient acts as $(y, \psi, \tilde{\phi}) \sim (y + 2\pi R/k, \psi, \tilde{\phi} + 2\pi m/k)$ in the coordinates appropriate near $r = 0$. This isometry has a fixed point at $r = 0$, $\theta = 0$, so the resulting orbifold has a $\mathbb{Z}_k$ orbifold singularity there. In addition, if $k$ and $m$ are not relatively prime, there will be a $\mathbb{Z}_j$ orbifold singularity at $r = 0$ for all $\theta$, where $j = \gcd(k, m)$. The supersymmetric orbifolds corresponding to $m = 1$ have previously been studied [10,15,19].

In the three-charge case, the discrete isometry becomes $(y, \tilde{\psi}, \tilde{\phi}) \sim (y + 2\pi R/k, \tilde{\psi} - 2\pi n/k, \tilde{\phi} + 2\pi m/k)$, and the $\mathbb{Z}_k$ will be freely acting if $m$ and $n$ are relatively prime to $k$. Thus, we get new smooth three-charged solutions by orbifolding by a $k$ which is relatively prime to $m$ and $n$. We could have found such solutions directly if we had allowed for the possibility that $y \to y + 2\pi Rk$ is the closed circle at $\rho = 0$, instead of insisting that it be $y \to y + 2\pi R$. We also have orbifolds similar to the two-charged ones if one or both of $m$ and $n$ are not relatively prime to $k$. In particular, the simple supersymmetric orbifolds studied in [23] correspond to taking $m = kn' + 1$, $n = kn'$ for some integer $n'$.[4] The preserved supersymmetries in the solutions with $m = n + 1$ correspond to Killing spinors which are invariant under translation in $y$ at fixed $\phi, \psi$, so all the orbifolds of cases with $m = n + 1$ will be supersymmetric. In particular, orbifolds where $k$ is relatively prime to $n$ and $n + 1$ will give new smooth supersymmetric solutions.

### D. Asymptotically AdS solutions

In order to understand the dual CFT interpretation of these solutions, it is interesting to see the effect of the constraints (3.20) and (3.22) on the asymptotically AdS solution (2.14). Consider first the two-charge case. If we set $a_2 = 0$, $\delta_p = 0$ and insert (3.13) and (3.14) in (2.14), we will have

$$\begin{aligned} ds^2 = &-\left(\frac{\rho^2}{\ell^2} + 1\right)d\tau^2 + \left(\frac{\rho^2}{\ell^2} + 1\right)^{-1} d\rho^2 + \rho^2 d\varphi^2 \\ &+ \ell^2[d\theta^2 + \sin^2\theta(d\phi + md\varphi)^2 \\ &+ \cos^2\theta(d\psi + md\tau/\ell)^2]. \end{aligned} \tag{3.24}$$

Thus, the asymptotically AdS version of the soliton is just global $AdS_3 \times S^3$, with a shift of the angular coordinates on the sphere determined by $m$.

In the general three-charge case, the interpretation of the dimensionless parameter $s$ changes in the asymptotically AdS solutions: it is now $s = \sqrt{\tanh\delta_p}$. The conditions (3.22) are unaffected, however, and inserting these and the value of the period (3.20) in (2.14), we will have

$$\begin{aligned} ds^2 = &-\left(\frac{\rho^2}{\ell^2} + 1\right)d\tau^2 + \left(\frac{\rho^2}{\ell^2} + 1\right)^{-1} d\rho^2 + \rho^2 d\varphi^2 \\ &+ \ell^2[d\theta^2 + \sin^2\theta(d\phi + md\varphi - nd\tau/\ell)^2 \\ &+ \cos^2\theta(d\psi - nd\varphi + md\tau/\ell)^2]. \end{aligned} \tag{3.25}$$

Thus, again, the asymptotically AdS version of the soliton is just global $AdS_3 \times S^3$, with shifts of the angular coordinates on the sphere determined by $m, n$.

Thus, in the cases where they have a large "core" region described by an asymptotically AdS geometry, the smooth solitons studied in the first two subsections above approach global $AdS_3 \times S^3$ in this region. As a consequence, the orbifolds studied in the previous section will have corresponding orbifolds of $AdS_3 \times S^3$; some of these orbifolds were discussed in [38,39]. The resulting quotient geometry is still asymptotically $AdS_3 \times S^3$, as can be seen by introducing new coordinates $\varphi' = k\varphi$, $\tau' = k\tau$, $\rho' = \rho/k$. The metric on the orbifold in these coordinates is then

$$\begin{aligned} ds^2 = &-\left(\frac{\rho'^2}{\ell^2} + \frac{1}{k^2}\right)d\tau'^2 + \left(\frac{\rho'^2}{\ell^2} + \frac{1}{k^2}\right)^{-1} d\rho'^2 + \rho'^2 d\varphi'^2 \\ &+ \ell^2\left[d\theta^2 + \sin^2\theta\left(d\phi + \frac{m}{k}d\varphi' - \frac{n}{k\ell}d\tau'\right)^2 \right. \\ &\left. + \cos^2\theta\left(d\psi - \frac{n}{k}d\varphi' + \frac{m}{k\ell}d\tau'\right)^2\right]. \end{aligned} \tag{3.26}$$

The redefined angular coordinate $\varphi'$ will have period $2\pi$ on the orbifold.

## IV. VERIFYING REGULARITY

In the previous section, we claim to have found a family of smooth solitonic solutions, by imposing three conditions on the parameters of the general metric. We should now verify that these solutions have no pathologies. In this section, we will use the radial coordinate $\rho^2 = r^2 - r_+^2$ (for the two-charge solutions, $\rho^2 = r^2$), which runs over $\rho \geq 0$.

The first step is to check that $\tilde{H}_1 > 0$, $\tilde{H}_5 > 0$ for all $\rho \geq 0$, as desired. In these coordinates,

$$f = \rho^2 + (a_1^2 - a_2^2)\sin^2\theta + (a_2^2 - a_1 a_2 s^2). \tag{4.1}$$

In the two-charge case, where $a_2 = 0$, the last term vanishes, so $f \geq 0$, and hence $\tilde{H}_1 > 0$, $\tilde{H}_5 > 0$ everywhere. In the more general case, however, the last term is

$$a_2^2 - a_1 a_2 s^2 = -a_1 a_2 (s^2 - j^2) < 0, \tag{4.2}$$

so we do not have $f \geq 0$. Examining $\tilde{H}_1$ directly,

$$\begin{aligned} \tilde{H}_1 = &\rho^2 + (a_1^2 - a_2^2)\sin^2\theta + a_1 a_2 (s^2 - j^2) \\ &\times (s^{-2} j^{-2} s_1^2 - c_1^2), \end{aligned} \tag{4.3}$$

so for $\tilde{H}_1 > 0$ everywhere, we need the last factor to be positive. We know $s^2 > j^2$, and we can rewrite the last bracket as

$$(s^{-2} j^{-2} s_1^2 - c_1^2) = \frac{c_1^2}{j^2}\left(s^2 \frac{c_5^2 c_p^2}{s_5^2 s_p^2} - j^2\right) > 0, \tag{4.4}$$

so we indeed have $\tilde{H}_1 > 0$. We can similarly show $\tilde{H}_5 > 0$. Thus, the metric in the $(t, \rho, \theta, \tilde{\phi}, \tilde{\psi}, z^i)$ coordinates is regular for all $\rho > 0$, apart from the coordinate singularities associated with the poles of the $S^3$ at $\theta = 0, \pi/2$, so the local geometry is smooth.

Next we check for global pathologies. We can easily see that these solutions have no event horizons. The determinant of the metric of a surface of constant $\rho$, (3.5), is negative for $\rho > 0$. That is, there is a timelike direction of constant $\rho$ for all $\rho > 0$, and hence by continuity there must be a timelike curve which reaches the asymptotic region from any fixed $\rho$. We will demonstrate the absence of closed timelike curves by proving a stronger statement, that the soliton solutions are stably causal. Using the expression for the inverse metric in Appendix A, we can evaluate

[4] In [23], other examples where $n \neq kn'$ are obtained by applying a chain of dualities to these ones. This is possible because while $(m, n)$ are U-duality invariant, $k$ is not, so this transformation can map us to new solutions.

$$\partial_\mu t \partial_\nu t g^{\mu\nu} = -\frac{1}{\sqrt{\tilde{H}_1 \tilde{H}_5}} \bigg[ f + M(1 + s_1^2 + s_5^2 + s_p^2) + \frac{M^2(c_1^2 c_5^2 c_p^2 - s_1^2 s_5^2 s_p^2)}{\rho^2 + r_+^2 - r_-^2} \bigg] < 0, \tag{4.5}$$

so $\partial_\mu t$ is a timelike covector, and $t$ is a global time function for the solitons. Hence the solitons are stably causal, and, in particular, free of closed timelike curves.

Finally, we should check regularity at $\rho = 0$. In the previous section, we chose $R$ so that the $\rho, y$ coordinates were the polar coordinates in a smooth $\mathbb{R}^2$. If we define new coordinates on this $\mathbb{R}^2$ regular at $\rho = 0$ by

$$x^1 = \rho \cos(y/R), \qquad x^2 = \rho \sin(y/R), \tag{4.6}$$

then

$$dy = \frac{1}{(x_1^2 + x_2^2)} (x^1 dx^2 - x^2 dx^1), \tag{4.7}$$

and we need the other $g_{\mu y}$ components in the metric to go to zero at least linearly in $\rho$ for the whole metric to be smooth at $\rho = 0$ once we pass to the Cartesian coordinates $x^1, x^2$. In fact, we find that the $g_{\mu y}$ go like $\rho^2$ for small $\rho$ in the $(t, \rho, \theta, \tilde{\phi}, \tilde{\psi}, z^i)$ coordinates.

We also need to verify the regularity of the matter fields. The dilaton is trivially regular, since $\tilde{H}_1 > 0$, $\tilde{H}_5 > 0$, but the Ramond-Ramond two-form requires checking. The nontrivial question is whether the $C_{y\mu}$ go to zero at $\rho^2 = 0$. In fact, in the gauge we used in (2.5), they do not: we find

$$\begin{aligned} C_{y\tilde{\phi}} &= \frac{M s_p c_p s_5 c_5}{a_1 c_1 c_5 c_p - a_2 s_1 s_5 s_p} + O(\rho^2), \\ C_{y\tilde{\psi}} &= \frac{M s_p c_p s_5 c_5}{a_2 c_1 c_5 c_p - a_1 s_1 s_5 s_p} + O(\rho^2), \\ C_{yt} &= \frac{1 + s_1^2 + s_p^2}{s_1 c_1} + O(\rho^2). \end{aligned} \tag{4.8}$$

We can remove these constant terms by a gauge transformation, so the Ramond-Ramond fields are regular at $\rho = 0$. The physical importance of the constant terms is that they correspond to electromagnetic potentials dual to the charges carried by the geometry, and their presence is presumably related to the first law satisfied by these soliton solutions, as in [40].

In summary, we have shown that the two integer parameter family of solutions identified in the previous section are all smooth solutions without closed timelike curves. In the next section, we will explore their relation to the CFT description of the D1-D5-P system.

## V. RELATION TO CFT

We have found new smooth solutions by considering the general family of charged rotating black hole solutions (2.1). These are labeled by the radius $R$, charges $(Q_1, Q_5)$, and three integers $(m, n, k)$. They include the previously known supersymmetric solutions as special cases, and add nonsupersymmetric solutions and new supersymmetric orbifold solutions. We would like to see if we can relate these solutions to the CFT description, as was done for the earlier supersymmetric cases in [15,16,22].

If we consider the asymptotically $AdS_3 \times S^3$ solutions constructed in Sec. III D, which describe the "core" region of the asymptotically flat solitons, we can use the powerful AdS/CFT correspondence machinery to identify the corresponding states in the CFT. The dual CFT for the asymptotically $AdS_3 \times S^3 \times T^4$ spaces with radius $\ell = (Q_1 Q_5)^{1/4}$ is a sigma model with target space a deformation of the orbifold $(T^4)^N/S_N$ [41–43], where

$$N = n_1 n_5 = \frac{\ell^4 V}{g^2 l_s^8}, \tag{5.1}$$

where $V$ is the volume of the $T^4$. This theory has $c = 6 n_1 n_5$. In Sec. III D, we showed that the corresponding asymptotically AdS solutions for a basic family of solitons were always global $AdS_3 \times S^3$, with a shift on the angular coordinates on the sphere specified by $n, m$. Following the proposal outlined in [15], we identify the geometries (3.25) with CFT states with charges

$$\begin{aligned} h &= \frac{c}{24}(m + n)^2, \qquad j = \frac{c}{12}(m + n), \\ \bar{h} &= \frac{c}{24}(m - n)^2, \qquad \bar{j} = \frac{c}{12}(m - n). \end{aligned} \tag{5.2}$$

Thus, these states have energy

$$E = h + \bar{h} = 2(m^2 + n^2)\frac{c}{24} = \frac{1}{2}(m^2 + n^2) n_1 n_5, \tag{5.3}$$

and momentum

$$q_p = h - \bar{h} = 4mn\frac{c}{24} = nm n_1 n_5. \tag{5.4}$$

Since the noncompact geometry is global $AdS_3$, there is a single spin structure on the spacetime. Because of the shifts in the angular coordinates, this spin structure can be either periodic or antiperiodic around $\varphi$ at fixed $\phi, \psi$: it will be periodic if $m + n$ is odd, and antiperiodic if $m + n$ is even. Thus, the geometry is identified with a RR state with the above charges if $m + n$ is odd, and with a NSNS state with these same charges if $m + n$ is even.

These states can be interpreted in terms of spectral flow. Recalling that spectral flow shifts the CFT charges by [44]

$$h' = h + \alpha j + \alpha^2 \frac{c}{24}, \qquad j' = j + \alpha \frac{c}{12}, \tag{5.5}$$

$$\bar{h}' = \bar{h} + \beta \bar{j} + \beta^2 \frac{c}{24}, \qquad \bar{j}' = \bar{j} + \beta \frac{c}{12}, \tag{5.6}$$

we can see that the required states can be obtained by spectral flow with $\alpha = m + n$, $\beta = m - n$ acting on the

NSNS ground state (for which $h = j = 0$, $\bar{h} = \bar{j} = 0$). This spectral flow can be identified with the coordinate transformation in spacetime which relates the $(\varphi, \phi, \psi)$ coordinates to the $(\varphi, \tilde{\phi}, \tilde{\psi})$ coordinates. Thus, we see that the nonsupersymmetric states corresponding to all the geometries labeled by $m, n$ are constructed by starting with the maximally supersymmetric NSNS vacuum and applying different amounts of spectral flow.

In [15], the special case $m = 1$, $n = 0$ was discussed. In this case, the spectral flow is by one unit on both the left and the right, and maps the Neveu-Schwarz (NS) vacuum to a Ramond (R) ground state both on the left and the right. We can see the supersymmetry of this state from the spacetime point of view: the covariantly constant Killing spinors in global AdS have the form

$$\epsilon_L^{\pm} = e^{\pm i(\tilde{\phi}_L/2)} e^{-i(\varphi/2)} \epsilon_0, \qquad \epsilon_R^{\pm} = e^{\pm i(\tilde{\phi}_R/2)} e^{-i(\varphi/2)} \epsilon_0, \tag{5.7}$$

so when we shift $\tilde{\phi}_L = \phi_L + \varphi$, $\tilde{\phi}_R = \phi_R + \varphi$, the Killing spinors $\epsilon_L^+$, $\epsilon_R^+$ become independent of $\varphi$, corresponding to the preserved Killing symmetries in the R ground state. If we consider $m = n + 1$, the spectral flow on the right is by one unit, so $\epsilon_R^+$ is still independent of $\varphi$. These are the supersymmetric states considered in [22], which are R ground states on the right, but the more general R states obtained by spectral flowing by $2n + 1$ units on the left. Our nonsupersymmetric solitons correspond to the more general nonsupersymmetric states obtained by spectral flowing the NSNS vacuum by $m - n$ units on the right and $m + n$ units on the left. In [22], an explicit representation for the R sector state obtained by spectral flow by $2r + 1$ units was given,[5]

$$|2r + 1\rangle_R = (J^+_{-(2r)})^{n_1 n_5} (J^+_{-(2r-4)})^{n_1 n_5} \ldots (J^+_{-2})^{n_1 n_5} |1\rangle, \tag{5.8}$$

where $J^+_{-k}$ is a mode of the $su(2)$ current of the full CFT which raises $h$ and $j$ by $\Delta h = k$, $\Delta j = 1$, and $|1\rangle$ is the R ground state with $j = +c/12$ obtained by spectral flow from the NS ground state. Similarly, one can give an explicit representation of the NS sector state obtained by spectral flow by $2r$ units, following [45],

$$|2r\rangle_{\rm NS} = (J^+_{-(2r-1)})^{n_1 n_5} (J^+_{-(2r-3)})^{n_1 n_5} \ldots (J^+_{-1})^{n_1 n_5} |0\rangle_{\rm NS}. \tag{5.9}$$

The CFT state corresponding to the geometry (3.25) is then $|m + n\rangle_R \times |m - n\rangle_R$ or $|m + n\rangle_{\rm NS} \times |m - n\rangle_{\rm NS}$, depending on the parity of $m + n$.

The situation is more interesting when we consider the orbifolds. The geometries (3.26) should be identified with CFT states with charges

$$h = \frac{c}{24}\left(1 + \frac{(m+n)^2 - 1}{k^2}\right), \qquad j = \frac{c}{12}\frac{m+n}{k},$$
$$\bar{h} = \frac{c}{24}\left(1 + \frac{(m-n)^2 - 1}{k^2}\right), \qquad \bar{j} = \frac{c}{12}\frac{m-n}{k}. \tag{5.10}$$

In the supersymmetric case, when $m = n + 1$, $\bar{h} = \frac{c}{24}$, $\bar{j} = \frac{c}{12}\frac{1}{k}$, so these geometries still have the charges of R ground states on the right. This particular R ground state corresponds to the spectral flow of the NS chiral primary state with $\bar{h} = \bar{j} = \frac{c}{24}\frac{k-1}{k}$. However, the charges of the state in the left-moving sector are, in general, not those of a R ground state or even the result of spectral flow on a R ground state. For general $m, n$, neither sector is the spectral flow of a ground state. Thus, these provide examples of geometries dual to more general CFT states.

To specify the CFT state completely, we need to say if (5.10) are the charges of a RR or a NSNS state. To do so, let us consider the spin structure on spacetime. When $m$ or $n$ is relatively prime to $k$, there is a contractible circle in the spacetime, and as a result the spin structure is fixed. The contractible circle is $(\varphi', \phi, \psi) \to (\varphi' + 2\pi k, \phi - 2\pi m, \psi - 2\pi n)$. The fermions must be antiperiodic around this circle. For the case where neither $m$ nor $n$ is relatively prime to $k$, we are not forced to make this choice, but we will assume that we still choose a spin structure such that the fermions are antiperiodic around this circle; this would correspond to the spin structure inherited from the covering space of the orbifold.

In the supersymmetric case $m = n + 1$, and more generally for $m + n$ odd, this implies that the fermions are periodic under $\varphi' \to \varphi' + 2\pi k$ at fixed $\phi, \psi$. For $k$ odd, this implies the fermions must be periodic under $\varphi' \to \varphi' + 2\pi$, while for $k$ even, they may be either periodic or antiperiodic. Thus, for $m = n + 1$, we can always choose the periodic spin structure for the fermions on spacetime. This spacetime will then be identified with the supersymmetric RR state with the charges (5.10). However, for $k$ even, we can choose the antiperiodic spin structure for the fermions on spacetime; this spacetime will then be identified with a NSNS state with the same charges (5.10). In this latter case, neither the spacetime solution nor the CFT state is supersymmetric.

The situation becomes stranger for $m + n$ even. The antiperiodicity around the contractible cycle implies that the fermions will be antiperiodic under $\varphi' \to \varphi' + 2\pi k$ at fixed $\phi, \psi$. If $k$ is odd, this is compatible with a spin structure antiperiodic in $\varphi'$, but if $k$ is even, there is no spin structure on the orbifold which satisfies this condition. The orbifold cannot be made into a spin manifold. The general conditions for such orbifolds $M/\Gamma$ to inherit a spin structure from the spin manifold $M$ were discussed in [46]; see also [47] for further discussion relevant to the case at hand. It will be interesting to see how this obstruction for $k$ even, $m + n$ even is reflected in the CFT dual.

[5] We use a slightly different notation than [22].

In the other cases, we can unambiguously identify the CFT state corresponding to the geometry as the state with charges (5.10) in the sector with the same periodicity conditions on the fermions as in the spacetime (choosing one of the two possible spin structures on spacetime in the case $k$ even, $m+n$ odd). It would be interesting to construct an explicit description of these states, as in the discussion in [22,23].

Thus, there is a clear CFT interpretation of the asymptotically $AdS_3 \times S^3$ geometries. However, the interesting discovery in this paper is that there are nonsupersymmetric asymptotically flat geometries, and we want to ask to what extent these can also be identified with individual microstates in the CFT. Clearly the appropriate CFT states to consider are the ones described above, but does the identification between state and geometry extend to the asymptotically flat spacetimes? In particular, does it make sense to identify the asymptotically flat spacetime with a CFT state in the general case where it does not have a large approximately $AdS_3 \times S^3$ core region, and there is no supersymmetry?[6] We would not in general expect the match to asymptotically flat geometries to be perfect, but there is one nontrivial piece of evidence for the identification of the full asymptotically flat geometries with the CFT states: the form of the charges still reflects the CFT structure. Plugging our parameters into (2.8), (2.10), and (2.11), gives

$$Q_p = nm\frac{Q_1 Q_5}{R^2}, \tag{5.11}$$

$$J_\phi = -m\frac{Q_1 Q_5}{R}, \tag{5.12}$$

$$J_\psi = n\frac{Q_1 Q_5}{R}. \tag{5.13}$$

These reproduce the quantization of the CFT charges in (5.2). In the orbifold case, we replace $R$ by $kR$, as the physical period of the asymptotic circle is $k$ times smaller, and these values now agree with the charges in (5.10). This seems to us like a very nontrivial consistency check, as it is very difficult to even express the parameters $M, a_1, a_2$ appearing in the metric (2.1) in terms of $Q_1$, $Q_5$, and $R$ and the integers $m, n$, so there is no reason why we would have expected to get such a simple result automatically. So this appears a good reason to believe properties of the full asymptotically flat geometries are connected to the CFT states. Note, however, that it does not seem to be possible to cast the ADM mass in such a simple form. In the next section, we will see also that the predicted time delay involved in scattering of probes does not quite match CFT expectations.

[6]The CFT state for some of the geometries is in the NSNS sector. We do not regard this as a serious obstruction to an identification at the classical level: we are considering nonsupersymmetric geometries, so we can allow the fermions to be antiperiodic around the asymptotic circle in spacetime. At the quantum level, one might worry that these antiperiodic boundary conditions lead to a constant energy density inconsistent with the assumed asymptotic flatness.

## VI. PROPERTIES OF THE SOLITONS

We will briefly discuss some properties of these solutions, solitons, and their relation to the dual CFT. We first discuss the solution of the massless scalar wave equation in these geometries, following the discussion in [10,19,23] closely. We then consider the most significant difference between our nonsupersymmetric solitons and the supersymmetric cases, the absence of an everywhere causal Killing vector.

### A. Wave equation

It is interesting to study the behavior of the massless wave equation on this geometry. This is a first step towards analyzing small perturbations and also allows us to address questions of scattering in the geometry which indicate how an exterior observer might probe the soliton. We consider the massless wave equation on the geometry,

$$\Box\Psi = 0. \tag{6.1}$$

It was shown in [33] that this equation is separable. Considering a separation ansatz

$$\Psi = \exp(-i\omega t/R + i\lambda y/R + im_\psi\psi + im_\phi\phi)\chi(\theta)h(r), \tag{6.2}$$

and using the inverse metric given in Appendix A, we find that the wave equation reduces to

$$\frac{1}{\sin2\theta}\frac{d}{d\theta}\left(\sin2\theta\frac{d}{d\theta}\chi\right) + \left[\frac{(\omega^2-\lambda^2)}{R^2}(a_1^2\sin^2\theta + a_2^2\cos^2\theta) - \frac{m_\psi^2}{\cos^2\theta} - \frac{m_\phi^2}{\sin^2\theta}\right]\chi = -\Lambda\chi, \tag{6.3}$$

$$\frac{1}{r}\frac{d}{dr}\left[\frac{g(r)}{r}\frac{d}{dr}h\right] - \Lambda h + \left[\frac{(\omega^2-\lambda^2)}{R^2}(r^2 + Ms_1^2 + Ms_5^2) + (\omega c_p + \lambda s_p)^2\frac{M}{R^2}\right]h - \frac{(\lambda - nm_\psi + mm_\phi)^2}{(r^2 - r_+^2)}h + \frac{(\omega\varrho + \lambda\vartheta - nm_\phi + mm_\psi)^2}{(r^2 - r_-^2)}h = 0, \tag{6.4}$$

where

$$\varrho = \frac{c_1^2c_5^2c_p^2 - s_1^2s_5^2s_p^2}{s_1c_1s_5c_5}, \qquad \vartheta = \frac{c_1^2c_5^2 - s_1^2s_5^2}{s_1c_1s_5c_5}s_pc_p. \tag{6.5}$$

We see that the singularity in the wave equation at $r^2 = r_+^2$ is controlled by the frequency around the circle which is shrinking to zero there. This is a valuable check on the algebra. If we introduce a dimensionless variable

$$x = \frac{r^2 - r_+^2}{r_+^2 - r_-^2}, \tag{6.6}$$

we can rewrite the radial equation in the form used in [22],

$$4\frac{d}{dx}\left[x(x+1)\frac{d}{dx}h\right] + \left(\sigma^{-2}x + 1 - \nu^2 + \frac{\xi^2}{x+1} - \frac{\zeta^2}{x}\right)h = 0, \tag{6.7}$$

where

$$\sigma^2 = \left[(\omega^2 - \lambda^2)\frac{(r_+^2 - r_-^2)}{R^2}\right]^{-1}, \tag{6.8}$$

$$\nu = \left[1 + \Lambda - \frac{(\omega^2-\lambda^2)}{R^2}(r_+^2 + Ms_1^2 + Ms_5^2) - (\omega c_p + \lambda s_p)^2\frac{M}{R^2}\right]^{1/2}, \tag{6.9}$$

$$\xi = \omega\varrho + \lambda\vartheta - nm_\phi + mm_\psi, \tag{6.10}$$

$$\zeta = \lambda - nm_\psi + mm_\phi. \tag{6.11}$$

We can then use the results of [22], where the matching of solutions of this equation in an inner and outer region was carried out in detail, to determine the reflection coefficient. This reflection coefficient can be used to determine the time $\Delta t$ it takes for a quantum scattering from the core region near $x = 0$ to return to the asymptotic region, by expanding $\mathcal{R} = a + b\sum_n e^{2\pi in(\omega/R)\Delta t}$. Their matching procedure is valid when

$$\sigma^2 \gg 1, \tag{6.12}$$

and

$$\Delta t \gg \frac{R}{(\omega^2 - \lambda^2)^{1/2}}. \tag{6.13}$$

Under these assumptions, their matching procedure gives

$$\Delta t = \pi R_s \varrho, \tag{6.14}$$

where $R_s$ is the radius (3.20) for a smooth solution; in the orbifolds, $R = R_s/k$. We note that this is in agreement with their result in the supersymmetric case, as in the limit $\delta_1, \delta_5, \delta_p \to \infty$,

$$\varrho = \frac{s_1^2s_5^2 + s_1^2s_p^2 + s_5^2s_p^2 + s_1^2 + s_5^2 + s_p^2 + 1}{s_1c_1s_5c_5} \approx \frac{Q_1Q_5 + Q_1Q_p + Q_5Q_p}{Q_1Q_5} = \frac{1}{\eta} \tag{6.15}$$

in the notation of [23].

In the CFT picture, this travel time is interpreted as the time required for two CFT modes on the brane to travel around its world volume and meet again. Thus, from the CFT point of view, the expected value is $\Delta t_{\rm CFT} = \pi R_s$. As in [23], there is a "redshift factor" $\varrho$ between our spacetime result and the expected answer from the CFT point of view. It was argued in [23] that such a factor must appear to make the spacetime result invariant under permutation of the three charges, and it was proposed that this factor could be understood as a scaling between the asymptotic time coordinate $t$ in the asymptotically flat space and the time coordinate appropriate to the CFT. Evidence for this point of view was found by noting that in the cases where the soliton had a large $AdS_3 \times S^3$ core region, the global AdS time $\tau$ was proportional to $\eta t$, so $\Delta\tau = \pi R_s$ in accordance with CFT expectations. In our nonsupersymmetric case, for fixed $m, n$, the appropriate limit in which we obtain a large AdS region is the limit $\delta_1, \delta_5 \gg 1$ for fixed $\delta_p$ considered in Sec. III D. We did not see any such scaling between the AdS and asymptotic coordinates there, but $\varrho \approx 1$ in this limit, so this is consistent with the interpretation proposed in [23]. However, we remain uncomfortable with this interpretation. It is hard to argue directly for such a redshift between the CFT and asymptotic time coordinates in the general case where the soliton does not have a large approximately $AdS_3 \times S^3$ core. Indeed, in the dual brane picture of the geometry, where we have a collection of D1 and D5 branes in a flat background, one would naïvely expect the two to be the same. A deeper understanding of this issue could shed interesting light on the limitations of the identification between CFT states and the asymptotically flat geometries.

### B. Ergoregion

Although our soliton solutions are free of event horizons, they typically have ergoregions. These already appear in the supersymmetric three-charge soliton solutions studied in [22,23], where the Killing vector $\partial_t$, which defines time translation in the asymptotic rest frame, becomes spacelike at $f = 0$ if $Q_p \neq 0$. However, in these supersymmetric cases, there is still a causal Killing vector (arising from the square of the covariantly constant Killing spinor), which corresponds asymptotically to the time translation with respect to some boosted frame. A striking difference in the nonsupersymmetric solitons is the absence of any such globally timelike or null Killing vector field.[7] The most general Killing vector field which is causal in the asymptotic region of the asymptotically flat solutions is

$$V = \partial_t + v^y\partial_y \tag{6.16}$$

for $|v^y| \leq 1$. However, when $f = 0$, the norm of this Killing vector is

[7]For the asymptotically AdS spacetimes, there is a globally timelike Killing vector field, given by $\partial_t$ at fixed $\tilde{\psi}$, $\tilde{\phi}$. In $(t, y, \psi, \phi)$ coordinates, this is of the form $V' = \ell\partial_t - m\partial_\psi + n\partial_\phi$, so it cannot be extended to a globally timelike Killing vector field in the asymptotically flat geometry.

$$|V|^2 = \frac{M}{\sqrt{\tilde{H}_1\tilde{H}_5}}(c_p - v^y s_p)^2 > 0. \tag{6.17}$$

The best we can do is to take $v^y = \tanh\delta_p$, for which this Killing vector is timelike for $f > M$. Note that as a consequence, the two-charge nonsupersymmetric solutions also have ergoregions.

In a rotating black hole solution, the existence of an ergoregion typically implies a classical instability when the black hole is coupled to massive fields [48,49]. This instability arises when we send in a wave packet which has positive energy less than the rest mass with respect to the asymptotic Killing time but negative energy in the ergoregion. The wave packet will be partially absorbed by the black hole, but because the absorbed portion has negative energy, the reflected portion will have a larger amplitude. This then reflects off the potential at large distances and repeats the process. This process causes the amplitude of the initial wave packet to grow indefinitely, until its back-reaction on the geometry becomes significant.

One might have thought that in the supersymmetric three-charge solitons, the instability would not appear as a consequence of the existence of a causal Killing vector, by a mechanism similar to that discussed in [50] for Kerr-AdS black holes. However, this instability is in fact absent for a different reason, which applies to both supersymmetric and nonsupersymmetric solitons. The instability in black holes is a result of the existence of both an ergoregion and an event horizon, so in the solitons, the absence of an event horizon can prevent such an instability from occurring. Indeed, from the discussion of the massless wave equation in the previous section, we can see that the net flux is always zero, and the amplitude of the reflected wave is the same as that of the incident wave. That is, although there is an ergoregion, no superradiant scattering of classical waves occurs in this geometry, and the mechanism that led to the black hole bomb does not apply here. There might be an instability if we considered some interacting theory, as the interactions might convert part of an incoming wave packet to negative-energy modes bound to the soliton, but we will not attempt to explore this issue in more detail.

Thus, for free fields, there is no stimulated emission at the classical level. We will now show that there is also no spontaneous quantum emission.[8] There is a natural basis of modes for this geometry; for the scalar field, (6.2). To establish which of these modes are associated with creation and which with annihilation operators, we need to consider the Klein-Gordon norm

$$(\Psi, \Psi) = \frac{i}{\hbar}\int_\Sigma d^5x\sqrt{h}n_\mu g^{\mu\nu}(\bar{\Psi}\partial_\nu\Psi - (\partial_\nu\bar{\Psi})\Psi), \tag{6.18}$$

where $\Sigma$ is a Cauchy surface, say for simplicity a surface $t = t_0$, and $n_\mu$ is the normal $n_\mu = \partial_\mu t$. The modes of positive norm $(\Psi, \Psi) > 0$ correspond to creation operators, while those of negative norm $(\Psi, \Psi) < 0$ correspond to annihilation operators. Because of the complicated form of the inverse metric (see appendix A), it is difficult to establish explicitly which are which. However, the main point is that we can define a vacuum state by requiring that it be annihilated by the annihilation operators corresponding to all the negative frequency modes in (6.2). This will then be the unique vacuum state on this geometry. Since the modes (6.2) are eigenmodes of both the asymptotic time-translation $\partial_t$ and of the timelike Killing vector in the near-core region,

$$V' = \ell\partial_t - m\partial_\psi + n\partial_\phi, \tag{6.19}$$

these will be the appropriate family of creation and annihilation operators for observers in both regions. That is, these observers who follow the orbits of the Killing symmetries will detect no particles in this state.

Thus, at the level of free fields, the solitons do not suffer from superradiance at either the classical or quantum level.

## VII. FUTURE DIRECTIONS

In this paper, we have found new nonsupersymmetric soliton solutions in the D1-D5 system and identified corresponding states in the CFT. These solitons can be viewed as interesting supergravity backgrounds in their own right. They also provide an interesting extension of the conjectured identity between CFT microstates and asymptotically flat spacetimes [9,10].

There are two corresponding classes of issues for further investigation: further study of the geometry itself and elucidating the relation to the dual CFT. In the first category, the classical stability of these solitons as solutions in IIB supergravity should be checked. As we discussed in Sec. VI B, although they have ergoregions, the usual black hole bomb instability will be absent at least for free fields, as there is no net flux in a scattering off the geometry. It would be interesting to study stability more generally; in particular, it would be interesting to know if the geometry suffers from a Gregory-Laflamme [51] type instability if we make the torus in the $z^i$ directions large.

It would be interesting to try to find asymptotically $AdS_5$ generalizations of these solitons, building on the studies of black holes in gauged supergravities in [40], as in AdS it might be possible to find nonsupersymmetric solitons with a globally timelike Killing vector. This is known to be possible for some Kerr black holes in AdS [50,52].

It would also be interesting to study these solutions as backgrounds for perturbative string theory. They provide new examples of smooth asymptotically flat geometries that do not have a global timelike Killing symmetry, of a rather different character from those presented in [53]. The

[8] We thank Don Marolf for pointing out that the argument for nontrivial quantum radiation in the original version of this paper was erroneous and for explaining the following argument to us.

existence of supersymmetric special cases may be a simplifying feature.

The most important direction of future work to elucidate the relation of these geometries to the dual CFT is to construct explicit CFT descriptions of the states dual to the generic orbifold spacetimes and study their properties from the CFT point of view. The charges for the dual states found in (5.10) show that these states are not simply the spectral flow of some chiral primary, so they do not maximize the $R$-charge for given conformal dimension. They should therefore be closer to representing the ‘‘typical‘‘ behavior of a CFT state (although they are clearly still very special), and we expect there will be new tests of the relation between geometry and CFT to be explored. It will also be interesting to see what happens in the CFT when we consider the orbifolds with $m + n$ even, $k$ even, where the spacetime is not a spin manifold.

Another important basic issue from this point of view is to understand the appearance of stationary geometries dual to nonsupersymmetric states coupled to bulk modes. We would have expected that the CFT states would decay by the emission of bulk closed string modes. Even in the simple cases where the near-core geometry is global $AdS_3 \times S^3$, the corresponding CFT state carries comparable numbers of left- and right-moving excitations, which we would expect can interact to produce bulk gravitons. This physics does not seem to be represented in our dual geometries. It will be important to study the decay of these nonsupersymmetric states in more detail and to try to understand the relation to the soliton.

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## APPENDIX A: INVERSE METRIC

To calculate the inverse metric, it is convenient to start from the fibred form of the metric (2.12), construct a corresponding orthonormal frame, and invert that. For this reason, it is simpler to give the inverse metric in terms of the boosted coordinates $\tilde{t} = t\cosh\delta_p - y\sinh\delta_p$, $\tilde{y} = y\cosh\delta_p - t\sinh\delta_p$.

The inverse metric is

$$g^{\tilde{t}\tilde{t}} = -\frac{1}{\sqrt{\tilde{H}_1\tilde{H}_5}}\bigg(f + M + M\sinh^2\delta_1 + M\sinh^2\delta_5 + \frac{M^2\cosh^2\delta_1\cosh^2\delta_5 r^2}{g(r)}\bigg), \tag{A1}$$

$$g^{\tilde{t}\tilde{y}} = \frac{1}{\sqrt{\tilde{H}_1\tilde{H}_5}}\frac{M^2\sinh\delta_1\sinh\delta_5\cosh\delta_1\cosh\delta_5 a_1 a_2}{g(r)}, \tag{A2}$$

$$g^{\tilde{t}\phi} = -\frac{1}{\sqrt{\tilde{H}_1\tilde{H}_5}}\frac{M\cosh\delta_1\cosh\delta_5 a_2(r^2 + a_1^2)}{g(r)}, \tag{A3}$$

$$g^{\tilde{t}\psi} = -\frac{1}{\sqrt{\tilde{H}_1\tilde{H}_5}}\frac{M\cosh\delta_1\cosh\delta_5 a_1(r^2 + a_2^2)}{g(r)}, \tag{A4}$$

$$g^{\tilde{y}\tilde{y}} = \frac{1}{\sqrt{\tilde{H}_1\tilde{H}_5}}\bigg(f + M\sinh^2\delta_1 + M\sinh^2\delta_5 + \frac{M^2\sinh^2\delta_1\sinh^2\delta_5(r^2 + a_1^2 + a_2^2 - M)}{g(r)}\bigg), \tag{A5}$$

$$g^{\tilde{y}\phi} = -\frac{1}{\sqrt{\tilde{H}_1\tilde{H}_5}}\frac{M\sinh\delta_1\sinh\delta_5 a_1(r^2 + a_1^2 - M)}{g(r)}, \tag{A6}$$

$$g^{\tilde{y}\psi} = -\frac{1}{\sqrt{\tilde{H}_1\tilde{H}_5}}\frac{M\sinh\delta_1\sinh\delta_5 a_2(r^2 + a_2^2 - M)}{g(r)}, \tag{A7}$$

$$g^{rr} = \frac{1}{\sqrt{\tilde{H}_1\tilde{H}_5}}\frac{g(r)}{r^2}, \tag{A8}$$

$$g^{\theta\theta} = \frac{1}{\sqrt{\tilde{H}_1\tilde{H}_5}}, \tag{A9}$$

$$g^{\phi\phi} = \frac{1}{\sqrt{\tilde{H}_1\tilde{H}_5}}\bigg(\frac{1}{\sin^2\theta} + \frac{(r^2 + a_1^2)(a_1^2 - a_2^2) - Ma_1^2}{g(r)}\bigg), \tag{A10}$$

$$g^{\phi\psi} = -\frac{1}{\sqrt{\tilde{H}_1\tilde{H}_5}}\frac{Ma_1a_2}{g(r)}, \tag{A11}$$

$$g^{\psi\psi} = \frac{1}{\sqrt{\tilde{H}_1\tilde{H}_5}}\bigg(\frac{1}{\cos^2\theta} + \frac{(r^2 + a_2^2)(a_1^2 - a_2^2) - Ma_2^2}{g(r)}\bigg). \tag{A12}$$

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